System identification of buildings by wave travel time analysis and layered shear beam models—Spatial resolution and accuracy

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ABSTRACT

A previously explored method for one-dimensional structural system identification and earthquake damage detection in buildings, based on measuring wave travel time through the structure and its changes, is formalized, and its spatial resolution and accuracy are analyzed. The method identifies the velocity of propagation of shear waves in the structure as function of height. The wave travel time is measured from impulse responses obtained from recorded response at different locations in the structure. The main advantages of this SHM method over other methods are its robustness in application to real buildings and large amplitude response, insensitivity to the effects of soil–structure interaction, and local in nature achieved with relatively small number of sensors. The identification is based on a layered shear beam model of the building. In this paper, analytical impulse response functions are presented for such model, which provide theoretical basis to define identification algorithms. The derivation of one such algorithm, which involves approximations, from the exact wave propagation solution of the model is presented, and its spatial resolution and accuracy are critically examined. Termed here ‘direct algorithm’, it involves measuring the pulse time shifts and amplitudes and identifies shear wave velocities and quality factor $Q$ in the layers. Various issues identified, e.g. the trade-off between accuracy and detail of the identification, are illustrated on a full-scale densely instrumented nine-story RC building (Millikan Library in Pasadena, CA, USA, excited by 2002 Yorba Linda earthquake). Copyright © 2012 John Wiley & Sons, Ltd.

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KEY WORDS: structural system identification; structural health monitoring; wave propagation; wave travel time; Millikan Library; impulse response; layered shear beam

1. INTRODUCTION

Buildings, traditionally analyzed using vibrational methods, can also be analyzed using wave propagation methods [1–20]. Although the vibrational and wave propagation representations of response are mathematically equivalent [1,12], one may be preferable for a particular problem. A building as a whole deforms generally in shear, and it can be characterized by its shear wave velocity $\beta = \sqrt{\mu/\rho}$ ($\rho$ = mass density, $\mu$ = shear modulus), which can be identified from observed wave travel time $\tau$ over distance $h$ as $\beta = h/\tau$. As $\beta$ is directly related to the stiffness, loss of stiffness because of damage would lead to reduction of the wave velocity in the damaged part and increase in wave travel time. This provides a basis for inference about the presence and location of damage in the structure [5,6,9,16,17,21]. Proof of concept studies of earthquake records in damaged buildings [16,17], with wave travel time measured from impulse response functions (IRFs) [12], showed that the detected changes in wave velocities are consistent with the location and degree of the observed damage and concluded that the method is promising and should be further developed. Important advantages of wave travel time-based methods for

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damage detection are their robustness when applied to real buildings and large amplitude response [16,17], local nature achieved with relatively small number of sensors [6,16,17], and not being sensitive to the effects of soil–structure interaction [12,18,19].

This paper further investigates structural system identification from wave travel time for use in structural health monitoring, with emphasis on the spatial resolution and accuracy of the identification. In the exploratory study in [16,17], the identification of the shear wave velocity was based on a conceptual model of the building as a layered medium and the ray theory interpretation of the wave travel time through inhomogeneous medium, which did not allow for error analysis. Analysis of accuracy requires a predictive theoretical model for the response that is derived from the complete wave propagation theory. In this paper, we present such a model, and exact analytical solutions for its transfer function and IRF.

The model is a layered shear beam, with piecewise constant material properties, no foundation rocking, and excited by vertically incident vertically polarized shear (SV) wave. Mathematically, this problem is equivalent to that of vertically incident horizontally polarized shear (SH) wave in a horizontally layered half-space, which has been solved in geophysics using the propagator matrix approach [22–24], and which we adopt. We review this elegant solution, little known among the engineering community, for the purpose of completeness of this paper, and also for convenience of others who may find it useful, e.g., for application to problems in nondestructive testing. We also review how the identification algorithm used in [16,17] is derived from the analytical model IRFs, based on ray theory 23, which we name the direct identification algorithm, and investigate its spatial resolution and accuracy for real buildings and data. In this paper, we derive the spatial resolution and error in the identified shear wave velocity profile, as function of IRF bandwidth, from the Heisenberg-Gabor uncertainty principle [25,26]. We show that there is a trade-off between resolution and accuracy, which is a consequence of the finite effective bandwidth of recorded structural response, and cannot be avoided even if a dense network of sensors is available. To the knowledge of the authors, this analysis is new. In the results section of this paper, we show simulated broadband (0–50 Hz) IRFs for layered building models and examine their features relevant for identification, such as reflected pulses and pulse amplitude attenuation. We demonstrate the trade-off between resolution and accuracy on a case study of a real building excited by an earthquake—Millikan Library NS response during Yorba Linda earthquake of 2002—which was recorded using a dense network of sensors.

The analytical model IRFs presented in this paper will be useful for deriving future, more effective algorithms for identification of buildings using IRFs. They will also be useful as a simulation tool to examine the effects of localized damage on the building response and to facilitate the interpretation of observed IRFs in full-scale buildings during earthquakes. Understanding and assessing the resolution and accuracy of structural system identification algorithms is important for their effectiveness as part of systems for early post-earthquake damage detection in buildings [27].

