DYNAMICS MODELING AND SIMULATION
OF A KIND OF WHEELED HUMANOID ROBOT
BASED ON SCREW THEORY

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Based on the screw theory and Lie group notations, this paper presents a modeling method for a
kind of wheeled humanoid robot whose upper human-like body is mounted on the top of a
mobile platform with three wheels. By combining the reciprocal product of the twist and wrench
with Jourdain variation principle, a general formulation method is proposed to model the whole
system’s dynamics that represents directly the relationship between the input and the resultant
external and inertial wrench. Both the system kinematics and dynamics are derived carefully.
The simulations are made to verify the proposed modeling methodology and the simulation
results are also compared with the results obtained from the multi-body dynamics software.

Keywords: Dynamics modeling; wheeled humanoid robot; nonholonomic constraints; screw
theory; twist and wrench.

1. Introduction

Regarding the various structures of the humanoid robots, one kind of general type is
the biped humanoid robot walking with two legs, like ASIMO1 and HRP,2 which
have been developed mainly to investigate the fundamental functions of biped
locomotion in terms of walking and jumping. Another kind of humanoid robot uses a
mobile platform instead of legs, which is called the Wheeled Humanoid Robot
(WHR).3 Considering its advantage in dexterity, stability and adaptivity, the WHR
has been widely used as a kind of service robot in indoor environment. There
are some successful examples for WHRs, which usually adopt the structure of

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the human-like upper body mounted on a mobile platform, such as HERMES, Robonaut, ARMAR, HARO-1, etc.

With respect to controlling the cooperative motion of two arms and the interactive motion of the arms with the mobile platform, the related works can be found in literature. Yamamoto and Yun studied the effect of the dynamic interactions between the manipulator and the mobile platform for a mobile manipulator on the task performance, thereafter developed a nonlinear feedback control algorithm that completely compensated the dynamic interactions. Tanner and Kyriakopoulos studied the interaction forces between the manipulator base and the vehicle and assumed them as a virtual relative movement for these forces which were useful in determining and maintaining contact stability of the wheels, although they did not contribute to the dynamics. Cheng and Tsai designed a tracking controller with fully dynamic compensation ability for the nonholonomic wheeled mobile manipulator via Lyapunov stability theory.

The dynamics modeling and analysis of the mobile manipulators will provide a solid basis for the study of the dynamic interaction of coupled systems, stability analysis, model-based control, and numerical implementation and simulation. The Lagrange method is mostly applied for the modeling of the whole system which is based on the work energy of the system, and its advantage is that the constraint or workless forces will be eliminated from the equation of motion (EOM), but it has high order O(N4). Efficient low-order algorithms are sought and the Newton–Euler formulation is found the way to the problem, which is the direct interpretation of Newton’s second law and the constraint forces appeared in the EOM. Luh et al. developed a very popular and efficient recursive Newton–Euler algorithm by referring most quantities to link coordinates. Li and Liu presented a method to establish the dynamics model of a mobile modular manipulator, then a neural-fuzzy control method was utilized to perform simulation. However, efficient recursive Newton–Euler formulation for open chain multi-body dynamics is not suitable for application in model-based controller design, which is based on the closed form dynamics model of dual manipulators as shown in Fig. 1.

Ploen and Park established the dynamics of cooperating robot systems using general notations from the theory of Lie groups, and the resulting closed-form equations provide a high-level description of the forward and inverse dynamics of cooperating robot systems that reduce the symbolic complexity without sacrificing computational efficiency. However, the whole arm manipulation and the mobile base are not discussed. Considering an omni-directional mobile base and the chassis without waist and neck parts, Qiu and Cao adopted the reciprocal product of twist and wrench, the variation form of Jourdain principle and Newton–Euler equation to formulate the self-contained motion equation of the omni-directional mobile manipulator system in such manipulation task as whole arm and cooperating manipulation.

As shown in Fig. 2, the potential applications for a dual-arm system with whole arm manipulation or cooperating manipulation in manufacturing and in robot-assisted applications have been approached by some researchers. Given a
Fig. 1. A series of wheeled humanoid robots compared to the biped humanoid robots.

Fig. 2. A scenario of the wheeled mobile robot doing the cooperative work.
multiple-loop closed chain, a method was proposed by Lee et al. who first converted it to a reduced system by selectively detaching some of the passive joints or cutting the loops into tree-topology structures, then the dynamics of the corresponding reduced system could be solved. Murray and Lovell presented a method for dynamic modeling of manipulators containing closed kinematic chains based on d’Alembert’s principle. Considering a general cooperating robot system with $N$ kinematic chains to manipulate a common workpiece, Ploen and Park assumed all contacts between the tips of the chains and the workpiece to be frictionless point contacts, hence the cooperating robot system was equivalent to $N$ open kinematic chains subjected to $N$ unknown constraint forces at the tip of each chain. A robust control method was developed for a planar dual-arm manipulator system, where contact and friction constraints for grasping conditions were considered.

In this paper, a WHR topology structure is shown in Fig. 3, which is configured with nonholonomic wheeled driven base, the chassis with two-DOF waist joints, the
head with two-DOF neck joints and dual redundant manipulators. For better understanding dynamic interaction of subsystems and further studying the application and control methods of this WHR system with complex topology structure, we need a geometrically intuitive closed form dynamics model. Considering the convenience of the task description and the large mobility space of the WHR, the left invariant body screw coordinate representation is adopted. By using the tool of Lie group and screw, a geometrical model can be obtained conveniently. The dynamic interactions and coupling of subsystems of the WHR exhibit clear form with intuitive physical meaning. At the same time, the numerical implementation efficiency and reduced symbolic complexity can be achieved by factorizing coefficient matrix and its subcomponents.

This paper is organized as follows. The system description of the wheeled humanoid robot is presented in Sec. 2. Kinematics analysis for the WHR system is performed in Sec. 3, and dynamics modeling and its theoretical foundation will be constructed in Sec. 4. The proposed modeling method applied to the WHR system is given in Sec. 5. Some simulations are made by our method and compared to the simulation results solved by the software of ADAMS. The paper is concluded in Sec. 6.

2. System Modeling of the Mobile Humanoid Robot

The configuration of a mobile humanoid robot is such that the upper human-like body is mounted on a top of mobile platform, which has two seven-DOF arms as shown in Fig. 1. In order to realize easy modeling, the whole upper body system can be divided into five subsystems in terms of mobile platform, chassis with waist joints, head with neck joints, and dual arms. For conveniently conducting the analysis and modeling of the robot, five kinematic chains are introduced in Fig. 3.

