Asymptotically optimal neighbour sum distinguishing colourings of graphs

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Consider a simple graph $G = (V, E)$ and its proper edge colouring $c$ with the elements of the set $\{1, 2, \ldots, k\}$. The colouring $c$ is said to be neighbour sum distinguishing if for every pair of vertices $u, v$ adjacent in $G$, the sum of colours of the edges incident with $u$ is distinct from the corresponding sum for $v$. The smallest integer $k$ for which such colouring exists is known as the neighbour sum distinguishing index of a graph and denoted by $\chi'_\Sigma(G)$. The definition of this parameter, which makes sense for graphs containing no isolated edges, immediately implies that $\chi'_\Sigma(G) \geq \Delta$, where $\Delta$ is the maximum degree of $G$. On the other hand, it was conjectured by Flandrin et al. that $\chi'_\Sigma(G) \leq \Delta + 2$ for all those graphs, except for $C_5$. We prove this bound to be asymptotically correct by showing that $\chi'_\Sigma(G) \leq \Delta(1 + o(1))$. The main idea of our argument relays on a random assignment of the colours, where the choice for every edge is biased by so called attractors, randomly assigned to the vertices.