2. METHODOLOGY

2.1. Model and analytical frequency domain solution

The building is modeled as elastic, layered shear beam, supported by a half-space, stress-free on the top, and excited by vertically incident plane shear waves (SV) (Figure 1a). Each layer represents a floor or a group of floors, and the building is assumed to move only horizontally, the foundation rocking because of soil–structure interaction being neglected. The layers, numbered from top to bottom, are homogeneous and isotropic, with thickness $h_i$, mass density $\rho_i$, shear modulus $\mu_i$, and shear wave velocities $\beta_i = \sqrt{\mu_i/\rho_i}$, $i = 1, \ldots, n$, and are perfectly bonded to each other. The displacements at the roof and at the consecutive layer interfaces are $u_1, u_2, \ldots$

The motion of this model is mathematically identical to that of a horizontally layered half-space, excited by vertically incident SH waves (Figure 1b), which has been solved in geophysics using the propagator matrix approach [22]. In the following, we outline the solution in the frequency domain for general incident angle $\gamma$ (Figure 1b). Let $U(x, z; t)$ be the y-component of displacement (Figure 1b), and $\tau_{zy} = \mu \partial U / \partial z$ be the corresponding shear stress. Then, $U(x, z; t)$ is a
propagating wave in the \( x \)-direction
\[
U(x, z; t) = u(z) \exp[i\omega(px - t)]
\] (1)
where \( \omega = \) circular frequency, and \( p = \sin \gamma/\beta_{n+1} = \) horizontal slowness (\( p = 1/c_x \) where \( c_x = \) horizontal phase velocity). Within each layer, the motion is governed by [28]
\[
\frac{\partial}{\partial z} \mathbf{f}(z) = \mathbf{A} \mathbf{f}(z)
\] (2)
where
\[
\mathbf{f}(z) = \begin{bmatrix} u(z) \\ \tau_\gamma(z) \end{bmatrix}
\] (3)
\[
\mathbf{A} = \begin{bmatrix} 0 & 1/\mu \\ \omega^2(\mu p^2 - \rho) & 0 \end{bmatrix}
\] (4)

The propagator operator (called matricant in mathematics), introduced to geophysics by Gilbert and Backus [22] as a matrix method for solving the layered half-space problem, is a generalization of the Thompson-Haskell method for surface waves [29,30]. According to [22], if matrix \( \mathbf{A} \) in Eqn (2) is a continuous function of \( z \), as is the case within each layer, then the solution at any point \( z \) in the layer can be projected from the solution at another point \( z_0 \) in the same layer as
\[
\mathbf{f}(z) = \mathbf{P}(z, z_0) \mathbf{f}(z_0)
\] (5)
where \( \mathbf{P}(z, z_0) = \) propagator from \( z_0 \). Let \( \mathbf{F}(z) \) be a fundamental matrix of the system of Eqn (2). Then, \( \mathbf{P}(z, z_0) \) can be obtained as
\[
\mathbf{P}(z, z_0) = \mathbf{F}(z)\mathbf{F}(z_0)^{-1}
\] (6)
and has entries
\[
\begin{align*}
\mathbf{P}_{11}(z, z_0) &= \cos \eta (z - z_0) \\
\mathbf{P}_{12}(z, z_0) &= \frac{i}{\kappa \eta \mu} \sin \eta (z - z_0) \\
\mathbf{P}_{21}(z, z_0) &= i(\kappa \eta \mu) \sin \eta (z - z_0) \\
\mathbf{P}_{22}(z, z_0) &= \cos \eta (z - z_0)
\end{align*}
\] (7)
where \( \eta = \sqrt{1/\beta^2 - p^2} = \) vertical slowness (\( = 1/c_z \), \( c_z = \) vertical phase velocity).

Figure 1. The model. (a) Layered shear-beam representing a building. (b) Layered half-space.
As \( f(z) \) must be continuous at the layer interfaces, if it is known at one point, e.g. at \( z = 0 \), where the stress is zero and unit displacement can be assumed, it can be projected to any other point in any of the layers. Starting from the top with \( f(0) = (0, 1) \), \( f(z) \) can be computed recursively at the layer interfaces \( z = z_1, z_2, \ldots \), and from there propagated inside the layers, which completely defines the displacements and stresses. Then, anywhere in the layered medium

\[
f(z) = P(z, 0) f(0), \quad z_{i-1} \leq z \leq z_i, \quad i = 1, \ldots, n
\]

\[
P(z, 0) = P(z, z_{i-1}) P(z, z_{i-2}) \cdots P(z, z_1) P(z, 0)
\]

where \( P(z, 0) \) is propagator directly from \( z=0 \) to \( z \), and \( P(z, z_{j-1}) \) is propagator for the \( j \)th layer (Eqn (7)).

For the one-dimensional building model (Figure 1a), \( f(0) = 0 \) and \( \eta = 1/\beta \). Furthermore, as the first entry of \( f(z) \) is \( u(z) \), \( P_{11}(z, 0) \) gives the transfer function (TF) of displacement at \( z \) with respect to the displacement at \( z = 0 \) (the top), and the inverse Fourier transform (FT) of \( P_{11}(z, 0) \) gives the IRF at \( z \) for virtual source at \( z = 0 \). This also completely defines the TF \( \hat{h}(z, z_{\text{ref}}; \omega) \) and IRF \( h(z, z_{\text{ref}}; t) \) with respect to any reference point \( z_{\text{ref}} \) as

\[
\hat{h}(z, z_{\text{ref}}; \omega) = P_{11}(z, 0)/P_{11}(z_{\text{ref}}, 0)
\]

\[
h(z, z_{\text{ref}}; t) = \text{FT}^{-1}\{\hat{h}(z, z_{\text{ref}}; \omega)\}
\]

where \( \text{FT}^{-1} \) indicates inverse FT, for which we follow the convention

\[
\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{j\omega t} dt \Leftrightarrow f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{-j\omega t} d\omega
\]

At the reference level, \( \hat{h}(z_{\text{ref}}, z_{\text{ref}}; \omega) = 1 \) and \( h(z_{\text{ref}}, z_{\text{ref}}; t) = \delta(t) \) represents a virtual source, whereas \( h(z, z_{\text{ref}}; t) \) represents the response at \( z \) to a virtual source at \( z_{\text{ref}} \). When the virtual source does not coincide with the physical source, there will be both causal and acausal pulses in the IRFs [12].