- **The serial chain of chassis**: From the global frame $G$ to the frame of the platform $C$;
- **The chassis with waist joints**: From the frame of the platform $C$ to the frame of the shoulder $M$;
- **The chain of the head**: From the frame of the shoulder $M$ to the frame of the head $H$;
- **The chain of the left arm**: From the frame of the shoulder $M$ to the frame of the left arm $i$th joint $L_i$;
- **The chain of the right arm**: From the frame of the shoulder $M$ to the frame of the right arm $i$th joint $R_i$.

Two assumptions are made in the modeling of the wheeled humanoid robot system:

- Two seven-DOF arms are installed laterally and symmetrically, in which each link of the robot is rigid;
- There are no slipping and sideways between the wheels and the floor.
The notations shown in Fig. 3 will be used in the derivation of the kinematics and
dynamics of the WHR, and their detailed information can be found in our previous
work. Some notations like \( s \varphi = \sin \varphi \), \( c \varphi = \cos \varphi \) are with the similar forms in the
following parts:

\[
G(O - X_o Y_o Z_o): \text{Global reference frame};
\]

\[
C(O_c - X_c Y_c Z_c), \quad e_i(O_i - X_i Y_i Z_i) \quad (i = 1, 2, 3), \quad e_b(O_b - X_b Y_b Z_b): \text{Body-fixed}
reference frame of the chassis, wheels and bracket;
\]

\[
M(O_m - X_m Y_m Z_m), \quad L_i(O_{Li} - X_{Li} Y_{Li} Z_{Li}), \quad R_i(O_{Ri} - X_{Ri} Y_{Ri} Z_{Ri}): \text{Body-fixed}
reference frame of the shoulder, the \( i \)th link of the left arm and right arm
(\( i = 1, 2, \ldots, n \));
\]

\( o_i \): Center of the \( i \)th wheel (\( i = 1, 2, 3 \));

\( \theta_i \): Rotation angle of the \( i \)th wheel (\( i = 1, 2, 3 \));

\( \eta_l, \eta_r, \eta_m, \eta_b \): Rotation angles of the joints in the left arm, right arm, the waist
and the neck;

\( r, r_c \): Radius of the driving wheels and caster wheel;

\( \varphi \): Rotating angle of frame \( C \) relative to frame \( G \);

\( \psi \): Rotating angle of frame \( e_b \) relative to frame \( C \);

\( 2l \): Distance between the two driving wheels.

3. Kinematic Analysis of the WHR System

Since the WHR has a kind of tree topological structure, it can be treated as a spatial
open-loop multi-body system. By applying recursive computation and reusing of the
components of the coefficient matrices, the kinematic parameters of any part or link
in terms of independent relative joint coordinates and their derivatives in the
chain or loop can be derived by the transformation from the reference frame of the
chassis \( C \).

In this section, the kinematics of the proposed WHR is deduced comprehensively
in left invariant form, and some related fundamental Lie group and screw theory can
be found in [25]. From the planning pose matrix of the body fixed frame \( C \) relative to
the global frame \( G \), the twist of the frame \( C \) has the following form:

\[
V_c = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{6 \times 1} = \begin{bmatrix} 0 & -\varphi & 0 & v_z^x \\ \varphi & 0 & 0 & v_z^y \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \varphi & 0 & 0 & 0 \end{bmatrix}_{6 \times 6} \begin{bmatrix} v_x^e \\ v_y^e \\ v_z^e \\ \omega_x^e \\ \omega_y^e \\ \omega_z^e \end{bmatrix}_{6 \times 1}.
\] (1)

\( ^a\) Similar to Murray
\( ^25\) we define the \( \vee \) operator to extract the six-dimensional twist coordinates which parameterize a twist.
The instantaneous tangential velocities of two driving wheels in the body frame $C$ can be expressed as:

\[
v_{o_1}^y = -l \dot{\varphi} + v_c^y = -r \dot{\theta}_1, \quad v_{o_2}^y = l \dot{\varphi} + v_c^y = -r \dot{\theta}_2.
\]

Due to the nonholonomic constraints, the velocity $v_{o_1}^x$ in the direction of $o_1o_2$ is zero and the relationship is:

\[
v_{o_1}^x = a \dot{\varphi} + v_c^x = 0.
\]

Let $\dot{\theta}_a = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$, and by combining (2) and (3), the velocity of the chassis $V_C$ in the body-fixed frame can be expressed as:

\[
V_c = \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = P_{p} \dot{\theta}_a,
\]

where $P_p \in \mathbb{R}^{6 \times 2}$, the spatial form is $P_p = \frac{r}{2l} \begin{bmatrix} -a & -l & 0 & 0 & 0 & 1 \\ a & -l & 0 & 0 & 0 & -1 \end{bmatrix}^T$.

By differentiating (4), the body acceleration of the chassis in the body-fixed frame will have the following form:

\[
\dot{V}_c = \begin{bmatrix} \dot{v}_c \\ \dot{\omega}_c \end{bmatrix} = P_{p} \ddot{\theta}_a.
\]

The body twist $V_{e_i}$ of the wheel fixed frame $e_i$ relative to $G$ can be obtained through the transformation of the $V_c$ and $V_{e_i}$,

\[
V_{e_i} = Ad_{g^{-1}} V_C + V_{ce_i}.
\]

With the chassis velocity $V_C$ and the transformation matrix $Ad_{g^{-1}} = \begin{bmatrix} R_{ce_1}^T & -R_{ce_2}^T \hat{p}_{o_1} \\ 0 & R_{ce_2}^T \end{bmatrix}$, the twist of the driving wheels $V_{e_i}(i = 1, 2)$ in the body-fixed frames will be

\[
V_{e_i} = \begin{bmatrix} v_{e_i} \\ \omega_{e_i} \end{bmatrix} = Ad_{g^{-1}} P_{d} \ddot{\theta}_a + B_d \dot{\theta}_i = P_{d} \ddot{\theta}_a + B_d \dot{\theta}_i,
\]

where $P_{d} \in \mathbb{R}^{6 \times 2}$,

\[
P_{d1} = \begin{bmatrix} 0 & -r & 0 & -\frac{r}{2l} & 0 & 1 \\ 0 & 0 & 0 & \frac{r}{2l} & 0 & 0 \end{bmatrix}^T \quad \text{and} \quad P_{d2} = \begin{bmatrix} 0 & 0 & 0 & -\frac{r}{2l} & 0 & 0 \\ 0 & -r & 0 & \frac{r}{2l} & 0 & 1 \end{bmatrix}^T.
\]

