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A fuzzy risk approach for seasonal water quality management of a river system

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[1] A fuzzy optimization model is developed for the seasonal water quality management of river systems. The model addresses the uncertainty in a water quality system in a fuzzy probability framework. The occurrence of low water quality is treated as a fuzzy event. Randomness associated with the water quality indicator is linked to this fuzzy event using the concept of probability of a fuzzy event. In most water quality management models the risk level for violation of a water quality standard is constrained by a preassigned value through a chance constraint. In the fuzzy risk approach a range of risk levels is specified by considering a fuzzy set of low risk, instead of using a chance constraint. Thus the two levels of uncertainty, one associated with low water quality and the other with low risk, are quantified and incorporated in the management model. The model takes into account the seasonal variations of river flow to specify seasonal fraction removal levels for the pollutants. Fuzzy sets of low water quality are considered in each season. The membership functions of these fuzzy sets represent the degree of low water quality associated with the discrete states of water quality in a season. The fuzzy risk of low water quality in a season at a checkpoint in the river system is expressed in terms of the degree of low water quality and the steady state probabilities associated with discrete river flows. Considering the fuzzy goals of the pollution control agency and dischargers, and the crisp constraints, the water quality management problem is formulated as a fuzzy optimization model. The model solution gives seasonal fraction removal levels for the pollutants. Application of the fuzzy optimization model is illustrated with a hypothetical river system. *INDEX TERMS:* 6309 Policy Sciences: Decision making under uncertainty, 6344 Policy Sciences: System operation and management, 1871 Hydrology: Surface water quality, 1869 Hydrology: Stochastic processes; *KEYWORDS:* fuzzy optimization, water quality, fuzzy goals, fuzzy probability

1. Introduction

[2] River water quality management problems have been addressed as multiobjective optimization problems in a number of earlier studies [e.g., *Loucks et al.*, 1981; *Loucks*, 1983; *Das and Haimes*, 1979; *Louie et al.*, 1984; *Tung and Hathhorn*, 1989]. Apart from being multiobjective in nature, water quality management problems are also characterized by imprecision in objectives and water quality standards. Fuzzy sets and fuzzy optimization provide a useful technique in addressing such imprecision. Uncertainties due to imprecision in objectives and model parameters in water resources problems have been modeled with fuzzy sets in some recent work [*Bogardi et al.*, 1983; *Duckstein et al.*, 1991; *Bardossy and Disse*, 1993; *Bardossy and Duckstein*, 1995; *Shreshta et al.*, 1996; *Fontane et al.*, 1997; *Teegavarapu and Simonovic*, 1999]. In a few recent studies, imprecision in water quality management problems has also been modeled using fuzzy sets. *Hathhorn and Tung* [1989] considered the waste load allocation problem for water quality management of a river system as a fuzzy optimization problem. The two objectives considered in their study are maximization of the waste discharge and minimization of the largest difference in an equity measure between the various discharges. The management problem is formulated as a fuzzy optimization problem and solved by fuzzy linear programming. *Chang et al.* [1997] combined the gray and fuzzy programming methods in an interactive multiobjective framework for water pollution control in a river basin. In their study the uncertainty due to the environmental and economic parameters is modeled using interval numbers and that due to the imprecise objectives of decision makers is modeled using fuzzy membership functions. Interactive fuzzy interval multiobjective mixed integer programming approach is used to evaluate optimal wastewater treatment strategies for pollution control in a river basin. *Sasikumar and Mujumdar* [1998] develop a fuzzy waste load allocation model for water quality management of a river system in a deterministic framework. The model incorporates conflicting objectives of the pollution control agency and dischargers in the system. Imprecision inherent in setting up the water quality criteria and the goals of the pollution control agency and dischargers are quantified using fuzzy sets. The management problem is formulated as a fuzzy multiobjective optimization problem.

[3] Apart from imprecision in objectives and standards, randomness of natural variables and model parameters also adds to the uncertainty in water quality problems. Uncertainty due to randomness has been addressed extensively in the models for water quality management of river systems, starting with the pioneering work of *Loucks and Lynn* [1966] [e.g., see *Lohani and Thanh*, 1978, 1979; *Whitehead and Young*, 1979; *Spear and Hornberger*, 1983; *Burn and McBean*, 1985, 1986; *Fugiwara et al.*, 1986, 1987, 1988; *Tung and Hathhorn*, 1988; *Ellis*, 1987; *Takyi and Lence*, 1995]. A study addressing uncertainties due to both randomness and fuzziness in water quality management of river systems is relevant from a modeling point of view. *Sasikumar and Mujumdar* [2000] have presented a theoretical framework to include both randomness and fuzziness in river water quality management models. A theoretical, closed form cumulative distribution function (CDF) is considered for river flow. The CDF is assumed to be continuously differentiable so that the probability distribution of the water quality indicator (dissolved oxygen (DO) deficit) could be derived, starting from the CDF of river flow. The concept of probability of a fuzzy event is used to link probability with fuzzy sets.

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Table 1. Description of the River System

Set	Description of the Set	Element Representation	Number of Elements
Q_L	water quality checkpoints	l	N_q
D_M	dischargers	m	N_d
P_N	pollutants	n	N_p
T_P	uncontrollable sources of pollutants	p	N_t
S_I	water quality indicators	i	N_s

[4] The problem of seasonal fraction removal of effluent discharges in streams has been dealt with in the works of *Boner and Furland* [1982], *Rheis et al.* [1982], *Eheart et al.* [1987], *Rossman* [1989], *Lence et al.* [1990], *Lence and Takyi* [1992], and *Takyi and Lence* [1995]. When the optimal fraction removal levels are significantly different across seasons, they could result in notable cost savings compared to a single, uniform fraction removal level all through the year. Innovative seasonal management programs, which consider the dynamic waste assimilative capacity of a river and allow for different levels of waste treatment during different seasons of the year, have been shown to be more cost-effective than nonseasonal uniform treatment programs [*Takyi and Lence*, 1995].

[5] In this paper, a water quality management model is developed addressing uncertainties due to randomness of river flow and fuzziness in management goals for obtaining seasonal fraction removal levels. Subjectivity in classification of the water quality into a "satisfactory (working) state" and a "nonsatisfactory (failed) state" is addressed by defining the low water quality as a fuzzy event. A fuzzy set with an appropriate membership function is defined to describe not only the working or failed states but any intermediate state also. The objectives related to the risk levels of low water quality at various checkpoints (locations in the river where water quality is of interest) and those related to the fraction removal levels are expressed as fuzzy goals in the management model. River flow in a season is represented by a finite number of discrete values, each associated with a known probability of occurrence. The imprecisely defined goals of pollution control agencies and dischargers are modeled using fuzzy sets. The concept of fuzzy risk is introduced in the context of water quality management problems. In general, several water quality indicators (e.g., DO defict, phosperous content, nutrient content, etc.) describe the state of water quality at a location. The influence of a pollutant (e.g., biochemical oxygen demand (BOD) loading) on a water quality indicator (e.g., DO deficit) at a downstream location is given by an appropriate water quality transport model, which defines a set of constraint in the water quality management model. Using such a transport model, the water quality concentrations are expressed as functions of the fraction removal levels, which form decision variables in the water quality management model. The model application is demonstrated with a simple hypothetical example problem.