To account for amplitude attenuation because of material friction, quality factor \( Q \) is introduced, describing the amplitude reduction of a propagating wave or a vibrating volume over one cycle. If \( A_0 \) is the initial amplitude, \( A(t; \omega) = A_0 \exp[-c\omega t/(2Q)] \) and \( A(z; \omega) = A_0 \exp[-c\omega z/(2Q\beta)] \) [28]. High \( Q \) implies small attenuation, and constant \( Q \) implies frequency proportional attenuation (increasing with frequency). It is incorporated in the solution by replacing the real valued vertical slowness \( \eta = 1/\beta \) by complex slowness \( \eta^{\text{complex}} \)

\[
\eta^{\text{complex}} = \eta + i \frac{1}{2Q}
\]

\( Q \) is related to the damping ratio \( \zeta \) (ratio of damping and critical damping of viscously damped oscillators) by \( \zeta = 1/(2Q) \). Then, \( \zeta = 1\% \) is equivalent to \( Q = 50 \), which is value typical for sediments. Although usually assumed to be frequency independent, \( Q \) and \( \zeta \) are in general strong functions of frequency [31]. As a consequence of \( Q \), the pulse propagation is dispersed, however, for lightly damped structures, this effect is small.

2.2. Analytical band-limited impulse response functions

In practice, we work with finite data bandwidth, i.e. \( |\omega| \ll \omega_{\text{max}} \), for which

\[
h(z, z_{\text{ref}}; t) = \frac{1}{2\pi} \int_{-\omega_{\text{max}}}^{\omega_{\text{max}}} \hat{h}(z, z_{\text{ref}}; \omega) e^{-j\omega t} d\omega
\]

The integral in Eqn (14) converges even for \( Q > 0 \), which is not otherwise the case [23]. Then, the source pulse is box function in the frequency domain, and sinc function in the time domain. The integration in Eqn (14) can be carried out both numerically, e.g. using fast Fourier transform, and analytically [23]. Although the former is more straightforward, the latter is more useful for devising and
analyzing identification algorithms, as shown later. In the following, we summarize the analytical IRFs derived in [23].

If \( z_{\text{ref}} = 0 \), then the kernel in Eqn (14) is the propagator \( P_{11}(z, 0) \) (Eqn (9)). Analytical integration gives at interface \( z_m, m \leq n \), IRFs normalized to unit source amplitude [23]

\[
h(z_m, 0; t) = \sum_{i=1}^{n-1} \left[ SC_i^{(m)}(t - t_i^{(m)}) + SA_i^{(m)}(t + t_i^{(m)}) \right]
\]

where \( SC_i^{(m)}(t) \) and \( SA_i^{(m)}(t) \) are inverse FTs of box functions for causal and acausal waves given by

\[
SC_i^{(m)}(t) = \frac{1}{\omega_{\text{max}}} \left\{ \exp\left(-z_i^{(m)} / \omega_{\text{max}} \right) \left[ -z_i^{(m)} \cos(\omega_{\text{max}} t) + t \sin(\omega_{\text{max}} t) \right] - \frac{-z_i^{(m)}}{\omega_{\text{max}}^2 + t^2} \right\}
\]

\[
SA_i^{(m)}(t) = \frac{1}{\omega_{\text{max}}} \left\{ \exp\left(-z_i^{(m)} / \omega_{\text{max}} \right) \left[ z_i^{(m)} \cos(\omega_{\text{max}} t) + t \sin(\omega_{\text{max}} t) \right] - \frac{-z_i^{(m)}}{\omega_{\text{max}}^2 + t^2} \right\}
\]

In Eqn (15), \( t_i^{(m)} \) are time shifts relative to source time, such that

\[
t_i^{(0)} = 0, t_{2i-1}^{(m)} = t_i^{(m-1)} + \eta_m h_m, t_{2i}^{(m)} = t_i^{(m-1)} - \eta_m h_m
\]

and the coefficients \( a_i^{(m)} \) and \( b_i^{(m)} \) are functions of the reflection and transmission coefficients \( R \) and \( T \) such that

\[
\begin{align*}
a_i^{(0)} &= 1 \\
a_{2i-1}^{(m-1)} &= a_i^{(m-1)}, \quad a_{2i}^{(m-1)} = a_i^{(m-1)} R_m, \quad i \text{ - odd} \\
a_{2i-1}^{(m)} &= a_i^{(m-1)} R_m, \quad a_{2i}^{(m)} = a_i^{(m-1)}, \quad i \text{ - even}
\end{align*}
\]

\[
b_i^{(m)} = \prod_{j=2}^{m} \left( T_j \right)
\]

where \( R_m \) is the reflection coefficient of waves in \( m \)th layer reflected from the interface with the \((m - 1)\)th layer and \( T_j \) is the transmission coefficient for waves from the \( j \)th layer into the \((j - 1)\)th layer defined as

\[
R_m = \frac{\eta_m h_m - \eta_{m-1} h_{m-1}}{\eta_m h_m + \eta_{m-1} h_{m-1}}
\]

\[
T_j = \frac{2 \eta_j \mu_j}{\eta_j \mu_j + \eta_{j-1} \mu_{j-1}}
\]

In Eqn (16) and (17), \( z_i^{(m)} \) are amplitude attenuation factors

\[
\begin{align*}
z_i^{(0)} &= 0 \\
z_{2i-1}^{(m-1)} &= z_i^{(m-1)} + \eta_m h_m \zeta_m, \quad z_{2i}^{(m-1)} = z_i^{(m-1)} - \eta_m h_m \zeta_m
\end{align*}
\]

The other parameters are same as defined earlier, i.e. \( h_m \) is the thickness, \( \eta_m = 1/\beta_m \) is the vertical slowness, \( \zeta_m = 1/(2Q_m) \), and \( \beta_m \) is the shear modulus, all of these of the \( m \)th layer.

As shown by Eqn (15), the IRF at each interface represents an assembly of shifted in time sinc functions, which include the transmitted causal and acausal pulses, and reflections from the layer interfaces. The TFs in Eqn (10) and the derived IRFs in Eqn (11) and (18) are exact.