In a similar way, the body twist of the bracket frame $e_b$ relative to the frame $C$ in the body-fixed reference frame is written as

\[
V_{e_b} = \begin{bmatrix} v_{e_b} \\ \omega_{e_b} \end{bmatrix} = Ad_{g^{-1}} P_{d} \ddot{\theta}_a + B_b \ddot{\psi} = P_{d} \ddot{\theta}_a + B_b \ddot{\psi},
\]
where $P'_{b}$ has the following form:

$$P'_{b} = \frac{r}{2l} \begin{bmatrix} (a + b) & -l & 0 & 0 & -\rho_1 c\psi - \rho_2 s\psi \\ a + b & -l & 0 & 0 & \rho_1 c\psi - \rho_2 s\psi \end{bmatrix}^T.$$  

The body twist of the castor wheel frame $e_3$ can be obtained through the transformation of body velocities between the frame $e_b$ and the frame $e_3$, and the relationship can be expressed as:

$$V_{e_3} = P_{c} \dot{\theta}_{a},$$  

(9)

where

$$P_{c} = \frac{r}{2l} \begin{bmatrix} 0 & -rc(-\rho_3 s\psi + \rho_4 c\psi) & 0 & \rho_1 c\psi + \rho_2 s\psi & 0 & -\rho_3 s\psi + \rho_4 c\psi \end{bmatrix}^T.$$  

By differentiating (7), (8) and (9), the body accelerations of the bracket frame $\dot{V}_{e_3}$, two driving wheels $V_{e_3}$ and the caster wheel frame $\dot{V}_{e_3}$ will be obtained and the detailed forms can be found in our previous work.23

The proposed WHR has only three-DOF waist, two-DOF head and seven-DOF left and right arms. Here, in order to make this work more generally, the number of waist joints, head joints and two arms can be set as $n_w$, $n_h$ and $n_a$. For the arbitrarily chosen link in any kinematic chain as discussed in Sec. 2, the similar method is used. Here, one link of the left arm is illustrated, the configuration of the frame $L_i$ relative to the chassis frame $C$ has the following form:

$$g_{cl} = e^{\xi_{w_i} q_{w_i}} \cdots e^{\xi_{w_m} q_{w_m}} e^{\xi_{h_i} q_{h_i}} \cdots e^{\xi_{h_i} q_{h_i}} g_{cl}(0), \quad (i \leq 7),$$  

(10)

where $\xi_{w_i} \in R^6$ is the twist of one waist joint, $\xi_{h_i} q_{h_i}$ is the twist of the $i$th link frame in the left arm, $g_{cl}(0)$ is the initial configuration of the frame $L_i$ relative to the chassis frame $C$ as shown in Fig. 3.

The body twist of the $i$th link frame relative to frame $C$ is given by

$$V_{cl} = J_{cl}(q_{i}^l) \dot{q}_{i}^l,$$  

(11)

where $q_{i}^l = [q_{w_i} \cdots q_{w_m} q_{h_i} \cdots q_{h_i}]^T$, $J_{cl}$ is the body Jacobian corresponding to $g_{cl}$.25 $J_{cl}$ has the form $J_{cl}(q_{i}^l) = [\xi_{w_i}^\dagger \cdots \xi_{w_m}^\dagger \xi_{h_i}^\dagger \cdots \xi_{h_i}^\dagger 0 \cdots 0]$. And $\xi_{h_i}^\dagger$ is the instantaneous joint twist for the $i$th joint in its reference frame with respect to the chassis frame $C$, $\xi_{h_i}^\dagger = Ad_{\theta_{h_{i-1}}}(0)\xi_{h_{i-1}}^\dagger$.

So the body twist of the $i$th link frame can be written as

$$V_{l_i} = \begin{bmatrix} v_{l_i} \\ \omega_{l_i} \end{bmatrix} = Ad_{g_{cl}}^{-1} V_{c} + V_{cl},$$  

(12)

Combining (4), (11) with (12) together, the body twist of the $f$th link frame can be obtained by

$$V_{l_f} = Ad_{g_{cl}}^{-1} P_{p} \dot{\theta}_{a} + J_{cl}(q_{i}^l) \dot{q}_{i}^l = P_{l} \dot{\theta}_{a} + J_{cl}(q_{i}^l) \dot{q}_{i}^l,$$  

(13)

where $P_{l} \in R^{6 \times 2}$, and $P_{l} = Ad_{g_{cl}}^{-1} P_{p}$.
Differentiating (13), the body acceleration twist of $i$th link frame $\dot{\mathbf{V}}_l$ will be obtained. In a similar way, any arbitrarily chosen link in any kinematic chain will be derived.

**Remark 1.** The advantage to using the screws and twists for describing WHR multi-rigid-body kinematics is obvious, a global description of rigid body motion is allowed, which does not suffer from singularities due to the local coordinates, the product of exponentials representation provides a geometric description of rigid motion. Since Lie theory of screws provides a very concise and complete description, offers more insight than the coordinate-based traditional methods like DH-based kinematics representation, Lie groups and the geometrical interpretation of screws are utilized in the study of the motion of WHR multi-rigid-body system.

4. Dynamics of the WHR

The modeling of the wheeled mobile manipulators provides a basis for the study of stability analysis, feedback control design, and computer simulations. As introduced in Sec. 1, recursive Newton–Euler formulation, the direct interpretation of Newton’s 2nd law, is quite efficient for open chain multi-body dynamics, but it is not suitable to give a closed form dynamics model for multi manipulators manipulating a common workpiece. There is another approach known as the natural orthogonal complement to handle with the forward dynamics problem, which is based on the reciprocity relations between the constraint wrenches and the feasible twists of a manipulator. In this paper, based on the d’Alembert’s principle, we have applied the reciprocal product of twist and wrench to deduct the WHR dynamics.

4.1. **Inverse dynamics**

The inverse dynamics is essential for the computed-torque control of robotic manipulators. In the inverse problem, a time-history of either the Cartesian or the joint coordinates is given, and from knowledge of these histories and the architecture and inertial parameters of the system at hand, the torque or force requirements at the different actuated joints are determined as time-histories as well. Newton–Euler recursive algorithm is mostly applied to obtain the inverse dynamics of the multi-body manipulator system.