[6] A description of the river system relevant to the present study is given in section 2. The procedure for evaluating the fuzzy risk in the case of discrete states of river flow is discussed in section 3. Section 4 describes the formulation of the fuzzy optimization model. In section 5 the model application is discussed through an example of a hypothetical river system. Section 6 presents a discussion of results. Section 7 provides general remarks related to the model, and section 8 gives the conclusions.

2. Description of the River System

[7] A general river system is considered for developing the water quality management model. In addition to the uncontrollable sources of pollution the river system includes a set of dischargers who are allowed to release the pollutants into the river after removing some fraction of the pollutants. A common practice of the pollution control agency to ensure acceptable water quality condition is to check the water quality at a finite number of locations in the river. These locations are called water quality checkpoints or simply checkpoints. The water quality at a checkpoint is described by means of some indicators called water quality indicators. The dissolved oxygen deficit (DO deficit) is an example of a water quality indicator. The concentration of a water quality indicator at a checkpoint is affected by the controllable as well as the uncontrollable sources of pollutants in the river system. The concentration of the water quality indicator at a checkpoint is obtained using a water quality transport model that determines the spatial distribution of the pollutants in the river system. In a water quality management model the concentration of the water quality indicator may be expressed as a function of the fraction removal levels, which form the decision variables of the optimization problem.

[8] Table 1 gives the description of a river system. The relevant components of the system are identified as sets. Set Q_L represents the collection of water quality checkpoints in the river system. Set D_M is the collection of dischargers (e.g., industries). Set P_N is the collection of the pollutants in the system (e.g., point sources of BOD, a mixture of toxic pollutants, etc.). Set T_P is the collection of uncontrollable sources of pollutants in the system (e.g., BOD addition due to runoff and scour in a stream). Set S_I is the collection of water quality indicators (e.g., dissolved oxygen deficit and toxic pollutant concentration). A water quality indicator is indexed as i (e.g., $i = 1$ may correspond to DO deficit, $i = 2$ may correspond to phosperous content, etc.). A pollutant is assumed to affect one or more water quality indicators in the set S_I. The pollution control agency specifies a desirable level, c_{il}^{Ds}, and maximum permissible level, c_{il}^{Hs}, of the water quality indicator i in season s at the checkpoint l. Most quality indicators of interest are such that $c_{il}^{Ds} < c_{il}^{Hs}$.

3. Evaluation of Fuzzy Risk

3.1. Discrete Water Quality States

[9] River flow is considered to be a discrete stochastic process, with known steady state probabilities associated with each discrete value of river flow in a season. The number of seasons within a year is denoted by n_s. A season in a year is represented by the index s. $\mathbf{Q}_l^s$ represents the set of possible discrete river flow values at checkpoint l in season s. Each element of the set $\mathbf{Q}_l^s$ represents a discrete state of flow and is associated with a probability of occurrence such that summation of the probabilities over all the states in the set $\mathbf{Q}_l^s$ is equal to 1. Corresponding to the set $\mathbf{Q}_l^s$, a set $\mathbf{C}_{il}^s$ may be identified to represent the state of water quality at the checkpoint l for the season s, with respect to the water quality indicator i. The elements of the $\mathbf{C}_{il}^s$ are represented by c_{il}^{ks}, where k corresponds to the kth discrete state of river flow. Each element of $\mathbf{C}_{il}^s$ represents the concentration of the water quality indicator i at the checkpoint l in the season s due

to a corresponding discrete state in $\mathbf{Q}_l^s$. It is possible to relate a discrete state in $\mathbf{Q}_l^s$ to a unique and corresponding state in $\mathbf{C}_{il}^s$ because all parameters other than the flow (e.g., the reaction rate coefficients, travel time of pollutants, etc.) are assumed to be known or computed using the functional relationships between the flow and those parameters. In other words, for a given state of river flow in $\mathbf{Q}_l^s$ the corresponding water quality states in $\mathbf{C}_{il}^s$ are uniquely defined. In general, an element in $\mathbf{C}_{il}^s$ and the corresponding element in $\mathbf{Q}_l^s$ are related using an appropriate water quality transport model. The water quality transport model relates the concentration of the water quality indicator with the river flow, pollutant loading, reaction rate coefficients, and travel time of the pollutant from its source to the checkpoint.

[10] Definition of low water quality as a fuzzy event, risk of low water quality, fuzzy risk, and fuzzy goals and their membership functions constitute the base formulation of the water quality management model. These concepts are introduced in section 3.2 with respect to a season s representing a discrete time stage in the time horizon of decision making.

3.2. Low Water Quality as a Fuzzy Event

[11] The conventional (crisp) definition of low water quality may be represented by the characteristic functions with respect to the concentration c_{il}^{ks} as follows:

$$\mu\left(c_{il}^{ks}\right) = \begin{cases} 0 & c_{il}^{ks} \le c_{il}^{Hs} \\ 1 & c_{il}^{ks} > c_{il}^{Hs}, \end{cases} \tag{1}$$

where c_{il}^{Hs} is the water quality standard at location l in season s. The crisp set, L_{il}^s, of concentration levels that belong to low water quality at checkpoint l in season s is defined as

$$L_{wl}^s = \left\{ c_{wl}^{ks} : \mu\left(c_{wl}^{ks}\right) = 1 \right\}. \tag{2}$$

The crisp set-based definition of low water quality may be generalized using a fuzzy set. The fuzzy set W_{il}^s of low water quality is expressed as follows:

$$W_{il}^s = \left\{ c_{il}^{ks} \ : \ 0 \ \le \ \mu_{W_{il}^s}\left(c_{il}^{ks}\right) \ \le \ 1 \right\}, \tag{3}$$

where $\mu_{W_{il}^s}\left(c_{il}^{ks}\right)$ is the membership function of the fuzzy set W_{il}^s. Unlike the crisp set of low water quality, L_{il}^s, the fuzzy set, W_{il}^s, allows partial membership also for the concentration level. The membership value indicates the degree of low water quality of the concentration c_{il}^{ks}. The membership function that assigns membership values to the elements of the fuzzy set of low water quality thus modifies the crisp definition of low water quality and makes it more realistic. A general class of fuzzy membership functions for use in water quality management models is discussed by *Sasikumar and Mujumdar* [1998]. To reduce the complexity of the model, only linear membership functions are used in the present study. This membership function may be expressed as

$$\mu_{W_{il}^s}\left(c_{il}^{ks}\right) = \begin{cases} 0 & c_{il}^{ks} < c_{il}^{Ds} \\ \frac{c_{il}^{ks} - c_{il}^{Ds}}{c_{il}^{Hs} - c_{il}^{Ds}} & c_{il}^{Ds} \le c_{il}^{ks} \le c_{il}^{Hs} \\ 1 & c_{il}^{ks} \ge c_{il}^{Hs}, \end{cases} \tag{4}$$

where c_{il}^{Ds} and c_{il}^{Hs} are the desirable and allowable levels of the water quality indicator i in season s, respectively.