2.3. Ray theory interpretation of impulse response functions and the direct identification algorithm

The ray theory is an approximate theory, based on the assumption that body waves travel through an inhomogeneous medium as a wave front, with the local propagation speeds, obeying the laws of
geometric optics, i.e. along ray paths determined by Snell’s law [28]. If the variations of material properties is smooth so that the effects of scattering from inhomogeneities along the path are small, then an approximate solution of the wave equation implies that the wave front travel time between points A and B, \(T_{AB}\) is the line integral

\[
T_{AB} = \int_{AB} \frac{1}{\beta(s)} \, ds
\]

Eqn (24) implies that the pulse travel time over the height of the building (Figure 1) is a sum of the travel time through the individual layers, and the travel time through the individual layers can be obtained directly from the pulse time shifts. This provides the basis for a simple identification algorithm, which we term direct, derived from the analytical IRFs in the previous section as follows. The velocities \(\beta_m\) can be obtained from the time shifts \(\tau_1^{(m)}\) of the pulses (relative to source time), and the quality factors \(Q_m\) can be obtained from the ratios of the peak amplitudes of the causal and acausal pulses, \(SC_i^{(m)}(0)\) and \(SA_i^{(m)}(0)\) (Eqn (15)) [23]. If there is a recording at every layer interface, then the transmitted pulse \((t = 1)\) is sufficient to identify all layers from the relationships

\[
t_1^{(m)} - t_1^{(m-1)} = h_m/\beta_m \quad (25)
\]

\[
\ln \frac{SA_1^{(m)}(0)}{SC_1^{(m)}(0)} - \ln \frac{SA_1^{(m-1)}(0)}{SC_1^{(m-1)}(0)} = \omega_{\text{max}} \zeta_m \left( t_1^{(m)} - t_1^{(m-1)} \right) \quad (26)
\]

where \(\zeta_m = 1/(2Q_m)\). Eqn (25) and (26) imply

\[
\beta_m = \frac{h_m}{\left( t_1^{(m)} - t_1^{(m-1)} \right)} \quad (27)
\]

\[
\frac{1}{2Q_m} = \left( \frac{1}{\omega_{\text{max}}} \right) \left( \frac{1}{t_1^{(m)} - t_1^{(m-1)}} \right) \left[ \ln \frac{SA_1^{(m)}(0)}{SC_1^{(m)}(0)} - \ln \frac{SA_1^{(m-1)}(0)}{SC_1^{(m-1)}(0)} \right] \quad (28)
\]

and overall \(Q\)

\[
\frac{1}{2Q} = \frac{1}{\omega_{\text{max}} t_1^{(m)}} \ln \frac{SA_1^{(n)}(0)}{SC_1^{(n)}(0)} \quad (29)
\]

where \(t_1^{(m)}\) is the travel time from base to roof. If there is not a sensor at each layer interface, then it is still possible, in principal, to identify all the layers, using the reflected pulses. However, the reflected pulses are practically useful only if they can be resolved.

2.4. Resolution and accuracy of the direct algorithm

The source pulse \(h(z_{\text{ref}}, \theta_{\text{ref}}; t) = \sin(\omega_{\text{max}}t)\) is localized both in time and in frequency, around time \(t = 0\) and frequency \(\omega = \omega_{\text{max}}/2\), but not perfectly. Its spread in frequency is \(\Delta \omega = \omega_{\text{max}}/2\). Let the half-width of the main lobe \(\Delta t = \pi/\omega_{\text{max}} = 1/(2f_{\text{max}})\) be a measure of its spread in time, where \(f_{\text{max}} = \omega_{\text{max}}/(2\pi)\). (The second moment, usually used to define the spread in time, cannot be used because it is infinite for this source function.) This implies that smaller \(\omega_{\text{max}}\) results in a wider source pulse, which worsens the time localization of the IRFs, and consequently, the spatial localization of the wave front. The product of \(\Delta \omega\) and \(\Delta t\) for the source pulse is always constant

\[
\Delta \omega \Delta t = \frac{\pi}{2} \quad (30)
\]

Eqn (30) represents the Heisenberg-Gabor uncertainty principle for signals [25,26], according to which a signal cannot be perfectly localized both in time and in frequency, and increased localization in one domain is at the expense of decrease localization in the other domain. (Its equivalent in physics states that both the position and velocity of a particle cannot be determined exactly.) An important consequence of this principle is the finite spatial resolution of the identification of the variations of the layer properties and the uncertainty in the identified properties at a particular location in the building. As demonstrated in the
results section, the limited resolving power implies that there is some minimum thickness of the top layer that can be resolved by the IRFs, even if there exists a recording within that layer.

The related uncertainties \( \Delta z \) and \( \Delta \beta \) can be derived from the Heisenberg-Gabor uncertainty principle and interpreted as follows. The disturbance at the virtual source will propagate through the medium as a pulse in space with half-width \( \Delta z = \beta \Delta t = \pi / o_{\text{max}} \). Furthermore, \( \Delta z \) can be expressed as \( \Delta z = \Delta \beta \tau \), where \( \Delta \beta \) is uncertainty in \( \beta \) and \( \tau \) is travel time, which gives \( \Delta \beta / \beta = \pi / (o_{\text{max}} \Delta t) \). This means that the point estimate of velocity \( \beta(z) \), measured from the time shift of a pulse in the IRF, is a weighted average over the interval \( [z - \Delta z, z + \Delta z] \), and that the identified value of \( \beta \) is an estimate, the error of which has distribution with spread \( \Delta \beta \). Recalling that \( \pi / o_{\text{max}} = \Delta t \), it follows that \( \Delta \beta / \beta = \Delta t / \tau \), which means that the relative error in the estimate of \( \beta \) will be proportional to the ratio of the source pulse half-width and the time lag \( \tau \) used to identify \( \beta \).