4.2. **Forward dynamics**

The forward dynamics is always required for the simulation and the real-time feedback control of the system. In the forward dynamics problem, current values of the joint coordinates and their first time-derivatives are known at a given instant, the time-history of the applied torques or forces is also known along with the architecture and the inertial parameters of the manipulator at hand. With these data, the values
of the joint coordinates and their time-derivatives are computed at a later sampling instant by integration of the underlying system of nonlinear ordinary differential equations.\textsuperscript{29}

For any rigid body of the WHR system in the arbitrarily chosen body-fixed frame, the inertial wrench and external wrench will keep balance at any instantaneous time and the external wrench includes not only the gravity forces but also the contact forces between the WHR system and the environment.

4.2.1. Preliminary knowledge

In consideration of the tasks of the WHR operated in most cases like grasping, picking and placing or doing the cooperative work by two arms, the coordinate frames of joints will change at every instant. The d’Alembert’s principle\textsuperscript{25,28} can be used in moving reference systems of multi-body systems. However, we prefer Jourdain principle, actually the second variation of d’Alembert’s principle, because it can be applied to eliminate the constraint forces from the dynamics equation for the nonholonomic multi-rigid-body system.\textsuperscript{30} There are detailed discussions and comparisons about d’Alembert’s principle and Jourdain’s principle both for linear and nonlinear nonholonomic constraint systems in literature.\textsuperscript{31}

According to the Jourdain principle,\textsuperscript{32} for an actual motion of a system constrained by ideal two-sided (restraining) constraints, the sum of the elements of work done by the active forces and inertial forces for arbitrary variations in the kinematically-possible velocities is zero at every moment of time, and the formulation of Jourdain principle is written as follows:

$$\delta W = \sum_i (F_i + C_i - m_i a_i) \cdot \delta r_i,$$

where the total forces are separated into applied forces $F_i$ and the constraint forces $C_i$, $\delta r_i$ is the virtual displacement of the system, consistent with the constraints, $m_i$ are the masses of the particles in the system, $a_i$ are the accelerations of the particles in the system, $m_i a_i$ together as products represent the time derivatives of the system momenta, and $i$ is an integer used to indicate (via subscript) a variable corresponding to a particular particle.

Considering the reciprocal product of twist and wrench (A wrench $F$ is said to be reciprocal to a twist $V$ if the instantaneous power is zero: $F \cdot V = 0$, see in \cite{25}) and the varied form of Jourdain’s principle, the dynamic equation of the multi-rigid-body systems can be rewritten as follows:

$$\sum_{i=1}^{n} (F_i + R_i - F^I_i) \cdot \delta V_i = 0,$$

where $F_i$ is the generalized external wrench (including active driving force/torque, dissipation force and external wrenches acted by the environment) applied on body $i$, $R_i$ is the ideal constraint wrench applied on body $i$ and $F^I_i$ is the inertial wrench of body $i$. $J$ is the Jacobian matrix of $i$th link in the body-fixed coordinate frame.
The generalized external wrench $F_i$ is decomposed into active driving wrench $F^A_i$, which is acted on the joint by driving torque/force $\tau_i$, dissipation wrench $F^C_i$ that is acted on the joint by friction and resistance $\varphi_i$, and the external wrench $F^E_i$, which is acted on the rigid link by the environment, including not only gravity, but also the external wrench acting on each body of the system by the environment. The ideal constraint force (or wrench) does not produce virtual work as stated in the Jourdain’s principle with the relationship $R_i \cdot \delta V_i = 0$, thus the further dynamics equation can be simplified as:

$$\sum_{i=1}^{n} (F^A_i + F^E_i + F^C_i - F^I_i) \cdot \delta V_i + \sum_{i=1}^{n} R_i \cdot \delta V_i = 0,$$

where $F^I \in \mathbb{R}^{6 \times 1}$ is the inertial wrench as defined in [25].

Let joint variables $q = [q_1, q_2, \ldots, q_n]^T \in \mathbb{R}^{n \times 1}$ represent the generalized coordinates of the multi-rigid body system. The virtual twist $\delta V_i$ has such a form as:

$$\delta V_i = J_i \cdot \delta \dot{q}_i,$$

where $J$ is the Jacobian matrix for the body-fixed frame of the $i$th link.

Substituting (17) into (16), the dynamics equation of the multi-rigid-body systems can be rewritten as:

$$\sum_{i=1}^{n} (\tau_i \cdot \delta \dot{q}_i + \varphi_i \cdot \delta \dot{q}_i + F^E_i \cdot \delta V_i - F^I_i \cdot \delta V_i) = 0.$$

4.2.2. Forward dynamics of the mobile platform

There are four kinds of subsystems in the mobile platform, two driving wheels, one caster wheel, one bracket and one chassis. Here, it is assumed that there is no active driving wrench on the bracket and the castor wheel since they are passive parts, but the dissipation wrench and the external wrench still exist on them. So the active driving wrench of the mobile platform can be expressed as: $F^A_i = [\tau_1 \quad \tau_2] \cdot \left[ \begin{array}{l} \delta \dot{\theta}_1 \\ \delta \dot{\theta}_2 \end{array} \right]$, and the dissipation wrench can be written as $F^C_i = [\varphi_1 \quad \varphi_2 \quad \varphi_b \quad \varphi_3] \cdot \left[ \begin{array}{cccc} \delta \dot{\theta}_1 & \delta \dot{\theta}_2 & \delta \psi & \delta \dot{\theta}_3 \end{array} \right]^T = [\varphi_1 \quad \varphi_2 \quad \varphi_b \quad \varphi_3] \cdot A \cdot \delta \dot{\theta}_a$, where $\dot{\theta}_a = \left[ \begin{array}{ccc} \dot{\theta}_1 & \dot{\theta}_2 & \dot{\psi} & \dot{\theta}_3 \end{array} \right]^T$.

The virtual power caused by the difference between the external wrench and inertial wrench has the following form:

$$\sum_{i=1}^{5} (F^E_i - F^I_i) \cdot \delta V_i,$$

---

b Kindly note that $\delta q$ is not the same as $\dot{q}$ because the generalized velocity $\dot{q}$ satisfies both the velocity constraints and the equations of motion, but the virtual displacement only satisfies the constraints.
where $F_E$ includes not only gravity, but also the external wrench acting on each body of the system by the environment. Also the dynamic interactions of the mobile platform caused by the motion of the upper body is included in the item of external wrench $F_E^i$.