[12] The desirable levels c_{il}^{Ds} and the allowable levels c_{il}^{Hs} are subjectively fixed by the decision makers (e.g., pollution control agencies). For example, the desirable level for DO deficit may be fixed at a very stringent value of 0, and the allowable level may be fixed at 4 mg L^{-1}. Specification of the desirable level and the permissible level depends on the perception of the decision makers. These levels are, in general, different for different pollutants and checkpoints. For modeling purpose they are also generalized to be different for different seasons. This generalization is included because the river water may be used for different purposes during different seasons, and therefore the desirable and allowable levels of the water quality indicators could be quite different across the seasons.

[13] The fuzzy definition of low water quality ensures that there is no single threshold value which defines a failed state. Indeed, all discrete levels of the water quality indicator are treated as "failures" of different degrees. The degree of failure itself is given by the membership function, which reflects the perception of the decision maker (the pollution control agency) as to how low the water quality is. A membership value of 0 in the fuzzy set of low water quality would mean that that level of water quality is "not low."

3.3. Risk of Low Water Quality

[14] The membership function of the fuzzy set W_{il}^s may be viewed as a representation of the degree of failure of the river system with respect to the water quality indicator i at the checkpoint l in the season s. The kth element c_{il}^{ks} of the set C_{il}^s corresponds to the kth discrete state of water quality. For example, $\mu_{W_{il}^s}\left(c_{il}^{ks}\right)$ represents the degree of failure of the river system corresponding to the kth element of the set C_{il}^s at the checkpoint l in the season s. Since each element, c_{il}^{ks}, has an associated probability, it is possible to define the expected degree of failure of the system. The expected degree of failure represents the fuzzy risk of low water quality at the checkpoint l. Thus the fuzzy risk, r_{il}^s, of low water quality is defined as

$$r_{il}^s = \sum_k \mu_{W_{il}^s}\left(c_{il}^{ks}\right) \ p\left(c_{il}^{ks}\right), \tag{5}$$

where $p(c_{il}^{ks})$ is the probability associated with the occurrence of the kth element of the set C_{il}^s. This definition is based on the probability of a fuzzy event proposed by *Zadeh* [1968]. Each element c_{il}^{ks} of the set C_{il}^s corresponds to an elementary fuzzy event of low water quality with the degree of occurrence of the event by $\mu_{W_{il}^s}\left(c_{il}^{ks}\right)$. The crisp definition of risk of low water quality, r'^s_{il}, is given as

$$r'_{il}s \ = \ P\left(c_{il}^s > c_{il}^{Hs}\right) \ = \ \sum_{\left\{k:\, c_{il}^{ks} > c_{il}^{Hs}\right\}} p\left(c_{il}^{ks}\right). \tag{6}$$

Using the characteristic functions given by (1), the right-hand sides of (6) may be rewritten as follows:

$$\sum_{\left\{k:\, c_{il}^{ks} > c_{il}^{Hs}\right\}} p\left(c_{il}^{ks}\right) \ = \ \sum_k \mu\left(c_{il}^{ks}\right) \ p\left(c_{il}^{ks}\right). \tag{7}$$

Thus the crisp definitions of risk of low water quality given by (6) may be considered as a particular case of the more general fuzzy set-definition given by (5). It may be noted that the crisp definition of risk denotes the probability of failure, while the fuzzy risk indicates the expected degree of failure.

4. Fuzzy Optimization Model

4.1. Fuzzy Decision

[15] The goals of the pollution control agency and dischargers are viewed as imprecisely defined linguistic statements. These goals are treated as fuzzy goals [*Sasikumar and Mujumdar*, 1998]. Two classes of goals related to the pollution control agency and dischargers are defined as follows: (1) The goals of the pollution control agency, G_{il}^s, are to make the risk of low water quality, r_{il}^s, with respect to the water quality indicator i at the checkpoint l as low as possible for all i, l, and s. (2) The goals of the dischargers, F_{imn}^s, are to make the fraction removal level, x_{imn}^s, as close as possible to the minimum level, x_{imn}^{Ls}, for all i, m, n, and s.

[16] The fuzzy decision Z for the water quality management problem may be defined using the concept of fuzzy decision proposed by *Bellman and Zadeh* [1970] and the fuzzy multi-objective optimization technique developed subsequently [*Zimmermann*, 1978, 1985; *Feng*, 1983; *Kindler*, 1992; *Rao*, 1993; *Sakawa*, 1995]. Noting that the decision space is defined by the intersection of the fuzzy goals and constraints, the fuzzy decision Z is written as follows:

$$Z = \left(\bigcap_{i,l,s} G_{il}^s\right) \cap \left(\bigcap_{i,m,n,s} F_{imn}^s\right), \tag{8}$$

where G_{il}^s represents the goals of the pollution control agency and F_{imn}^s represents the goals of the dischargers. The membership function of the fuzzy decision Z is given by

$$\mu_Z(X) = \min_{i,l,m,n,s}\left[\mu_{G_{il}^s}\left(r_{il}^s\right),\ \mu_{F_{imn}^s}\left(x_{imn}^s\right)\right], \tag{9}$$

where X is the space of alternatives composed of r_{il}^s and x_{imn}^s. The corresponding optimal decision, X^*, is given by

$$\mu_Z(X^*) = \lambda^* = \max_{X \in Z}\left[\mu_Z(X)\right]. \tag{10}$$

Details of the membership functions of the various fuzzy goals are described in section 4.2.

4.2. Membership Functions of the Fuzzy Goals

4.2.1. Goal G_{il}^s. [17] This goal of the pollution control agency requires the risk of low water quality to be as low as possible. To quantify the notion of low risk, a fuzzy set of low risk is used. For convenience the fuzzy set of low risk is also referred to as G_{il}^s. Different risk levels belonging to the fuzzy set G_{il}^s are assigned membership values between 0 and 1 depending on how high the risk levels are. A very high risk, for example, will have a membership value close to 0 in the fuzzy set of low risks.

[18] The membership function, denoted by $\mu_{G_{il}^s}(r_{il}^s)$, of the fuzzy set of low risk is essentially a mapping from the set of risk levels [0,1] to the set membership values [0,1]. Different membership functions that are monotonically nonincreasing functions of risk levels may be assigned for the fuzzy set G_{il}^s. Mathematically, a linear membership function can be expressed as follows:

$$\mu_{G_{il}^s}\left(r_{il}^s\right) = \begin{cases} 1 & 0 \le r_{il}^s < r_{il}^{Ls} \\ \frac{r_{il}^{Ms} - r_{il}^s}{r_{il}^{Ms} - r_{il}^{Ls}} & r_{il}^{Ls} \le r_{il}^s \le r_{il}^{Ms} \\ 0 & r_{il}^s \ge r_{il}^{Ms}. \end{cases} \tag{11}$$

The quantities r_{il}^{Ls} and r_{il}^{Ms} are the minimum acceptable and maximum permissible risk levels. The argument r_{il}^s of the membership function can be substituted from (5). Equation (5) evaluates the risk of low water quality, and the membership function defined by (11) assigns a membership value to this risk in the fuzzy set of low risk. It may be noted that two levels of fuzziness, one of low water quality and the other of low risk, have been quantified using fuzzy sets and the associated membership functions.