The aforementioned discussion implies that detecting localized variations in \( \beta(z) \) from readings of the pulse shift requires high enough signal bandwidth \( o_{\text{max}} \) and that for given \( o_{\text{max}} \), the error of the identified \( \beta(z) \) is smaller if it is identified from larger travel time \( \tau \), i.e. from larger distances. It also implies that the error in the identified \( \beta \) is smaller in more flexible buildings, and within the building, in its softer parts (e.g. near the top or in the soft first floor). As an example, if \( \beta \approx 200 \text{m/s}, f_{\text{max}} = 25 \text{Hz} \Rightarrow \Delta z \approx 4 \text{m} \approx h, \) and \( f_{\text{max}} = 10 \text{Hz} \Rightarrow \Delta z \approx 10 \text{m} \approx 2.5 h \), where \( h \) is the story height. This uncertainty in measuring \( \beta(z) \) is similar to the uncertainty in identifying instantaneous frequency of signals using wavelet transform, Gabor transform, or any other type of moving window technique, where instant refers to a time window, and there is uncertainty in the identified frequency, which is larger if the time window is shorter [32].

The bandwidth for strong motion data is typically 25 or 50 Hz [33,34]. However, the effective \( o_{\text{max}} \), which determines the width of the pulses in observed IRFs in real buildings, is considerably smaller, because of the physical nature of the response, which becomes very small beyond some frequency, relative to the regularization parameter \( \varepsilon \) in

\[
\hat{h}(z, z_{\text{ref}}; \omega) = \frac{\hat{u}(z)\hat{u}(z_{\text{ref}})}{|\hat{u}(z_{\text{ref}})|^2 + \varepsilon} \tag{31}
\]

used to compute TFs from observed response [12]. Introduced to prevent division by accidental zeros in \( |\hat{u}(z_{\text{ref}})| \), \( \varepsilon \) also sets to zero the TF beyond some frequency, resulting in pulses in the IRFs that have width larger than theoretically expected for chosen \( o_{\text{max}} \). Another reason to limit \( o_{\text{max}} \) in the identification is to exclude in the observed IRFs the effects from details (elements) not captured by the model (e.g. effects of bending deformation).

### 3. RESULTS AND ANALYSIS

Results are shown of simulated IRFs (0–50 Hz) of a model of a nine-story RC building and of identified shear wave velocity profiles of Millikan Library NS response from recorded response during Yorba Linda earthquake of 2002, using the direct algorithm. The model IRFs aim to illustrate the effect of layering on the pulse propagation and IRFs, and the use of reflected pulses for identification. Furthermore, it is examined if the direct algorithm, which is based on simplifying assumptions, is valid for buildings, and if the pulse amplitudes can be used to identify the structural damping. The results of the identification aim to show the magnitude of the identification error for the direct algorithm, for a real building and data, as function of the detail of the model fitted.

#### 3.1. Simulated broadband impulse response functions

To avoid unnecessary complexity, a three-layer model of a nine-story RC building, with three storeys per layer, is chosen, with mass density and shear wave velocity profiles as shown in Figure 2 (the solid lines). The dashed lines show the profiles for an equivalent uniform model approximation, which has same travel time from level ground to roof. Without loss of generality, the model corresponds to Millikan Library NS response. Tables I and II show the layer parameters, ordered from top to bottom.
The theoretical domain travel time $t_i = h_i / b_i$ is also shown, as well as the sum of the domain travel times $\sum t_i$. The quality factor $Q = 25(\zeta = 2\%)$.

### 3.1.1. Pulse travel time

Impulse response functions for the three-layer model are shown in Figure 3. In part Figure 3a, the virtual source is at ground level (point of energy entry), and all pulses are causal. Most prominent is the pulse that is transmitted through all of the layer interfaces. It arrives at the roof first at $t = 0.1\, \text{s}$, where it is reflected with doubled amplitude, and propagates back to ground level. The pulses reflected from the interface between the top and middle layers can also be clearly seen, especially the one reflected back to the top layer. It has opposite sign because it reflects from a stiffer layer and arrives at the roof at $t = 0.2\, \text{s}$. The top layer can be identified from its time shift $\tau$ relative to the first arrival of the transmitted pulse at the roof, which is $0.1\, \text{s}$, and equals twice the travel time through the top layer, $2h_1 / \beta_1$. This gives $\beta_1 = 2 \times 12.8 / 0.1 = 256\, \text{m/s}$, which is close to the input value ($242\, \text{m/s}$).

In Figure 3b, the virtual source is at the roof, not coinciding with the physical source. Therefore, there is an acausal transmitted pulse propagating downwards, which is due to the physical wave propagating upwards, in addition to a causal pulse propagating downwards [12]. As in the case of a uniform model, the transmitted pulse does not reflect from the base (because all reflections from the roof are suppressed) [12]. Similarly, the pulses from the internal reflections also all propagate only downward and are never reflected back. The reflections of the acausal pulse, which physically propagate downward, are all causal. In contrast, the reflections of the causal pulse, which physically propagate upward, are all acausal. For example, the pulse that is physically reflected back into the top layer now is an acausal pulse propagating through the middle layer and can be used to identify

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**Table I. Input parameters for the equivalent uniform model**

<table>
<thead>
<tr>
<th>Layer</th>
<th>Floors</th>
<th>$h_i$ (m)</th>
<th>$\rho_i$ (kg/m$^3$)</th>
<th>$\beta_i$ (m/s)</th>
<th>Domain travel time $t_i = h_i / \beta_i$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-9</td>
<td>39</td>
<td>496</td>
<td>390</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**Table II. Input parameters for the three-layer model**

<table>
<thead>
<tr>
<th>Layer</th>
<th>Floors</th>
<th>$h_i$ (m)</th>
<th>$\rho_i$ (kg/m$^3$)</th>
<th>$\beta_i$ (m/s)</th>
<th>Domain travel time $t_i = h_i / \beta_i$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7-9</td>
<td>12.8</td>
<td>526</td>
<td>242</td>
<td>0.0529</td>
</tr>
<tr>
<td>2</td>
<td>4-6</td>
<td>12.8</td>
<td>473</td>
<td>569</td>
<td>0.0225</td>
</tr>
<tr>
<td>3</td>
<td>1-3</td>
<td>13.4</td>
<td>490</td>
<td>536</td>
<td>0.0250</td>
</tr>
</tbody>
</table>

$\sum_{i=1}^{\mu} t_i = 0.103\, \text{s}$. The sum of the domain travel times $\sum_{i=1}^{\mu} t_i$.

