Stable Adaptive Fuzzy PID Controller for Nonlinear Uncertain Systems

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Abstract

This paper investigates an improved fuzzy adaptive control scheme for a class of nonlinear uncertain systems. A fuzzy system, whose inputs are the feedback error signals instead of the state variables, is used to cancel the nonlinear function. By using the feedback errors as the inputs of the fuzzy system, the disturbance rejection performance can be achieved because the controller will become more sensitive to the presence of the tracking errors. To prove the validity of the proposed method, the simulation results are illustrated in the end.

Keywords*: Fuzzy Controller, PID control, Adaptive Fuzzy Systems*

1. Introduction

In practical control engineering, the fuzzy control has been successfully used in a lot of practical systems. Fuzzy system-based adaptive control methodologies have received much attention. Based on the universal approximation theorem [1-3], some adaptive fuzzy control schemes have been developed for a class of single-input single-output (SISO) nonlinear uncertain systems [4], and multi-input multi-output (MIMO) nonlinear uncertain systems are explored in [7–13]. Stability analysis in such schemes is achieved by using the Lyapunov synthesis method.

Because fuzzy systems are universal approximators, some stable adaptive controls for the single input and single output new uncertain systems, based on fuzzy systems have been developed, including fuzzy supervisory control [6], Direct and indirect adaptive fuzzy control [7][8], the stable direct and indirect adaptive fuzzy control scheme are also developed for the multiple input multiple output (MIMO) nonlinear system [9][10], recently reported that direct and indirect adaptive fuzzy decentralized control method of course is one of [11] [12]. Lee and Zak has proposed the use of a high-gain feedback controller to generate a supervisory signal [13]. Shahnazi, Akbarzadeh-T. have improved the robustness of the system, replaced by supervision having a PI controller a control signal, once the tracking error is driven to a predefined area [14] [15]. A design method of the stable fuzzy controller for T-S fuzzy control systems based on the Lyapunov's method was proposed [16]. The Fuzzy-PID controller was designed in rolling vertical system [17]. The main advantage of this adaptive fuzzy control method is developed controller can deal with the increasing complexity of systems, and the controller does not accurately model the knowledge structure underlying dynamic system.

In this paper, the proposed design scheme guarantees that all the signals in the resulting closed-loop system are UUB(uniformly ultimately bounded). Moreover, the tracking errors can be reduced by adjusting the value of the designed parameter. We present a stable indirect adaptive fuzzy controller that can be designed without prior knowledge about the unknown function. The proposed adaptive fuzzy system can be provided a smaller tracking error than the adaptive algorithms. The proposed adaptive fuzzy controller can be written in the form of fuzzy PID-like controller, basically as a integrator, drive tracking error to zero. The rest of this paper is organized as follows. The second part, we describe the problem. The third section the direct adaptive fuzzy control schemes are presented, simulation results in the fourth quarter, with conclusions given in Section fifth.

2. Overview of stable adaptive control

2.1 Introduction of the model

We introduce the following differential equations

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x,t) + d(t) + u(t) \end{cases} \tag{1}$$

where $x = [x_1, x_2]^T$ is the state vector which is available for measurement, $u(t)$ is the control input, $d(t)$ is the time-varying external disturbance, $f(x,t)$ is an unknown function.

(1) can be rewritten as

$$\dot{x} = Ax + b(f(x,t) + d(t) + u(t)) \tag{2}$$

where $b = \begin{pmatrix} 0 \\ 1 \end{pmatrix} A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

The objective of this paper is to design a control law u that:

◇drive the output variable x_1 follows the specified desired trajectory r, with all signals involved in the closed loop system remain bounded.

◇control signals and state variables are bounded can be guaranteed.

Assumption 1. The desired trajectory r is a known bounded function with bounded known two times derivatives.

Assumption 2. $\|f(x,t)\|_\infty \le M$, $\|d(t)\|_\infty \le \beta$, where M and β are two positive constants and are unknown.