4.2.2. Goal F_{imn}^s. [19] The fraction removal level, x_{imn}^{Ls}, corresponding to the aspiration level of the discharger m with respect to x_{imn}^s is assigned a membership value of 1. The maximum acceptable level, x_{imn}^{Ms}, is assigned a membership value of 0. The membership function for the fuzzy goal F_{imn}^s is expressed as follows:

$$\mu_{F_{imn}^s}\left(x_{imn}^s\right) = \begin{cases} 1 & 0 \le x_{imn}^s < x_{imn}^{Ls} \\ \frac{x_{imn}^{Ms} - x_{imn}^s}{x_{imn}^{Ms} - x_{imn}^{Ls}} & x_{imn}^{Ls} \le x_{imn}^s \le x_{imn}^{Ms} \\ 0 & x_{imn}^s \le x_{imn}^{Ms}. \end{cases} \tag{12}$$

This membership function may be interpreted as the variation of satisfaction level of the discharger m in treating the pollutant n to control the water quality indicator i in the river system in season s. Using the membership functions of the fuzzy goals, G_{il}^s and F_{imn}^s, the water quality management model is expressed as follows: Maximize λ (lambda) subject to

$$\mu_{G_{il}^s}\left(r_{il}^s\right) \ge \lambda \ \forall\, i,\, l,\, s, \tag{13}$$

$$\mu_{F_{imn}^s}\left(x_{imn}^s\right) \ge \lambda \ \forall\, i,\, m,\, n,\, s, \tag{14}$$

$$r_{il}^{Ls} \le r_{il}^{Ls} \le r_{il}^{Ms} \ \forall\, i,\, l,\, s, \tag{15}$$

$$c_{il}^{Ds} \le c_{il}^{ks} \le c_{il}^{Hs} \ \forall\, i,\, k,\, l,\, s, \tag{16}$$

$$x_{imn}^{Ls} \le x_{imn}^s \le x_{imn}^{Ms} \ \forall\, i,\, m,\, n,\, s, \tag{17}$$

$$x_{imn}^{\mathrm{MIN}s} \le x_{imn}^s \le x_{imn}^{\mathrm{MAX}s} \ \forall\, i,\, m,\, n,\, s, \tag{18}$$

$$0 \le \lambda \le 1. \tag{19}$$

The constraints (13) and (14) define the parameter λ as the minimum membership value in the system. When the membership functions given by (11) and (12) are viewed as the representation of the variation of satisfaction level of the pollution control agency and dischargers, then the parameter λ may be interpreted as the minimum satisfaction level in the system [*Kindler*, 1992]. The parameter λ is also a decision variable in addition to the fraction removal levels, x_{imn}^s, in the optimization problem. Constraints (15) are based on the water quality requirements in terms of the minimum acceptable and maximum permissible risk levels set by the pollution control agency. Constraints (16) arise from the definition of the fuzzy set of low water quality given by (4). The minimum and maximum acceptable fraction removal levels set by the dischargers are expressed in constraints (17). However, the pollution control agency imposes minimum fraction removal levels that are expressed as the lower bounds, x_{imn}^{MINs}, constraints (18). The upper bounds, x_{imn}^{MAXs} in these constraints represent the technologycally possible maximum fraction removal levels. Observing that the maximum acceptable level of pollutant treatment cannot exceed the technologycally possible upper limit, constraints (17) and (18) can be simplified to a single constraint given by (20) as follows:

$$\max\left[x_{imn}^{Ls},\ x_{imn}^{\mathrm{MIN}s}\right] \le x_{imn}^s \le x_{imn}^{Ms} \ \forall\, i,\, m,\, n,\, s. \tag{20}$$

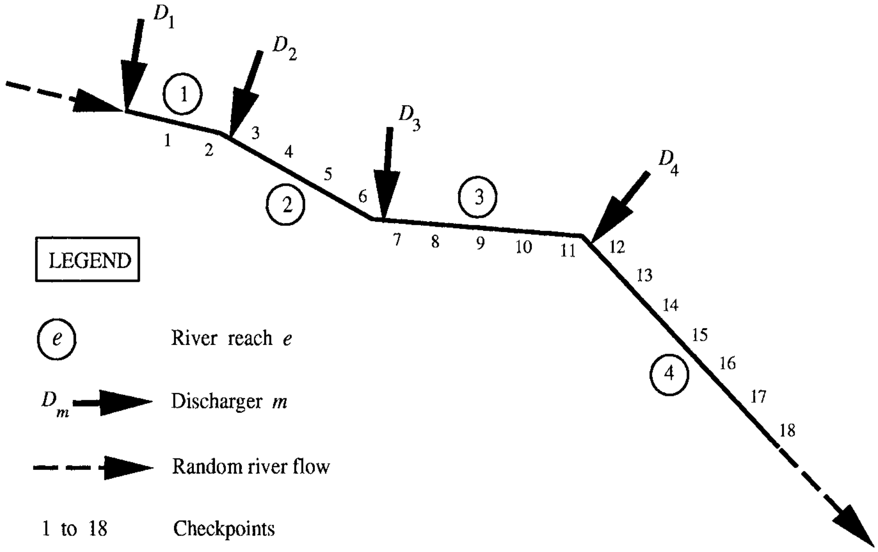

Figure 1. A hypothetical river system.

Substituting the expression for the membership functions from (11) and (12), the optimization model is given as follows: Maximize λ subject to

$$\frac{r_{il}^{Ms} - r_{il}^{s}}{r_{il}^{Ms} - r_{il}^{Ls}} \geq \lambda \; \forall \, i, \, l, \, s, \tag{21}$$

$$\frac{x_{imn}^{Ms} - x_{imn}^{s}}{x_{imn}^{Ms} - x_{imn}^{Ls}} \geq \lambda \; \forall \, i, \, m, \, n, \, s, \tag{22}$$

$$r_{il}^{Ls} \leq r_{il}^{s} \leq r_{il}^{Ms} \; \lambda \, \forall \, i, \, l, \, s, \tag{23}$$

$$c_{il}^{Ds} \leq c_{il}^{ks} \leq c_{il}^{Hs} \; \lambda \, \forall \, i, \, k, \, l, \, s, \tag{24}$$

$$\max\left[x_{imn,}^{Ls} \; x_{imn}^{\mathrm{MIN}s}\right] \leq x_{imn}^{s} \leq x_{imn}^{Ms} \; \lambda \, \forall \, i, \, m, \, n, \, s, \tag{25}$$

$$0 \leq \lambda \leq 1. \tag{26}$$

The decisions x_{imn}^{s} in the season s result in a pollutant concentration which forms the initial condition for the next season (s + 1). As a season is of significantly longer duration than the travel time of the pollutants, the effect of this initial condition will die down relatively quickly in season (s +1). Thus the water quality state in season (s +1) depends only on the decision x_{imn}^{s+1} and not on x_{imn}^{s}. It may, however, be noted that decisions across the seasons are related through the objective function (Maximize λ) and the constraints (13) and (14). As may be noted from (5), the risk r_{il}^{s} may be expressed in terms of the discrete states c_{il}^{ks} of the system in the season s. Each state c_{il}^{ks}, that is, the concentration of the water quality indicator i at the checkpoint l corresponding to the discrete state k in season s, is related to the fraction removal levels using transfer functions. These transfer functions are obtained using appropriate mathematical models that determine the spatial distribution of the water quality indicators due to the pollutants in the river system. Use of the transfer functions transforms all the constraints in terms of the fraction removal levels. The solution of the model gives **X*** and λ*, where **X*** is the vector of optimum fraction removal levels and λ* is the maximum-minimum satisfaction level in the system. Application of the fuzzy optimization model to a river system is discussed in section 5.