From Sec. 3, some equations can be found that the twist equation of the chassis (4), $V_c = P_p \dot{\theta}_a$, the twist equation of driving wheels (7), $V_{e_i} = P_d \dot{\theta}_a$, the twist equation of the bracket (8), $V_{e_b} = P_b \dot{\theta}_a$, and the twist equation of castor wheel (9), $V_{e_3} = P_c \dot{\theta}_a$, so the $\delta V_i$ can be expressed as:

$$\delta V_i = J_p \delta \dot{\theta}_a = \begin{bmatrix} P_p \\ P_d \\ P_{d_i} \\ P_{e_i} \\ P_c \end{bmatrix}_{30 \times 2} \cdot \delta \dot{\theta}_a.$$  \hspace{1cm} (20)

Then the dynamics equation of the mobile platform has such a form:

$$\sum_{i=1}^{5} (F_i - F_i^l) \cdot \delta V_i = \sum_{i=1}^{5} \left( \tau_i + \varphi_i \right) \cdot \delta \dot{q}_i + \sum_{i=1}^{5} \left( F_{E_i}^E - F_{E_i}^l \right) \cdot \delta V_i$$

$$= \delta \dot{\theta}_a^T \cdot \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} + \delta \dot{\theta}_a^T \cdot A^T \cdot \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_b \\ \varphi_3 \end{bmatrix}$$

$$+ \delta \dot{\theta}_a^T \cdot J_p^T \cdot \sum_{i=1}^{5} (F_{E_i}^E - F_{E_i}^l),$$ \hspace{1cm} (21)

where the former part is the virtual power produced by active driving torque/force and dissipation force for the dynamics system, the latter part is the virtual power produced by external wrench and inertial wrench on the mobile platform.

From the kinematics equation as derived in the Sec. 3, the latter part of the mobile platform’s dynamics equation can be rewritten as

$$\sum_{i=1}^{5} (F_{E_i}^E - F_{E_i}^l) \cdot \delta V_i = \delta \dot{\theta}_a^T (T_{p1} + T_{p2})(F_{p}^E - F_{p}^l),$$ \hspace{1cm} (22)

where

$$T_{p1} = \begin{bmatrix} P_p^T \\ Ad_{g^{-1}(ce)g}^T \\ Ad_{g^{-1}(cc2)g}^T \\ Ad_{g^{-1}(cc3)g}^T \\ I_6 \end{bmatrix} \in R^{2 \times 30},$$

$$T_{p2} = \begin{bmatrix} \xi_{cc1}^T Ad_{g^{-1}(ce)g}^T & 0 & 0 & 0 \\ 0 & \xi_{cc2}^T Ad_{g^{-1}(cc2)g}^T & 0 & 0 \end{bmatrix} \in R^{2 \times 30},$$

$$F_{p}^E = \begin{bmatrix} F_{1}^E \\ F_{2}^E \\ F_{3}^E \\ F_{4}^E \\ F_{5}^E \end{bmatrix}^T \in R^{30 \times 1},$$

$$F_{p}^l = \begin{bmatrix} F_{1}^l \\ F_{2}^l \\ F_{3}^l \\ F_{4}^l \\ F_{5}^l \end{bmatrix}^T \in R^{30 \times 1}.$$
4.2.3. Forward dynamics of the upper body

There are four subsystems in the upper body, the waist, the head, left arm and right arm. Considering the generality of dynamic modeling method and the convenience of the expression, it is assumed that there are \( n_w \) DoFs in the waist, \( n_h \) DoFs in the head, and \( n_a \) DoFs in each arm respectively. Let \( n_s \) denote the sum of DoFs in the system and \( n_s = n_w + 2n_a + n_h \). In a similar way, the dynamics equation for the upper body can be obtained by

\[
\sum_{i=1}^{n_s} (F_i - F_i^I) \cdot \delta V_i = \sum_{i=1}^{n_s} (\tau_i \cdot \delta \dot{q}_{ui} + \varphi_i \cdot \delta \dot{q}_{ui} + F_i^E \cdot \delta V_i - F_i^I \cdot \delta V_i)
\]

\[
= \delta \dot{q}_T^T \sum_{i=1}^{n_s} (\tau_i + \varphi_i) + \sum_{i=1}^{n_s} (F_i^E - F_i^I) \cdot \delta V_i,
\]

(23)

where \( F_i^E \) includes not only gravity, but also the external wrench acting on each body of the system by the environment.

For an arbitrarily chosen joint in the upper body subsystem from Sec. 3, based on its twist equation (13), \( V_i = Ad_{g_i} P_i \dot{\theta}_a + J_{cl_i}(q_i) \dot{q}_i' = P_i \dot{\theta}_a + J_{cl_i}(q_i) \dot{q}_i' \), the \( \delta V_i \) can be expressed as:

\[
\delta V_i = T_i \left[ \begin{array}{c} \delta \dot{q}_i \\ \delta \dot{q}_{ui} \end{array} \right] = \left[ P_i \quad J_{cl_i} \right]_{q(n_s) \times (2+n_s)} \cdot \delta \dot{q}_{gi},
\]

(24)

where \( T_i \) is the diagonal matrix composed of \( P_i \) and \( J_{cl_i} \), \( q_{gi} \) represents the general joints of the upper body: \( q_{gi} = [\theta_1 \ \theta_2 \ q_{w1} \ \cdots \ q_{wn} \ q_{l1} \ \cdots \ q_{ln}]^T \).

Therefore, by combining (23) and (24), the dynamic equation of the WHR upper body can be rewritten as

\[
\sum_{i=1}^{n_s} (F_i - F_i^I) \cdot \delta V_i = \delta \dot{q}_T^T \sum_{i=1}^{n_s} (\tau_i + \varphi_i) + \delta \dot{q}_T^T \cdot T_i \cdot \sum_{i=1}^{n_s} (F_i^E - F_i^I),
\]

(25)

where the former and the latter parts are the virtual power produced by active driving torque/force and dissipation force and by external wrench and inertial wrench on the upper body respectively. The dynamic interactions of the upper body caused by the motion of the mobile platform are also included in the item of external wrench \( F_i^E \) as discussed before.