Suppose $f(x,t)$ is known, then the ideal controller can be expressed as

$$u = -f(x,t) + Ke + \ddot{r} - d(t) \tag{3}$$

where $e = \begin{pmatrix} r - x_1 \\ \dot{r} - \dot{x}_1 \end{pmatrix} = \begin{pmatrix} e_1 \\ \dot{e}_1 \end{pmatrix}$, $K = [k_1, k_2 \cdots, k_n]^T$, then the roots of the equation $s^n + k_1 s^{n-1} + \cdots + k_n = 0$ are all in the complex plane of the left half-plane.

The resulting error model is

$$\begin{aligned} \dot{e} &= Ae + b(\ddot{r} - f(x,t) - d(t) - u(t)) \\ &= (A - bK^T)e \end{aligned} \tag{4}$$

Since $f(x,t) + d(t) = F(x,t)$ is the unknown, one solution is the use of fuzzy logic system to approximate the unknown function $F(x,t)$. As $\hat{F}(x,t)$ are only approximate, a control signal is appended to the stability problem caused to prevent any modeling errors. Hence, a stable indirect adaptive fuzzy controller signal is

$$u = u_c + u_s \tag{5}$$

$$u = -\hat{F}(x,t) + K^T e + \ddot{r} + u_s \tag{6}$$

$\hat{F}(x,t)$ is the estimation of the unknown nonlinear function $F(x,t)$ by using the fuzzy system. u_c is an approximate ideal control law and u_s is a supervisory control signal. The following part of this

section describes the fuzzy system is used to approximate the unknown function and review various oversight reports in the literature of the control signal.

2.2 Design of the fuzzy logic system

Since the function $F(x)$ is unknown, the control system design to asymptotically stabilize the dynamics is very difficult. In this paper, we employ fuzzy systems to approximate the stable indirect adaptive fuzzy controller. The fuzzy logic system that employ singleton fuzzification, sum-product inference and center-of-sets defuzzification is modeled by

$$F(x) = \frac{\sum_{j=1}^{N} \theta_j \Pi_{i=1}^{n} \mu_{F_i^j}(x_i)}{\sum_{j=1}^{N} [\Pi_{i=1}^{n} \mu_{F_i^j}(x_i)]} \tag{7}$$

where $F(x)$ is the output of the fuzzy system, x is the input vector, $\mu_{F_i^j}(x_i)$ is $x_i's$ membership of jth rule and θ_j is the centroid of the jth consequent set. (7) can be rewritten as following equation:

$$F(x) = \theta^T \psi(x) \tag{8}$$

with $\theta = [\theta_1, \cdots, \theta_N]^T$, $\psi(x) = [p_1(x), p_2(x), \cdots, p_N(x)]^T$, and the fuzzy basis function can be writing as

$$p_j(x) = \frac{\Pi_{i=1}^{n} \mu_{F_i^j}(x_i)}{\sum_{j=1}^{N} [\Pi_{i=1}^{n} \mu_{F_i^j}(x_i)]} \tag{9}$$

Suppose there are N rules of fuzzy system used to approximate the unknown function $F(x)$:
Rule i :if x_1 is A_1^i and ... and x_n is A_n^i, then

$$u_F = \theta_{Fi}, i = 1, 2, \cdots, N. \tag{10}$$

Let us define the ideal parameters of θ_F as:

$$\theta_F^* = \arg \min_{\theta_F} [\sup_{X \in R^n} \left| \hat{F}(x|\theta_F) - F(x) \right|] \tag{11}$$

Note that the ideal parameter θ_F^* is introduced only for the purposes of analysis, and its value is not needed when implementing the controller.

Hence, the approximation error D_w^* is given by

$$D_w^* = F(x) - \hat{F}(x|\theta_F^*) \tag{12}$$

Define

$$\phi_F = \theta_F^* - \theta_F \qquad (13)$$

as the parameter estimation error, when $\theta_F = [\theta_{F1} \cdots \theta_{Fi} \cdots]^T$ is vector containing the consequent parameters of the fuzzy rules.

3. Main results

Our proposed control law includes an approximate ideal control signal u_c and a supervisory control signal u_s. The controller can be written in the form of a PID controller:

$$u = u_c + u_s = -\hat{F}(e) + K^T e + \ddot{r} + \int_{t=0}^{t} \rho_w e^T Pb dt \qquad (14)$$

where $\hat{F}(e)$ is fuzzy logic system and $\hat{D}_w$ is an estimate of the approximation error D_w defined by

$$D_w = F(x) - \hat{F}(e|\theta_F^*) \qquad (15)$$

The proposed adaptive controller is an approximation of the nonlinear function $F(x)$, the use of the fuzzy system $F(e)$, whose inputs are the feedback error signals instead of the state variables.