5. Model Application

[20] The model is applied to a hypothetical river system shown in Figure 1. The river system consists of four river reaches. A point source of biochemical oxygen demand (BOD) is located at the beginning of each reach. The water quality indicator of interest is the dissolved oxygen deficit (DO deficit) at a finite number of checkpoints in the river system due to the point sources of BOD. The saturation DO concentration is 10 mg L^{-1} for all the reaches. Water quality is checked at 18 checkpoints as shown in Figure 1. The distance between any two adjacent checkpoints is 5 km. The details of the effluent flow are given in Table 2. The last column of Table 2 gives the dissolved oxygen concentration in the effluent from the point sources of pollutants. A bimonthly seasonal river flow is considered in this illustrative example. A year thus consists of six seasons. Each season has three discrete states of river flow. The river flow values corresponding to discrete states of the six seasons are given in Table 3. The steady state probabilities of these

Table 2. Effluent Flow Data

Discharger	Effluent Flow Rate 10^4 m^3d^{-1}	BOD Concentration mg L^{-1}	DO Concentration mg L^{-1}
D_1	2.134	1250	1.23
D_2	6.321	1415	2.40
D_3	7.554	1040	1.70
D_4x	5.180	935	2.16

BOD is biochemical oxygen demand; DO is dissolved oxygen.

Table 3. Discrete States of River Flow[a]

States	Seasons					
	1	2	3	4	5	6
1	6.0	6.0	5.0	3.0	3.0	4.0
2	7.0	7.0	6.0	4.0	4.0	5.0
3	8.0	8.0	7.0	5.0	5.0	6.0

[a] Values are given in 10^6 m^3 d^{-1}.

Table 4. Steady State Probabilities

States	Seasons					
	1	2	3	4	5	6
1	0.1212	0.1745	0.2389	0.1621	0.6824	0.2118
2	0.2548	0.5631	0.5699	0.6901	0.2176	0.4518
3	0.6240	0.2624	0.1912	0.1478	0.1000	0.3364

discrete states of river flow are given in Table 4. These steady state probabilities are computed for the example problem based on assumed transition probabilities, considering the seasonal flow to follow a single step Markov chain. In real situations these simply may be computed from available flow data. The DO deficit at a checkpoint is expressed in terms of the fraction removal levels of BOD associated with various dischargers located upstream of the checkpoint under consideration. The transfer function that expresses the DO deficit at a checkpoint in terms of the concentration of point source of BOD and the fraction removal levels can be obtained using the one-dimensional steady state Streeter-Phelps BOD-DO equations [*Gromiec et al.*, 1983; *Thomann and Mueller*, 1987; *James and Elliot*, 1993]. Different methods that employ the Street-Phelps equations to predict the spatial distribution of DO deficit in a river network with multiple reaches are discussed in the literature [e.g., *Arbabi and Elzinga*, 1975; *Fugiwara et al.*, 1986]. A suitable adoption of the model proposed by *Fugiwara et al.* [1986] is used in the present study.

[21] The deoxygenation rate coefficient, reaeration rate coefficient, and pollutant travel time may each be functionally related to the discrete state q_l^{ks} of the river flow. Such functional relationships are reported in the literature [e.g., *Burn and McBean*, 1985; *Thomann and Mueller*, 1987; *Fugiwara et al.*, 1987] and are generally of the following forms:

$$K_1 = A_d\left[q_l^{ks}\right]^{a_d} \tag{27}$$

$$K_2 = A_r\left[q_l^{ks}\right]^{a_r}, \tag{28}$$

$$t = A_t\, d\left[q_l^{ks}\right]^{a_t}, \tag{29}$$

where A_d, A_r, A_t, a_d, a_r, and a_t are constants, d is the distance between the point source of the pollutant and the checkpoint l in kilometers, and q_l^{ks} is the river flow value (m^3 s^{-1}) corresponding to the kth discrete state. For the illustrative example considered, it is assumed, with some judgement based on literature, that $A_d = 2$, $a_d = -0.5$, $A_r = 1.2$, $a_r = -0.15$, $A_t = 0.2$, and $a_t = -0.5$.

[22] Since only a single water quality indicator (DO deficit) and only one type of pollutant (point source of BOD from the dischargers) are considered in this illustrative example, the notations of different variables are simplified by retaining only the suffixes l (checkpoint), s (season), k (discrete state of water quality), and m (discharger) with the various quantities of interest. For example, c_l^{ks}, r_l^s, and x_m^s denote the DO deficit at checkpoint l corresponding to the kth discrete state of river flow in season s, risk of low water quality at checkpoint l in season s, and the fraction removal level of BOD for the discharger m in season s, respectively. Similarly, the goals of the pollution control agency are denoted by G_l^s, and the goals of the dischargers are denoted by F_m^s. The fuzzy set of low water quality at the checkpoint l is denoted by W_l^s.

[23] Linear membership functions are used for the fuzzy sets W_l^s, G_l^s, and F_m^s. Use of the linear membership functions renders the optimization problem to be a linear programming problem. The membership functions for the fuzzy sets W_l^s, G_l^s, and F_m^s are given as follows:

$$\mu_{W_l^s}\left(c_l^{ks}\right) = \begin{cases} \left[c_l^{ks}/c_l^{Hs}\right] & 0 \le c_l^{ks} \le c_l^{Hs} \\ 1 & c_l^{ks} \ge c_l^{Hs} \end{cases} \tag{30}$$

$$\mu_{G_l^s}\left(r_l^s\right) = \begin{cases} 1-\left[r_l^s/r_l^{Ms}\right] & 0 \le r_l^s \le r_l^{Ms} \\ 0 & r_l^s \ge r_l^{Ms} \end{cases} \tag{31}$$

$$\mu_{F_m^s}\left(x_m^s\right) = \begin{cases} 1 & 0 \le x_m^s \le x_l^{Ls} \\ \left[x_m^{Ms}-x_m^s\right]/\left[x_m^{Ms}-x_m^{Ls}\right] & x_m^{Ls} \le x_m^s \le x_m^{Ms} \\ 0 & x_m^s \ge x_m^{Ms}. \end{cases} \tag{32}$$

The constants appearing in the linear membership functions given by (30), (31), and (32) are as follows: $c_l^{Hs} = 5$ mg L^{-1}; $r_l^{Ms} = 0.5\ \forall\ l, s$; $x_m^{Ls} = 0.30$; and $x_m^{Ms} = 0.95\ \forall\ m, s$. Value for the minimum fraction removal level, x_m^{MINs}, imposed by the pollution control agency is assumed as 0.3 for all m and s. The fuzzy optimization model for the river system shown in Figure 1 is given as follows: Maximize λ subject to

$$\mu_{G_l^s}\left(r_l^s\right) \ge \lambda\ \forall\ l,\ s, \tag{33}$$

$$\mu_{F_m^s}\left(x_m^s\right) \ge \lambda\ \forall\ m,\ s, \tag{34}$$

$$0 \le r_l^s \le r_l^{Ms}\ \forall\ l,\ s, \tag{35}$$

$$0 \le c_l^{ks} \le c_l^{Hs}\ \forall\ l,\ k,\ s, \tag{36}$$

$$\max\left(x_m^{Ls},\ x_m^{\mathrm{MIN}s}\right) \le x_m^s \le x_m^{Ms}\ \forall\ m,\ s, \tag{37}$$

$$0 \le \lambda \le 1. \tag{38}$$

The argument r_l^s appearing in the membership function $\mu_{G_l^s}(r_l^s)$ of the fuzzy goal G_l^s, (31), is substituted by the following expression (see (5)) for the risk of low water quality:

$$r_l^s = \sum_k \mu_{W_l^s}\left(c_l^{ks}\right)\ p_s^k, \tag{39}$$

Table 5. Data for Computation of DO Deficit[a]