The dynamics equation of the upper body is divided into four subsystems, the waist part has such a relationship:

\[
\sum_{i=1}^{n_w} (F_i - F_i^I) \cdot \delta V_i = \delta \dot{\theta}_a^T T_w (F_w^E - F_w^I) + \delta \dot{q}_w^T J_w (F_w^E - F_w^I),
\]

(26)

where

\[
T_w = P^T Ad_{g_{cm}}^T \left[ T_{m_1 m_2}^T A_d_{y_{min}}^{T-1}(0) \cdots T_{m_1 m_w}^T A_d_{y_{min}}^{T-1}(0) \right] \in \mathbb{R}^{2 \times 6n_w},
\]
\[
J_w = \begin{bmatrix} J_{mm_1}^T & \cdots & J_{mm_{n_w}}^T \end{bmatrix} \in R^{n_w \times 6n_w}, \quad F_w^E = \begin{bmatrix} F_{m_1}^E & \cdots & F_{m_{n_w}}^E \end{bmatrix}^T \in R^{6n_w \times 1},
\]

\[
F_w^I = \begin{bmatrix} F_{m_1}^I & \cdots & F_{m_{n_w}}^I \end{bmatrix}^T \in R^{6n_w \times 1}.
\]

In a similar method, for other subsystems of the upper body, there are following relationships:

\[
\sum_{i=1}^{n_a} (F_i - F_i^I) \cdot \delta V_i = \delta \dot{q}_a^T T_a(F_h^E - F_h^I) + \delta \dot{q}_h^T J_h(F_h^E - F_h^I),
\]

(27)

\[
\sum_{i=1}^{n_a} (F_i - F_i^I) \cdot \delta V_i = \delta \dot{q}_i^T T_i(F_i^E - F_i^I) + \delta \dot{q}_i^T J_i(F_i^E - F_i^I),
\]

(28)

\[
\sum_{i=1}^{n_a} (F_i - F_i^I) \cdot \delta V_i = \delta \dot{q}_i^T T_r(F_r^E - F_r^I) + \delta \dot{q}_r^T J_r(F_r^E - F_r^I).
\]

(29)

4.2.4. Forward dynamics of the whole WHR system

As discussed before, the dynamic interactions of the mobile platform caused by the motion of the upper body and the dynamic interactions of the motion of the upper body caused by the mobile platform are both included, they will be taken as the internal wrench when the whole WHR system is considered.

\[
\sum_{i=1}^{5+n_a} (F - F^I) \cdot \delta V = 0.
\]

(30)

The dynamic equation of the whole WHR body can be expressed as:

\[
\sum_{i=1}^{5+n_a} (\tau_i \cdot \delta \dot{q}_i + \varphi_i \cdot \delta \dot{q}_i) + \sum_{i=1}^{5+n_a} (F_i^E \cdot \delta V_i - F_i^I \cdot \delta V_i) = 0.
\]

(31)

From the equations of the mobile platform (21) and the upper body (25) as derived before, the whole dynamic equation can be derived by

\[
\delta \dot{q}_a^T \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} + \delta \dot{q}_a^T A^T \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix} + \delta \dot{q}_a^T J_p^T \sum_{i=1}^{5} (F_i^E - F_i^I)
\]

\[
+ \delta \dot{q}_a^T \sum_{i=1}^{n_a} (\tau_i + \varphi_i) + \delta \dot{q}_a^T \sum_{i=1}^{n_a} (F_i^E - F_i^I) = 0.
\]

(32)

The general joints of the whole WHR body \( qgi \) can be expressed as

\[
qgi = \begin{bmatrix} q_1 & q_2 & q_{w_1} & \cdots & q_{w_n} & q_i & \cdots & q_b \end{bmatrix}^T = \begin{bmatrix} q_a^T & q_{u_1}^T \end{bmatrix}^T.
\]

(33)
Then the whole WHR dynamic equation can be expressed by the general coordinates as follows:

$$
\delta \dot{q}_g^T \cdot \sum_{i=1}^{2+n_p} \tau_i + \delta \dot{q}_g^T \cdot A^T_\varphi \cdot \sum_{i=1}^{4+n_p} \varphi_i + \delta \dot{q}_g^T \cdot T^T_\varphi \cdot \sum_{i=1}^{5+n_p} (F^E_i - F^I_i) = 0,
$$

(34)

where $A_\varphi$ has the form as

$$
\begin{bmatrix}
A_{i=1}^p & 0_{i=1}^p \\
0_{i=1}^p & I_{(4+n_p) \times (4+n_p)}
\end{bmatrix},
$$

and the matrix $T_\varphi$ has the form as

$$
\begin{bmatrix}
J^p_{i=1} & 0_{i=1}^p \\
0_{i=1}^p & T_{i=1}^p(4+n_p) \times (4+n_p)
\end{bmatrix}.
$$

It can also be easily found that the matrix $T_\varphi$ is the generalized Jacobian matrix, hence

$$\quad V_G = T_\varphi \cdot \dot{q}_g = J_\varphi \cdot \dot{q}_g,
$$

(35)

where $V_G$ is the generalized twist and $J_\varphi$ is the generalized Jacobian matrix.

In a simple form, the whole WHR dynamic equation can be expressed as:

$$
\sum_{i=1}^{2+n_p} \tau_i + A^T_\varphi \cdot \sum_{i=1}^{4+n_p} \varphi_i + J^T_\varphi \cdot \sum_{i=1}^{5+n_p} (F^E_i - F^I_i) = 0.
$$

(36)

Generally, the active driving torque $\tau$, the dissipation force $\varphi$, the external and inertial wrench $F^E$ and $F^I$ are all expressed in concise forms, thus the whole WHR dynamic equation can be expressed as:

$$\tau + A^T_\varphi \varphi + J^T_\varphi (F^E - F^I) = 0.
$$

(37)

4.2.5. Forward dynamics of the whole WHR system in a canonical form

In another way, the forward dynamics of the whole WHR can be derived in a general form and a canonical form will be formulated by substituting (22), (26), (27), (28) and (29) into (31), the canonical-form equation is

$$
G^T(F^I - F^E) - \varphi - \tau = 0,
$$

(38)

where

$$
G = \begin{bmatrix}
T^T_{p1} + T^T_{p2} & 0 & 0 & 0 \\
T^T_w & J^T_w & 0 & 0 \\
T^T_h & 0 & J^T_h & 0 \\
T^T_l & 0 & 0 & J^T_l \\
T^T_r & 0 & 0 & J^T_r
\end{bmatrix} \in \mathbb{R}^{(6 \times n_p + 30) \times (n_p + 2)},
$$

$$
F^E = \begin{bmatrix}
F^E_p & F^E_w & F^E_h & F^E_l & F^E_r
\end{bmatrix}^T \in \mathbb{R}^{(6 \times n_p + 30) \times 1},
$$

$$
F^I = \begin{bmatrix}
F^I_p & F^I_w & F^I_h & F^I_l & F^I_r
\end{bmatrix}^T \in \mathbb{R}^{(6 \times n_p + 30) \times 1}.
$$
In (38), the matrix $G$ is a stacked form of the generalized Jacobian matrix in the body coordinates for the system. The whole WHR dynamics is expressed in a general form in (37), both of them have concise forms and are intuitive in reflecting the dynamic relationship between the input and the output.