Theorem 1. An adaptive controller for the system described in Eq. (2) that comprises the controller in Eq. (14), the rules in Eqs. (10) designed to cancel $\hat{F}(x)$, and the adaptive laws

$$\dot{\theta}_F = proj(-\rho_F e^T Pb\varsigma_F) = \begin{cases} 0 & if\ \theta_F \leq \underline{\theta}_F,\ \ \theta_F \geq \overline{\theta}_F \\ -\rho_F e^T Pb\varsigma_F & otherwise \end{cases} \qquad (16)$$

$$\dot{\hat{D}}_w = -\rho_w e^T Pb \qquad (17)$$

has the following properties:

(1) The tracking error $e(t) = r(t) - x(t)$ is uniformly ultimately bounded(UUB). $\hat{D}_w$ and θ_F are bounded.

(2) The tracking error $e(t)$ and the parameter error vectors ϕ_F, ϕ_w will converge to

$$\|e(t)\| \leq \sqrt{\frac{2V(0)\exp(-\mu t)}{\lambda_{\min}(Q)}} \qquad (18)$$

$$\|\phi_F\| \leq \sqrt{\rho_F V(0)\exp(-\mu t)} \qquad (19)$$

$$\|\phi_w\| \leq \sqrt{\rho_w V(0)\exp(-\mu t)} \qquad (20)$$

where

$$V(0) = \frac{1}{2}e(0)^T Pe(0) + \frac{1}{\rho_F}\phi(0)^T \phi(0) + \frac{1}{\rho_w}\phi(0)^2 \text{ and } \mu = \lambda_{\min}(Q)/\lambda_{\max}(P).$$

(3) The control signal u is also bounded by

$$|u| < |\overline{F}| + \|K^T\|\sqrt{\frac{2V(0)\exp(-\mu t)}{\lambda_{\min}(Q)}} + \ddot{r} + \overline{D}_w. \tag{21}$$

Proff.
(1) The system model in Eq. (2) can be written as
$$\dot{x} = Ax + b(f(x,t) + d(t) + u)$$
$$= AX + b(F(x,t) + u)$$
$$= AX + b(\hat{F}(e|\theta_F^*) + F(x,t) - \hat{F}(e) + u)$$
$$= AX + b(\hat{F}(e|\theta_F^*) - \hat{F}(e|\theta_F) + K^T e + \ddot{r} + D_w - \hat{D}_w)$$
$$= AX + b(\phi_F \varsigma_F + K^T e + \ddot{r} + D_w - \hat{D}_w) \quad (\phi_F = \theta_F^* - \theta_F)$$
Then
$$\dot{e} = Ae + b(\ddot{r} - F(x,t) - u)$$
$$= Ae - bK^T e + b(-\phi_F \varsigma_F + \hat{D}_w - D_w)$$
$$= (A - bK^T)e + b(-\phi_F \varsigma_F + \phi_w) \; (\phi_w = \hat{D}_w - D_w)$$
Consider the following Lyapunov candidate function:

$$V = \frac{1}{2}(e^T Pe + \frac{1}{\rho_F}\phi_F^T \phi_F + \frac{1}{\rho_w}\phi_w^2) \tag{22}$$

The time derivative of V is

$$\dot{V} = -\frac{1}{2}e^T Qe + \phi_F(\frac{1}{\rho_F}\dot{\phi}_F - e^T Pb\varsigma_F) + \phi_w(\frac{1}{\rho_w}\dot{\phi}_w + e^T Pb) \tag{23}$$

where ρ_F and ρ_w are positive scalars ,and Q is an $n \times n$ positive definite matrix. Therefore, $\dot{V} = -\frac{1}{2}e^T Qe$ will be positive definite under the adaptive law:

$$\dot{\phi}_F = \rho_F e^T Pb\varsigma_F \tag{24}$$

$$\dot{\phi}_w = -\rho_w e^T Pb \tag{25}$$

(2)From the result $\dot{V} = -\frac{1}{2}e^T Qe$, we have

$$\dot{V} \le -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}e^T Pe = -\mu V \tag{26}$$