Checkpoint l	Constant Term	Coefficients Multiplied by −1			
		x_1	x_2	x_3	x_4
1	0.0907	0.0688	0.0000	0.0000	0.0000
2	0.1522	0.1317	0.0000	0.0000	0.0000
3	0.2105	0.1307	0.0000	0.0000	0.0000
4	0.4896	0.1879	0.2268	0.0000	0.0000
5	0.7449	0.2398	0.4348	0.0000	0.0000
6	0.9796	0.2873	0.6262	0.0000	0.0000
7	1.0465	0.2847	0.6195	0.0000	0.0000
8	1.4474	0.3272	0.7910	0.1954	0.0000
9	1.8150	0.3658	0.9481	0.3755	0.0000
10	2.1500	0.4011	1.0908	0.5401	0.0000
11	2.4544	0.4330	1.2203	0.6903	0.0000
12	2.4897	0.4300	1.2125	0.6864	0.0000
13	2.8793	0.4583	1.3287	0.8222	0.1191
14	3.2353	0.4839	1.4349	0.9464	0.2282
15	3.5593	0.5068	1.5301	1.0600	0.3291
16	3.8502	0.5271	1.6152	1.1620	0.4205
17	4.1158	0.5450	1.6927	1.2554	0.5049
18	4.3537	0.5610	1.7613	1.3392	0.5815

[a] River flow is 8×10^6 m^3 d^{-1}.

where p_s^k is the steady state probability of flow in state k in season s. Using (30) for $\mu w_l^s(c_l^{ks})$ in (39), for the range of c_l^{ks} considered in the model, the expression for risk r_l^s may be rewritten as

$$r_l^s = \sum_k \left[\left(c_l^{ks} / c_l^{Hs} \right) \; p_s^k \right]. \tag{40}$$

The optimization model given by maximize λ and (33) through (38) is then expressed as follows: Maximize λ subject to

$$1 - \left(1/r_l^{Ms}\right) \sum\nolimits_k \left[\left(c_l^{ks} / c_l^{Hs} \right) \; p_s^k \right] \quad \geq \quad \lambda \, \forall \, l, \, s, \tag{41}$$

$$\left(x_m^{Ms} - x_m^s \right) / \left(x_m^{Ms} - x_m^{Ls} \right) \quad \geq \quad \lambda \, \forall \, m, \, s, \tag{42}$$

$$0 \quad \leq \quad \sum\nolimits_k \left[\left(c_l^{ks} / c_l^{Hs} \right) \; p_s^k \right] \quad \leq \quad r_l^{Ms} \, \forall \, l, \, s, \tag{43}$$

$$0 \quad \leq \quad c_l^{ks} \quad \leq \quad c_l^{Hs} \, \forall \, l, \, k, \, s, \tag{44}$$

$$\max\left[x_m^{Ls}, \; x_m^{\mathrm{MIN}s} \right] \quad \leq \quad x_m^s \quad \leq \quad x_m^{Ms} \, \forall \, m, \, s, \tag{45}$$

$$0 \quad \leq \quad \lambda \quad \leq \quad 1. \tag{46}$$

The DO deficit c_l^{ks} corresponding to the kth discrete state of river flow at location l in season s is expressed as a linear function of fraction removal levels x_m^s using the method given by *Fugiwara et al.* [1986]. For example, the constant term and coefficients of the fraction removal levels in the expression for DO deficit for the discrete river flow state of 8×10^6 m^3 d^{-1} are given in Table 5. Similarly, these values are computed for the other five discrete states of river flow (i.e., for river flow rates 3×10^6, 4×10^6, 5×10^6, 6×10^6, and 7×10^6 m^3 d^{-1}). Out of the six river flow states, three states occur in a season as given in Table 3. For example, the three river flow states in season 1 are 6×10^6, 7×10^6, and 8×10^6 m^3 d^{-1}. Then in season 1, the state of DO deficit corresponding to $k = 3$ (i.e., river flow of 8×10^6 m^3 d^{-1}) at checkpoint 10 may be given as follows:

$$c_{10}^{3,\,1} = 2.1500 - 0.4011 \, x_1^1 - 1.0908 \, x_2^1 - 0.5401 \, x_3^1, \tag{47}$$

where $x_1{}^1$, $x_2{}^1$, $x_3{}^1$ are the fraction removal levels in season 1 for the dischargers D_1, D_2, and D_3, respectively. Substitution of c_l^{ks} in terms of the fraction removal levels in the optimization model results in a linear programming problem which is solved to obtain the optimum fraction removal levels. Results obtained by solving the linear programming problem are discussed in section 6.

6. Results and Discussion

[24] The optimum value, λ*, of the objective function is 0.1259. This corresponds to the maximum-minimum satisfaction level in the system. A further increase in this value of λ* is possible only with a reduction in the satisfaction level of at least one of the decision makers who is presently at a higher satisfaction level than λ*. Also, the value of the new λ* will be less than that of the old λ*, as the old λ* corresponds to the maximized minimum satisfaction level in the system. On the basis of such an argument the parameter λ may be viewed as a measure of compromise between the decision makers in the system [*Kindler*, 1992]. Various other interpretations for λ* are also possible. The value of λ* may be considered as a measure of conflict existing in the system. A value of 0 for λ* indicates a strong conflict scenario, whereas the value of 1 corresponds to a no-conflict scenario. Usually, for a water quality management problem a value of λ* close to 0 may be expected, as illustrated in the example in the present study, indicating that a conflict scenario cannot be avoided in the water quality management problem. Another interpretation for λ* as a measure of equity may be also worth mentioning. Maximization of λ tries to attain an equitable distribution of satisfaction levels that are represented by means of membership functions. A value of λ* equal to 1 indicates the most equitable distribution of satisfaction levels of the decision makers within the binding constraints of the management problem.