---

Figure 2. Profiles of mass density and shear wave velocity distributions for the three-layer (solid line) and the uniform model (dashed line).
the middle layer, as follows. In the IRF at the interface between bottom and middle layers, its time shift relative to the arrival of the transmitted causal pulse is \( t = 2h_2/\beta_2 \) and can be used to find \( \beta_2 \).

Another observation is that the pulses from the internal reflections all occur in the IRFs between the transmitted acausal and causal pulses. As shown on the model IRFs, the top and middle layer can be resolved as separate layers even if there is no recording at their interface. However, in practice, this can be done only if the reflected pulses can be distinguished from the ‘noise’, i.e. from the side lobes of the transmitted and other reflected pulses, as well as pulses because of foundation rocking [18].

Figure 3. Impulse response functions at each floor for the three-layer model, for virtual source (a) at ground floor and (b) at roof.
Figure 4 shows the pulse arrival times at the layer interfaces, as measured from the IRFs. The differences between the arrival times at different floors closely match the domain travel times and the total travel time (Tables I and II). This demonstrates that the direct identification algorithm, which is based on the approximate ray theory interpretation of the IRFs, is valid for layered building models in which the layers correspond to a group of floors (such as used in [16,17,20]). However, this is not the case for layered models with slabs (modeled as thin and heavy layers between soft layers). Our analysis (not presented here because of space limitation) shows that for such models, the pulse travel time is larger than the sum of the domain travel times, indicating additional phase delays because of scattering of the pulse from the slabs. The direct algorithm does not work for this case because the assumption of smoothness of the variation of material properties across the model is violated.

3.1.2. Pulse amplitudes. Next, we analyze the attenuation of pulse amplitudes as function of the IRF bandwidth \( f_{\text{max}} \). Let \( A^a \), \( A^{\text{top}} \), and \( A^c \) be the amplitudes of the acausal pulse at ground level, half of the amplitude of the source pulse at the roof, and the amplitude of the causal pulse at ground level, respectively. Figure 5a shows ratios \( A^{\text{top}}/A^a \), \( A^c/A^{\text{top}} \), and \( A^c/A^a \) versus \( f_{\text{max}} \) for the three-layer model, which are measures of the pulse attenuation on the way up, on the way down, and the total attenuation, respectively. It can be seen that the pulse is strongly amplified on its way up (\( A^{\text{top}}/A^a > 1 \)) and deamplified on its way down (\( A^c/A^{\text{top}} < 1 \)), which indicates that the pulse amplitude is governed primarily by the impedance of the layers, and to a minor degree by the material attenuation. The ratio \( A^c/A^a \), however, is a measure of the pulse attenuation because of \( Q \) because the effect of the impedance cancels out over the total path (up and down), and the model has no foundation rocking [18]. Figure 5b shows \( A^c/A^a \) versus \( f_{\text{max}} \) for the three-layer and equivalent uniform models, which are
fixed base, and also for the uniform model with foundation rocking [18]. It can be seen that the pulse attenuation because of $Q$ is practically identical for the uniform and three-layer models and increases with $f_{\text{max}}$, as expected from the assumed attenuation model. However, the attenuation for the model with rocking, which has same $Q$ in the structure, is much larger, demonstrating that the pulse amplitudes in IRFs reflect the combined attenuation—because of $Q$ in the building and because of radiation damping via foundation rocking. This makes it impossible, in general, to identify $Q$ of the fixed-base structure from IRFs, except for building with very large foundation rocking stiffness.

3.2. Identification of Millikan Library NS response

Millikan Library (Figure 6) is a nine-story RC building in Pasadena, CA, USA, instrumented over a period of 40 years, and tested extensively, in particular for soil–structure interaction studies [12,19,35,37–43]. The building is $21 \times 23$ m in plan and vertically extends 43.9 m above grade and 48.2 m above basement level (Figure 6). Resistance to lateral forces in the NS direction is provided by RC shear walls on the east and west sides of the building. The RC central core houses the elevators and provides resistance to lateral forces in the EW direction. The local soil can be characterized as alluvium, with average shear wave velocity in the top 30 m of about 300 m/s, and depth to ‘bedrock’ of about 275 m [35,39–41]. Published work suggests uniform mass distribution over the first three, middle three, and top three storeys, which we use to construct our models [35].

Figure 6. Millikan Library: (a) photo (courtesy of M. Trifunac), (b) vertical cross section and (c) typical floor layout (redrawn from [12]), and (d) sensor locations at basement.
Yorba Linda earthquake of September 3, 2002 (M = 4.8, epicentral distance R = 40 km), was recorded using a dense network of sensors (Figure 6c and d). The building response was small, with maximum rocking angle of $0.012 \times 10^{-3}$ rad. Figure 7 shows the observed NS response (at west wall): (a) the recorded acceleration time histories (low pass filtered at 25 Hz); (b) the corresponding IRFs for $f_{\text{max}} = 15$ Hz and 25 Hz (solid and dashed lines); and (c) the TF between roof ground floor accelerations. As seen in Figure 7c, mostly the first two modes contribute to the recorded response, and the TF amplitudes beyond 15 Hz are very small. This reflects on the widths of the pulses in Figure 7b, which are practically the same for the different bandwidths, while theoretically should have differed by factor of 1.7, and suggests that the effective bandwidth $f_{\text{max}}$ for these records is about 15 Hz, much smaller than the capability of the recording instruments (typically 25–50 Hz [36]). This effective $f_{\text{max}}$ is critical for the resolving power of the IRFs. As seen in Figure 7b, the transmitted causal and acausal pulses are too wide relative to their time shifts and cannot be resolved at the ninth floor, even though there is a sensor there. It is also seen that no reflected pulses can be resolved in the observed IRFs.