Integrating over $[0,t]$ leads to

$$0 \le V(t) \le V(0)\exp(-\mu t) \tag{27}$$

where $V(0) = \frac{1}{2}e(0)^T Pe(0) + \frac{1}{\rho_F}\phi(0)^T\phi(0) + \frac{1}{\rho_w}\phi(0)^2$

We can deduce

$$\frac{1}{\rho_F}\|\phi_F\|^2 \le V(t) \tag{28}$$

$$\frac{1}{\rho_w}\|\phi_w\|^2 \le V(t) \tag{29}$$

It yields

$$\|e(t)\| \le \sqrt{\frac{2V(0)\exp(-\mu t)}{\lambda_{\min}(Q)}} \tag{30}$$

$$\|\phi_F\| \le \sqrt{\rho_F V(0)\exp(-\mu t)} \tag{31}$$

$$\|\phi_w\| \le \sqrt{\rho_w V(0)\exp(-\mu t)} \tag{32}$$

(3) The tracking error is bounded, the upper limit on the control signal can be used as the lower bound by the parameters in the denominator replace it. The control signal $u^{'}$ s upper bound is

$$|u| < |\overline{F}| + \|K^T\|\sqrt{\frac{2V(0)\exp(-\mu t)}{\lambda_{\min}(Q)}} + \ddot{r} + \overline{D}_w \tag{33}$$

The proposed adaptive controller has two design parameters, ρ_F determines the fuzzy logic system convergence rate, ρ_w determines the size of the supervision and control signal. As the size of the tracking error convergence speed to supervision and control signal. The tracking error will decrease as one of ρ_F or ρ_w increase.

4. Simulation results

In this section, we will provide a simulation example to illustrate the feasibility of the control scheme proposed, we let the mentioned system to track a desired trajectory.

Consider the system with the following definitions

$$\dot{x} = f(x,t) + d(t) + u(t) \tag{34}$$

where $\quad f(x,t) = \begin{bmatrix} \sin(x_1 x_2) \\ \dfrac{1 - \exp(-x_2)}{1 + \exp(-x_2)} \end{bmatrix}$, $d(t) = \begin{bmatrix} 0.1 \times \sin(0.86t) \\ 0.2 \times \cos(0.86t) \end{bmatrix}$, $u(t) = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$.

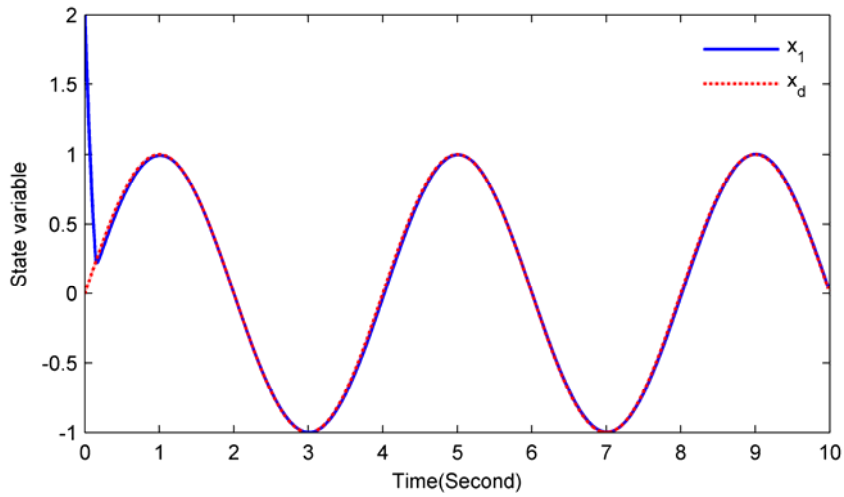The reference signal is given as follows: $r = x_d = \sin(\dfrac{\pi t}{2})$.