Table 6. Optimal Fraction Removal Levels

Season s	$x_1{}^s$	$x_2{}^s$	$x_3{}^s$	$x_4{}^s$
1	0.30	0.87	0.41	0.30
2	0.30	0.87	0.52	0.30
3	0.30	0.87	0.83	0.30
4	0.60	0.87	0.87	0.87
5	0.87	0.87	0.87	0.87
6	0.30	0.87	0.87	0.66

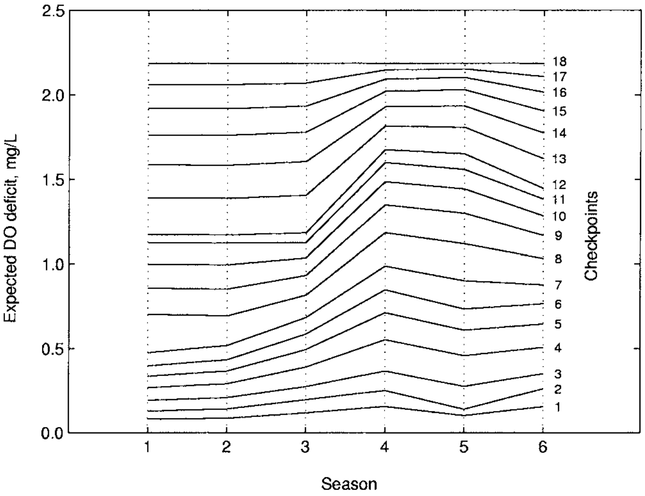

Figure 2. Expected dissolved oxygen (DO) deficits at the checkpoints.

[25] The optimum fraction removal levels at the four point sources in six seasons are given in Table 6. Since the river flows in seasons 4 and 5 are comparatively lower than those in the other seasons, higher fraction removal levels result in these seasons. The fraction removal levels for the dischargers 2 and 3 are higher than those for the dischargers 1 and 4 in all the seasons. This is because of the high effluent flows with relatively high BOD concentration from the two dischargers. A fraction removal level of 0.87, which occurs most often, corresponds to a binding constraint in the set of constraints (41) and (43). The optimum fraction removal levels in each season are used for computing the expected DO deficit at various checkpoints for that season. The expected DO deficit, $\bar{c}_{ls}$, at a checkpoint l in the season s is given by

$$\bar{c}_l^s = \sum_k c_l^{ks} \, p_s^k, \tag{48}$$

where p_s^k is the steady state probability of the discrete water quality state c_l^{ks} in season s.

[26] The expected DO deficit at all the checkpoints is shown in Figure 2 for all seasons. The expected values of the DO deficit at

all the checkpoints are less than 2.2 mg L^{-1} for all the seasons. The expected values of the DO deficit are higher in seasons 4 and 5 compared with the other seasons of the year. Even though the fraction removal levels are the highest in seasons 4 and 5 for all the dischargers, the worst water quality condition, in terms of DO deficit, occurs because of the low river flows in these seasons.

7. General Remarks

[27] In the model presented in this paper, uncertainties due to imprecision and randomness in a water quality management problem have been addressed. To account for the uncertainty in the standards for determining a failure of water quality, occurrence of failure has been treated as a fuzzy event in the paper. The fuzzy definition of low water quality ensures that there is no single threshold value which defines a failed state. The fuzzy set of low water quality maps all water quality levels to "low water quality" and its membership function denotes the degree to which the water quality is low. The question, How low is low water quality? is being addressed through such a fuzzy set. A high water quality, for example, will have a membership value of 0 in the fuzzy set of low water quality. Similarly, through the fuzzy set of low risk the question, How low is low risk? is addressed. The membership functions represent the perceptions of the decision makers. The bounds of the membership functions are subjective and may depend on the particular problem being solved. To address uncertainty in fixing the lower and upper bounds of the membership functions, the fuzzy membership functions themselves may be treated as fuzzy in the model and may be modeled using gray numbers [e.g., *Chang et al.*, 1997]. However, this is not done in the present model.

[28] A limitation of the model presented in this paper is that the dependence of treatment among various pollutants is not considered. As seen from (12) and (14), treatment of each pollutant is assumed to be independent of the other. That is, the joint product effect for treatment of multiple pollutants is not considered in the model. Considering such an effect in the model would render the model extremely complex, and the model framework would have to depend on the particular set of pollutants. The model would thus be applicable only to pollutants whose treatments are independent of each other. Also, for practical utility, considering the fraction removal levels at discrete levels, such as 30%, 40% etc., will be more advantageous than treating them as continuous variables in the model.

[29] Evaluation of the risk of low water quality (equation (5)) and incorporation of this risk into the goals of the pollution control agency (equation (11)) may be considered more reasonable than the use of chance constraints on water quality in the management model. The fuzzy risk approach as illustrated in the present study not only allows a range for the risk of low water quality but provides an opportunity for the pollution control agency to specify its perceptions of risk of low water quality. The model presented in the paper may be made more comprehensive by considering randomness in other parameters of water quality, namely, the rate coefficients related to the pollutant transfer and decay, the tributary flows, pollutant loading, etc., and also by introducing model uncertainties. However, such a generalization would result in a much more complex model and may become computationally intractable. Explicitly considering randomness in all variables in the model is indeed impossible, and an implicit approach using Monte Carlo simulations of variables may provide useful results for applications to real situations.

8. Conclusions

[30] A fuzzy optimization model is developed for the seasonal water quality management of river systems. The model addresses the uncertainty in a water quality system in a fuzzy probabilistic framework. The concepts of fuzzy decision and fuzzy probability provide the bases for the management model. The occurrence of the event of low water quality is treated as a fuzzy event. Randomness associated with the water quality indicator is linked to this fuzzy event using the concept of probability of a fuzzy event. This linking essentially defines the risk of low water quality. The two levels of uncertainty, one associated with low water quality and the other with low risk, are quantified and incorporated in the management model. The membership function for the fuzzy set of low water quality represents the degree of low water quality associated with each discrete water quality state. Using the membership function for low water quality and the steady state probability distribution for the water quality states, the fuzzy risk of low water quality is evaluated. This fuzzy risk forms the argument of the membership function of the fuzzy goals of the pollution control agency. Considering the goals of the pollution control agency and dischargers as fuzzy goals with appropriate membership functions, the water quality management model is formulated as a fuzzy optimization model.

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References

Arbabi, M., and J. Elzinga, A general linear approach to stream water quality modeling, *Water Resour. Res.*, *11*(2), 191–196, 1975.
Bardossy, A., and M. Disse, Fuzzy rule-based models for infiltration, *Water Resour. Res.*, *29*(2), 373–382, 1993.
Bardossy, A., and L. Duckstein, *Fuzzy Ruled-Based Modeling With Applications to Geophysical, Biological and Engineering Systems*, CRC Press, Boca Raton, Fla., 1995.
Bellman, R. E., and L. A. Zadeh, Decision-making in a fuzzy environment, *Manage. Sci.*, *17*(4), B141–B164, 1970.
Bogardi, I., A. Bardossy, and L. Duckstein, Regional management of an aquifer for mining under fuzzy environmental objectives, *Water Resour. Res.*, *19*(6), 1394–1402, 1983.
Boner, M. C., and L. P. Furland, Seasonal treatment and variable efficient quality based on assimilative capacity, *J. Water Pollution Control Fed.*, *54*, 1408–1416, 1982.
Burn, D. H., and E. A. McBean, Optimization modeling of water quality in an uncertain environment, *Water Resour. Res.*, *21*(7), 934–940, 1985.
Burn, D. H., and E. A. McBean, Linear stochastic optimization applied to biochemical oxygen demand–dissolved oxygen modelling, *Can. J. Civ. Eng.*, *13*, 249–254, 1986.
Chang, N., H. W. Chen, D. G. Shaw, and C. H. Yang, Water pollution control in river basin by interactive fuzzy interval multiobjective programming, *J. Environ. Eng.*, *123*(12), 1208–1216, 1997.
Das, P., and Y. Y. Haimes, Multiobjective optimization in water quality and land management, *Water Resour. Res.*, *15*(6), 1313–1322, 1979.
Duckstein, L., B. P. Shreshta, and E. Z. Stakhiv, Multicriterion risk and reliability analysis in hydrologic system design, in *Water Resources Engineering Risk Assessment,* edited by J. Ganoulis, *NATO ASI Ser., Ser. G.*, vol. 29, pp. 363–392, Springer-Verlag, New York, 1991.
Eheart, J. W., E. D. Brill Jr., B. J. Lence, J. D. Kilgore, and J. G. Uber, Cost efficiency of time varying discharge permit programs for water quality management, *Water Resour. Res.*, *23*(2), 245–251, 1987.
Ellis, J. H., Stochastic water quality optimization using imbedded chance constraints, *Water Resour. Res.*, *23*(12), 2227–2238, 1987.
Feng, Y., A method using fuzzy mathematics to solve the vector maximum problem, *Fuzzy Sets Syst.*, *9*, 129–136, 1983.
Fontane, D. G., T. K. Gates, and E. Moncada, Planning reservior operations with imprecise objectives, *J. Water Resour. Plann. Manage.*, *123*, 154–162, 1997.
Fugiwara, O., S. K. Gnanendran, and S. Ohgaki, River quality management under stochastic streamflow, *J. Environ. Eng.*, *112*(2), 185–198, 1986.