As discussed in [19, 25], the d’Alembert’s inertial wrench $FI$ of rigid body can be extended to inertial wrench vector of multi-rigid body systems, then the derivation of (38) has the following form:

$$G^T MG \ddot{q} + (G^T MG - G^T AMG) \dot{q} - G^T F^E - \varphi = \tau,$$

where $M = \text{diag}[M_p, M_w, M_h, M_l, M_r] \in \mathbb{R}^{(6 \times n_0 + 30) \times (6 \times n_0 + 30)}$,

$$\dot{G} = \begin{bmatrix}
H_p & 0 & 0 & 0 & 0 \\
H_m & H_l & 0 & 0 & 0 \\
H_h & 0 & H_t & 0 & 0 \\
H_r & 0 & 0 & H_r & 0 \\
H_t & 0 & 0 & 0 & H_r
\end{bmatrix},$$

$$A = \text{diag}[ad_T^{V_p} \ldots ad_T^{V_n} \ldots ad_T^{V_r}] \in \mathbb{R}^{(6 \times n_0 + 30) \times (6 \times n_0 + 30)}.$$

The parameters of $M_p, M_t, H_p, H_t, \ldots$ are not derived in detailed forms here.

Let $L(q) = G^T MG$ represent the generalized system inertia matrix, $K(q, \dot{q}) = G^T MG - G^T AMG$ represents the generalized Coriolis and Centrifugal force term in the system, the whole dynamics equation of the WHR system will have a canonical form as

$$L(q) \ddot{q} + K(q, \dot{q}) \dot{q} - G^T F^E - \varphi = \tau.$$

It can easily get the detailed form of $L(q)$ through $L(q) = G^T MG$,

$$L(q) = \begin{bmatrix}
L_p & L_p^T & L_p^T & L_p^T & L_p^T \\
L_p & 0 & 0 & 0 & 0 \\
L_p & 0 & L_p & 0 & 0 \\
L_p & 0 & 0 & L_p & 0 \\
L_p & 0 & 0 & 0 & L_p
\end{bmatrix} = \begin{bmatrix}
L_p & T_w M_w J_w^T & T_h M_h J_h^T & T_l M_l J_l^T & T_r M_r J_r^T \\
J_w & 0 & 0 & 0 \\
J_h & T_h M_h J_h^T & 0 & 0 \\
J_l & T_l M_l J_l^T & 0 & 0 \\
J_r & T_r M_r J_r^T & 0 & 0 & J_r M_r J_r^T
\end{bmatrix},$$

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\[ L_p = (T_{p1} + T_{p2}) M_p (T_{p1} + T_{p2})^T + T_w M_w T_w^T + T_h M_h T_h^T + T_l M_l T_l^T + T_r M_r T_r^T. \]

From the detailed structure of the generalized inertia matrix \( L(q) \), the symmetric and positive definite property of the matrix \( L(q) \) and the skew-symmetric property of the matrix \( \dot{L}(q) - 2K(q, \dot{q}) \) can be easily proved. The matrix shows clearly the inertia interactions among different parts of the system. \( L_p, L_w, L_h, L_l \) and \( L_r \) are the generalized system inertial matrices of the driving wheels, the waist, the head, the left arm, and right arm, respectively. \( L_{pw}, L_{ph}, L_{pl} \) and \( L_{pr} \) denote the generalized interaction inertia matrices for the movements caused by waist, head, left arm and right arm. There is no more analysis on the matrix \( K(q, \dot{q}) \) and the same conclusion will be obtained. In this part, the dynamic interactions and coupled subsystems of the WHR exhibit clear form with intuitive physical meaning.

### 4.3. Further clarifications

According to the Newton–Euler equation, a rigid body with twist is utilized to formulate the dynamic equation, \( M \ddot{V} + GV = F_E \), where \( M \) is the generalized inertia matrix, \( G \) is the transformation matrix about the Coriolis and centrifugal forces, \( F_E \) is the external wrench applied at the center of mass and specified with respect to the body coordinate frame. From the basic idea of d’Alembert’s principle, there is a relationship \( (F_E - F^I) \cdot \delta V = 0 \), and the inertial wrench has the form as
\[ M \ddot{V} + GV = F^I. \] (42)

As the kinematics equations derived in the Sec. 3, the twist can be expressed in a general form with the generalized Jacobian matrix \( J_G \).
\[ V = J_G \dot{q}, \quad \dot{V} = J_G \ddot{q} + \dot{J}_G \dot{q}. \] (43)

Substituting (43) into (42), the inertial wrench can be rewritten as \( F^I = M(J_G \ddot{q} + \dot{J}_G \dot{q}) + G(J_G \dot{q}) = MJ_G \ddot{q} + (M\dot{J}_G + GJ_G)\dot{q}. \) With the new form of inertial wrench, the general WHR dynamics equation will be expressed as:
\[ J_G^T M J_G \ddot{q} + J_G^T (M\dot{J}_G + GJ_G)\dot{q} - J_G^T F_E - A_G^T \dot{\varphi} = \tau. \] (44)

Therefore, it can be expressed in a simple form using the most used dynamics equation:
\[ M_G(q) \ddot{q} + C_G(q, \dot{q})\dot{q} + N_G(q, \dot{q}) = \tau, \] (45)

where the generalized inertia matrix has the form \( M_G = J_G^T MJ_G \), the Coriolis matrix has the form \( C_G = J_G^T (MJ_G + GJ_G) \), and \( N_G \) is a term including external forces and other forces (like dissipation force) that can be written as \( N_G = -J_G^T F_E - A_G^T \dot{\varphi} \).

As it can be easily deduced that (45) satisfies two properties, \( M_G(q) \) is the symmetric and positive definite matrix, and \( M_G - 2C_G \) is a skew-symmetric matrix, which are important in the proof of many control laws for multi-rigid body dynamics system.
Remark 2. For an open-chain multi-body system with the tree topology structure, other methods, such as the recursive Newton–Euler algorithm, are very efficient for the dynamics modeling also. However, in this paper, although our proposed WHR has the tree topology form, it will be mainly applied in the cases of whole-arm manipulation like picking or placing a common object, or doing some cooperative work by two arms. Our proposed approach of dynamics modeling has the advantage in building up the closed-form dynamics model of the multi-rigid body system, which will be more attractive than the traditional methods, such as Lagrange method, Newton–Euler method, and Kane’s method. Also the system dynamics are built up with screw theory, thus the computational efficiency can be improved largely by properly computing the coefficient matrices of the system dynamics equation.