Three models with different spatial resolution were fitted using the direct algorithm: one-layer, three-layer (with three floors per layer), and eight-layer model in which the top layer consists of the eighth and ninth floors. The assumed mass distribution is same as in Figure 2 [35], and $Q = 25$.
Table III. Shear wave velocities of Millikan Library NS response identified by the direct algorithm based on one-layer model, using accelerations of Yorba Linda earthquake of 2002 recorded at west wall (0–15 Hz)

<table>
<thead>
<tr>
<th>Layer</th>
<th>Floors</th>
<th>$h_i$ (m)</th>
<th>$\bar{z}_i$ (m)</th>
<th>Observed $\tau_i$ (s)</th>
<th>Identified $\beta_i$ (m/s)</th>
<th>Predicted $\tau_i$ (s)</th>
<th>$\Delta\tau/\tau$ (%)</th>
<th>$\Delta\beta/\beta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1–9</td>
<td>38.9</td>
<td>36.45</td>
<td>0.096</td>
<td>405.2</td>
<td>0.097</td>
<td>1.04</td>
<td>−1.04</td>
</tr>
</tbody>
</table>

Table IV. Shear wave velocities of Millikan Library NS response identified by the direct algorithm based on a three-layer model, using accelerations of Yorba Linda earthquake of 2002 recorded at west wall (0–15 Hz)

<table>
<thead>
<tr>
<th>Layer</th>
<th>Floors</th>
<th>$h_i$ (m)</th>
<th>$\bar{z}_i$ (m)</th>
<th>Observed $\tau_i$ (s)</th>
<th>Identified $\beta_i$ (m/s)</th>
<th>Predicted $\tau_i$ (s)</th>
<th>$\Delta\tau/\tau$ (%)</th>
<th>$\Delta\beta/\beta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7–9</td>
<td>12.75</td>
<td>6.38</td>
<td>0.048</td>
<td>265.63</td>
<td>0.045</td>
<td>−6.3</td>
<td>6.3</td>
</tr>
<tr>
<td>2</td>
<td>4–6</td>
<td>12.75</td>
<td>19.13</td>
<td>0.025</td>
<td>510</td>
<td>0.0285</td>
<td>14</td>
<td>−14</td>
</tr>
<tr>
<td>3</td>
<td>1–3</td>
<td>13.4</td>
<td>32.2</td>
<td>0.023</td>
<td>582.61</td>
<td>0.0215</td>
<td>−6.5</td>
<td>6.5</td>
</tr>
</tbody>
</table>

($\zeta = 2\%$), which is close to the first mode apparent (soil–structure interaction system) damping $\zeta_{app} = 1.74\%$ as identified from the TF. The average of the time shifts for the causal and acausal pulses was used, as our study of several buildings showed that it leads to consistently smaller identification error. The goodness of fit is assessed from the general agreement between the predicted by the identified model and the observed pulse arrival times at the different levels, and travel times through the layers. In particular, the relative error in pulse travel time was computed, $\Delta\tau/\tau = (\tau_{pred} - \tau_{obs})/\tau_{obs}$, where $\tau_{pred}$ and $\tau_{obs}$ are the predicted by the model and the observed travel times through the layer, respectively. This enabled to estimate the relative error in the identified $\beta$ as $\Delta\beta/\beta = -\Delta\tau/\tau$ (see the methodology section). The different resolution models were fitted for data and model bandwidth $f_{max} = 15$ Hz. The highest resolution model (eight layers) was also fitted for $f_{max} = 25$ Hz. The results are shown in Tables III–V and Figure 8. The different columns in the tables show the layer number (top to bottom), width $h_i$, average $z$-coordinate $\bar{z}_i$ (Figure 1), observed $\tau_i$, identified $\beta_i$, predicted $\tau_i$, and the errors $\Delta\tau/\tau$ and $\Delta\beta/\beta$. Figure 8 shows the identified $\beta$ profiles (a) and the agreement of the observed and predicted pulse arrival times (b) and TFs (c). The agreement of TFs is an indicator of how realistic the fitted model is.

The results confirm that the identification error $\Delta\beta/\beta$ is larger for more detailed models (eight layers in this case) than for coarser models (one layer and three layers in this case) as predicted theoretically in the methodology section (Figure 8b). The error for the identified eight-layer model is considerable

Table V. Shear wave velocities of Millikan Library NS response identified by the direct algorithm based on 8-layer model and using accelerations of Yorba Linda earthquake of 2002 recorded at west wall