Fugiwara, O., S. K. Gnanendran, and S. Ohgaki, Chance constrained model for water quality management, *J. Environ. Eng.*, *113*(5), 1018–1031, 1987.
Fugiwara, O., W. Puangmaha, and K. Hanaki, River basin water quality management in stochastic environment, *J. Environ. Eng.*, *114*(4), 864–877, 1988.
Gromiec, M. J., D. P. Loucks, and G. T. Orlob, Stream quality modeling, in *Mathematical Modeling for Water Quality: Streams, Lakes and Reservoirs, IIASA Ser.*, vol. 12, edited by G. T. Orlob, pp. 176–226, John Wiley, New York, 1983.
Hathhorn, W. E., and Y. K. Tung, Bi-objective analysis of waste load allocation using fuzzy linear programming, *Water Resour. Manage.*, *3*, 243–257, 1989.
James, A., and D. T. Elliot, Models of water quality in rivers, in *An Introduction to Water Quality Modeling*, 2nd ed., edited by A. James, pp. 141–181, John Wiley, New York, 1993.
Kindler, J., Rationalizing water requirements with aid of fuzzy allocation model, *J. Water Resour. Plann. Manage.*, *118*, 308–323, 1992.
Lence, B. J., and A. K. Takyi, Data requirements for seasonal discharge programs: An application of a regionalized sensitivity analysis, *Water Resour. Res.*, *28*(7), 1781–1789, 1992.
Lence, B. J., J. W. Eheart, and E. D. Brill Jr., Risk equivalent seasonal discharge programs for multidischarger streams, *J. Water Resour. Plann. Manage.*, *116*, 170–186, 1990.
Lohani, B. N., and N. C. Thanh, Stochastic programming model for water quality management in a river, *J. Water Pollut. Control Fed.*, *50*, 2175–2182, 1978.
Lohani, B. N., and N. C. Thanh, Probabilistic water quality control policies, *J. Environ. Eng. Div. Am. Soc. Civ. Eng.*, *105*(4), 713–725, 1979.
Loucks, D. P., Models for management applications, in *Mathematical Modeling for Water Quality: Streams, Lakes and Reservoirs, ILASA Ser.*, vol. 12, edited by G. T. Orlob, pp. 468–509, John Wiley, New York, 1983.
Loucks, D. P., and W. R. Lynn, Probabilistic models for predicting stream quality, *Water Resour. Res.*, *2*(3), 593–605, 1966.
Loucks, D. P., J. R. Stedinger, and D. A. Haith, *Water Resource Systems Planning and Analysis*, pp. 427–545, Prentice-Hall, Old Tappan, N. J., 1981.
Louie, P. W. F., W. W.-G. Yeh, and N. S. Hsu, Multiobjective water resources management planning, *J. Water Resour. Plann. Manage.*, *110*, 39–56, 1984.
Rao, S. S., Optimization using fuzzy set theory, in *Structural Optimization: Status and Promise*, edited by M. P. Kamat, Prog. Astronaut. Aeronaut., vol. 150, edited by A. R. Seebass, pp. 637–661, Am. Inst. of Aeronaut. and Astronaut., New York, 1993.
Rheis, H. F., J. C. Dozier, D. M. Wood, and J. R. Holland, Treatment cost savings through monthly variable effluent limits, *J. Water Pollut. Control Fed.*, *54*(8), 1224–1230, 1982.
Rossman, A. L., Risk equivalent seasonal waste load allocation, *Water Resour. Res.*, *25*(10), 2083–2091, 1989.
Sakawa, M., Fuzzy programming, in *Fuzzy Systems for Management*, edited by K. Asai, pp. 57–78, IOS Press, Amsterdam, 1995.
Sasikumar, K., and P. P. Mujumdar, Fuzzy optimization model for water quality management of a river system, *J. Water Resour. Plann. Manage.*, *124*, 79–88, 1998.
Sasikumar, K., and P. P. Mujumdar, Application of fuzzy probability in water quality management of a river system, *Int. J. Sys.Sci.*, *31*(5), 575–591, 2000.
Shreshta, B. P., L. Duckstein, and E. Z. Stakhiv, Fuzzy rule-based modeling of resevior operation, *J. Water Resour. Plann. Manage.*, *122*, 262–268, 1996.
Spear, R. C., and G. M. Hornberger, Control of DO level in a river under uncertainty, *Water Resour. Res.*, *19*(5), 1266–1270, 1983.
Takyi, A. K., and B. J. Lence, Markov chain model for seasonal water quality management, *J. Water Resour. Plann. Manage.*, *121*, 144–157, 1995.
Teegavarapu, R. S. V., and S. P. Simonovic, Modeling uncertainty in reservior loss functions using fuzzy sets, *Water Resour. Res.*, *35*(9), 2815–2823, 1999.
Thomann, R. V., and J. A. Mueller, *Principles of Water Quality Modeling and Control,* pp. 1–89, 261–384, Harper and Row, New York, 1987.
Tung, Y. K., and W. E. Hathhorn, Assessment of probability distribution of dissolved oxygen deficit, *J. Environ. Eng.*, *114*(6), 1421–1435, 1988.
Tung, Y. K., and W. E. Hathhorn, Multi-objective waste load allocation, *Water Resour. Manage.*, *3*, 129–140, 1989.
Whitehead, P., and P. Young, Water quality in river system: Monte Carlo analysis, *Water Resour. Res.*, *15*(2), 451–459, 1979.
Zadeh, L. A., Probability measures of fuzzy events, *J. Math. Anal. Appl.*, *23*, 421–427, 1968.
Zimmermann, H. J., Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets Syst.*, *1*, 45–55, 1978.
Zimmermann, H. J., Application of fuzzy set theory to mathematical programming, *Inf. Sci.*, *36*, 29–58, 1985.

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