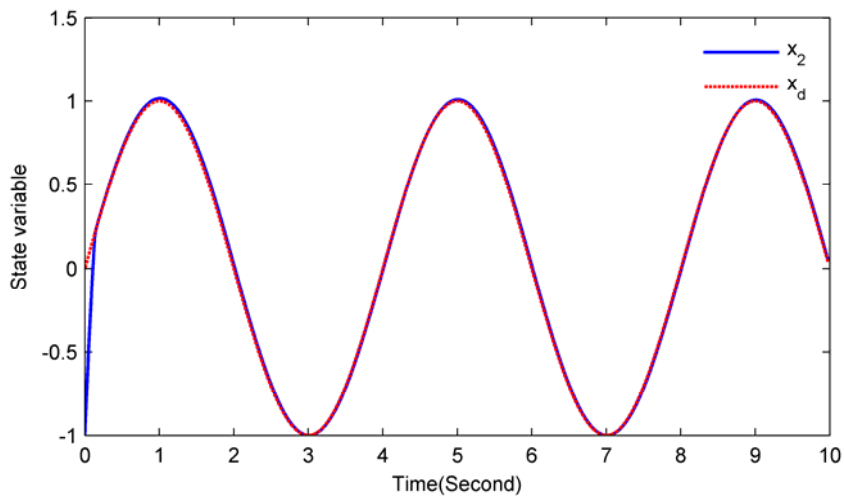**Figure 1.** The time response of the state variable x_1

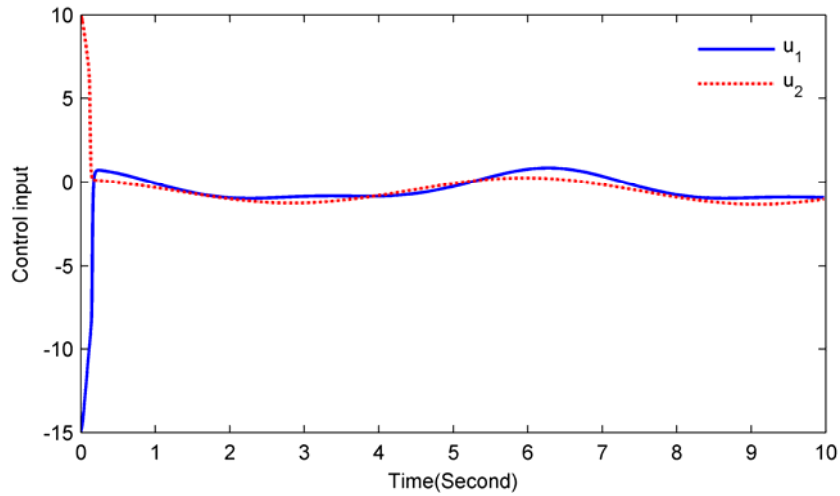**Figure 2.** The time response of the state variable x_2

Figure 3. The control input $u(t)$

In this test, system (34) is used and in simulations, we used $x_1 = 2, x_2 = -1$ as the initial values of states. $\rho_F = 100$, θ_F is set to $-100 \le \theta_F \le 100$,the supervisory controller u_s, ρ_F is set to 1000.Simulation results in Figures 1-3 are obtained by applying the design algorithm to the system (34). The time response of the state variables of the uncertain system are given in Figure 1 and Figure 2, a better tracking performance is achieved for the desired trajectories x_d . Boundedness and smoothness of the controllers u_1 and u_2 are described in Figure 3. The simulation results show that the proposed method is successful in control unknown system and the states (x_1, x_2) quickly converge to respective desired trajectories x_d is nice.

5. Conclusion

This paper introduces a kind of stable adaptive fuzzy controller for a class of nonlinear system. The proposed adaptive fuzzy controller for the main features is used to cancel the non-linear function of a fuzzy PID-like system and a management control device, basically as an integrator, to drive the tracking error tends to zero. The efficiency of the system through numerical simulation proved that the proposed adaptive fuzzy. Another advantage of the adaptive fuzzy controller, it may work in two modes, one to reduce the computational burden online tuning. In the operating mode, on-line adaptation is terminated. The simulation results show that, in the deterioration of the tracking performance of the proposed controller, the controller according to the previously reported.

6. ACKNOWLEDGEMENTS

7. References

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