<table>
<thead>
<tr>
<th>Layer</th>
<th>Floor</th>
<th>$h_i$ (m)</th>
<th>$\bar{z}_i$ (m)</th>
<th>Observed $\tau_i$ (s)</th>
<th>Identified $\beta_i$ (m/s)</th>
<th>Predicted $\tau_i$ (s)</th>
<th>$\Delta\tau/\tau$ (%)</th>
<th>$\Delta\beta/\beta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–15 Hz</td>
<td>1</td>
<td>8</td>
<td>8.50</td>
<td>4.25</td>
<td>0.04</td>
<td>212.5</td>
<td>0.042</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>4.25</td>
<td>10.63</td>
<td>0.008</td>
<td>531.3</td>
<td>0.0015</td>
<td>−81</td>
<td>81</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>4.25</td>
<td>14.88</td>
<td>0.0055</td>
<td>772.7</td>
<td>0.002</td>
<td>−64</td>
<td>64</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4.25</td>
<td>19.13</td>
<td>0.008</td>
<td>531.3</td>
<td>0.019</td>
<td>137</td>
<td>−137</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>4.25</td>
<td>23.38</td>
<td>0.0115</td>
<td>369.6</td>
<td>0.014</td>
<td>22</td>
<td>−22</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>4.25</td>
<td>27.63</td>
<td>0.0055</td>
<td>772.7</td>
<td>0.015</td>
<td>−73</td>
<td>73</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>4.25</td>
<td>31.88</td>
<td>0.005</td>
<td>850.0</td>
<td>0.001</td>
<td>−80</td>
<td>80</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>4.90</td>
<td>36.45</td>
<td>0.0125</td>
<td>392.0</td>
<td>0.025</td>
<td>100</td>
<td>−100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Layer</th>
<th>Floor</th>
<th>$h_i$ (m)</th>
<th>$\bar{z}_i$ (m)</th>
<th>Observed $\tau_i$ (s)</th>
<th>Identified $\beta_i$ (m/s)</th>
<th>Predicted $\tau_i$ (s)</th>
<th>$\Delta\tau/\tau$ (%)</th>
<th>$\Delta\beta/\beta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–25 Hz</td>
<td>1</td>
<td>8</td>
<td>8.50</td>
<td>4.25</td>
<td>0.0395</td>
<td>215.2</td>
<td>0.04</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>4.25</td>
<td>10.63</td>
<td>0.0075</td>
<td>566.7</td>
<td>0.003</td>
<td>−60</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>4.25</td>
<td>14.88</td>
<td>0.006</td>
<td>708.3</td>
<td>0.0055</td>
<td>−8.3</td>
<td>8.3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4.25</td>
<td>19.13</td>
<td>0.0085</td>
<td>500.0</td>
<td>0.0155</td>
<td>82</td>
<td>−82</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>4.25</td>
<td>23.38</td>
<td>0.0111</td>
<td>386.4</td>
<td>0.007</td>
<td>−37</td>
<td>37</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>4.25</td>
<td>27.63</td>
<td>0.0055</td>
<td>772.7</td>
<td>0.004</td>
<td>−27</td>
<td>27</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>4.25</td>
<td>31.88</td>
<td>0.005</td>
<td>850.0</td>
<td>0.0055</td>
<td>10</td>
<td>−10</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>4.90</td>
<td>36.45</td>
<td>0.013</td>
<td>376.9</td>
<td>0.0155</td>
<td>19</td>
<td>−19</td>
</tr>
</tbody>
</table>
(Table V), and the three-layer model is the optimal for the direct algorithm and this building, both in terms of detail and accuracy. Further development of this method of identification by fitting IRFs is necessary to improve its accuracy for detailed models, e.g. by introducing iteration and/or least squares fit.

In examining the agreement of TFs (Figure 8c), it is important to note that the observed TFs are those of the soil–structure system (without foundation translation but with foundation rocking), whereas the model TFs are those of the fixed-base structure. Furthermore, the modal frequencies are sensitive to the variation of shear wave velocity and mass density with height, because of which more detailed models would predict better the modal frequencies than coarser models, but only if they are accurate. (In contrast, the total travel time depends on the properties of the layers but not on their order and would be the same for any order of the layers.) Figure 8c shows that the frequency of the fundamental mode is significantly higher for all the models (2.5–3 Hz) than for the observed response (~1.7 Hz), as can be expected for this building NS response for which it is known that foundation rocking contributes considerably to the total response (about 30%) and the effects of the soil–structure interaction are significant [19,39–41]. The predicted fixed-base fundamental frequency agrees generally with other studies [39–41] and disagrees with [12]. The frequencies of the higher modes are not affected much by soil–structure interaction, and their agreement can be used as a measure of how realistically the identified models represent the building. Figure 8c shows that the predicted frequency of the second mode differs between the three models but is close to the observed one, which indicates that the models, identified solely from wave travel times, represent realistically the dynamic behavior of the fixed-base building.

3.3. Uncertainty in pulse localization and identification error

Figure 9 further clarifies how the uncertainty in the localization of the pulses in the IRFs affects the accuracy of the identification. It compares the observed pulse arrival times at each floor in the IRFs...
of the one-layer model (Table III), (the symbols connected by a solid line) with the physical arrival time (the straight line). The properties are as in Table III, and $f_{max} = 15$ Hz. As it can be seen, the symbols do not lie on the straight line but are scattered around it. The error is considerable even for this simple model with constant impedance and no internal reflections and is much larger than what is inferred from the precision of the readings of the time of the peaks of the pulses. In principle, this error occurs because the time of the peaks of the pulses does not coincide with the true, physical pulse arrival time (consequence of Heisenberg-Gabor uncertainty principle). In this case, it is practically realized by the interference of the main lobe of each pulse with the side lobes of the other pulse, which leads to some distortion of the main lobes and shift of the pulse peak time. In general, pulses from internal reflections and foundation rocking [18] further contribute to the error. These errors are present in IRFs of real data and lead to significant relative error in the identified shear wave velocities if the layer thickness is small and if the data effective bandwidth is small.

4. DISCUSSION AND CONCLUSIONS

The most important findings of this study are the following: (1) The resolving power of IRFs is not only limited by the density of sensors but also by the effective bandwidth of the data, which determines the width of the pulses in the IRFs. This is a consequence of the Heisenberg-Gabor uncertainty principle. (2) There is a trade-off between detail and accuracy of the identified velocity profile, which also follows from the same principle. The error is smaller for smoother models (with smaller number of thicker layers). (3) The pulse amplitudes are affected by the soil–structure interaction (through foundation rocking) and cannot be used alone to determine the structural damping. (4) In principle, reflected pulses can also be used for identification, compensating for lack of sensors at some floors. However, in practice, this would be possible only if the effective data bandwidth is sufficiently large. (5) The application of the direct identification algorithm to Millikan Library NS response produced robust and realistic results. (6) For this building, the direct algorithm is optimal for a three-layer model, in terms of both detail and accuracy (the error $\Delta/\beta \approx 6\%$ in the top and bottom layers and 14% in the middle layer). (6) The error of this algorithm in identifying the detailed models model (one story per layer) is large in some of the layers and is unacceptable for structural health monitoring.

These results and analysis in this paper confirm that the wave method for structural system identification and health monitoring using IRFs is robust when applied to real buildings and data, insensitive to the effects of soil–structure interaction (except for the damping) and local in nature, and therefore is promising for actual implementation in structural health monitoring systems. However, further developments are needed to increase its accuracy at high resolution so it can be effective in detecting localized damage. Such developments are, e.g., inclusion of information on the pulse amplitudes, in addition to the time shifts, and iterative optimization schemes to minimize the error in all layers. We report on such an algorithm in [44].
ACKNOWLEDGEMENTS

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