5. Case Study

The proposed WHR is mainly designed to caster for a service robot used in the people’s daily life, it is assumed that the robot will serve a person in an indoor environment. The robot moves toward a desk, then pick up a cup by two robotic hands. Like many cases in the robot-assisted applications, the robot should complete the work by two arms with two hands simultaneously as shown in Fig. 4. Here in order to verify our modeling method applied to whole arm manipulation, the robotic cooperative manipulation is investigated as a typical case.

5.1. Initial parameters of the wheeled humanoid robot

The WHR is composed of a mobile platform and an upper humanoid body with two seven-DOF arms. The main material used to build up the upper body is aluminum alloy because of its lower in mass and higher in strength. The mass and related inertial parameters of each WHR component is listed in the Table 1.

The initial configuration of the WHR is that the upper body is vertically installed on the mobile base and two robotic arms with the same structure are placed symmetrically at the two sides of upper main body. The initial configuration of two arms is that two arms point to the ground vertically and the initial joint angles of left arm and right arm are \( q_{L0} = [0, -90, 90, 0, 0, 0, 0](\text{deg}) \) and \( q_{R0} = [0, 90, -90, 0, 0, 0, 0](\text{deg}) \), respectively. The initial joints velocities are set to zero. The joint limits of the waist joints is \( q_{m1}, q_{m2} \in [-15, 15](\text{deg}) \). The lower joint limits of the left arm are \( q_{L_l} = [-60, -120, -120, 0, -90, -80, -60](\text{deg}) \) and its upper limits are \( q_{L_u} = [120, 0, 120, 150, 120, 80, 60](\text{deg}) \). The lower joint limits of the right arm are \( q_{R_l} = [-120, 0, -120, -150, -120, -80, -60](\text{deg}) \) and its upper limits are \( q_{R_u} = [60, 120, 120, 0, 90, 80, 60](\text{deg}) \). The velocity limits of joints for both arms are \( q_{L_u} = q_{R_u} = [50, 50, 60, 50, 60, 60, 60](\text{deg/s}) \).

The lower torque limits of joints for both arms are \( \tau_{L_l} = \tau_{R_l} = [-30, -30, -40, -30, -20, -20, -10](N \times m) \) and their upper limits are \( \tau_{L_u} = \tau_{R_u} = [30, 30, 40, 30, 20, 20, 10](N \times m) \). The torque limits of two driving wheels are \( q_{w1}, q_{w2} \in [-300, 300](N \times m) \).
The link lengths of two robotic arms are $d_1 = 0.205 \text{ m}$, $d_3 = 0.290 \text{ m}$, $d_5 = 0.235 \text{ m}$, $d_7 = 0.125 \text{ m}$.

5.2. Task performance

As shown in Fig. 2, the wheeled humanoid robot comes closer to the desk and will grasp the object on the desk. Here, the lower part of mobile base will no longer move and the waist part will keep the configuration as before. The main task is focused on the cooperative manipulation by the dual arms. For realizing arm manipulation, the dual arms should avoid obstacles in the environment, then adjust the position and orientation of two hands to approach the object. Since the object to be grasped cannot be twisted or turned, the forces exerted by the two robotic hands should be consistent. The change of the object’s position or configuration should be synchronously reflected then controlled by the two robotic arms as shown in the sequential snapshots in Fig. 4.
Two arms will perform the cooperative task, so the trajectory of the object is decided by the motions of both left arm and the right arm. In order to investigate the cooperating motion of two arms, the simulations are made in the MATLAB software as shown in Fig. 5. The object is grasped by two robotic hands denoted as the bar from points $A$ to $B$, and it will be grasped with the orientation unchanged, like moving from $AB$ to $A'B'$. A closed trajectory of object moving is mainly designed to demonstrate the whole arm manipulation. The trajectory to be followed by the left arm is the circle with the part $AA'$ and the trajectory of the right arm is the circle with the part $BB'$ as shown in Fig. 5(a). Considering the gravity force of the object and the reaction force between the robotic hand and the object, a force $5N$ is exerted on two hands of both arms, which can be taken as the resultant force. The workspace and the continuing plotting of two robotic arms doing the cooperative work is shown in Fig. 5(b). Different colors shows different workspace of two arms and the crossed workspace denotes the workspace of end-effector. Two crossed circles are the actual trajectories of the end-effectors of two arms. It is noticed that only the motions of two arms are concerned since the mobile base and the upper body are not required to move.

5.3. Simulation of the WHR performing cooperative work

The desired task is to implement the cooperative work by two robotic arms and the related simulations are made. From 0 s to 1 s, two arms move from the initial configurations to the starting configurations. The left arm starts from the point $A$ while the corresponding joint angles of left arm is $q_{Ls} = [90, 0, -90, 30, 0, 0, 0]$(deg) and the right arm starts from the point $B$ while the corresponding joint angles of left arm is $q_{Rs} = [-90, 0, 90, 30, 0, 0, 0]$(deg). The radius of the tracked circles is 0.05 m.
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(a) Projecting the trajectories of the cooperative work on the XZ plane.  
(b) The sequential snapshots of dual-arm doing the cooperative work.

Fig. 5. Simulation results of two arms during the cooperative work.

(a) Joints angles of left arm.

(b) Joints angles of right arm.

Fig. 6. The joint angles of dual-arm in doing the cooperative work.
The joint angles of left arm to track a circle are shown in Fig. 6(a) and their curves of velocities are shown in Fig. 7(a). The joint angles and velocities of right arm are shown in Figs. 6(b) and 7(b), respectively.

In order to verify our proposed modeling method, the first step of work is to run the model in the ADAMS software since different dynamics methods are used by ADAMS. The task is assumed that WHR should complete the cooperative work by the whole arms manipulation. The joint torques of left arm and right arm generated from the ADAMS software are shown in Figs. 8 and 9, respectively.

Then the same simulations of the wheeled humanoid robot performing the cooperative work are made by our proposed method. The mass and inertia properties...
used in our simulation are kept consistent with the simulation in the ADAMS software, and other features like geometrical and initial configuration are also set the same as in the ADAMS. The joint torques obtained from the simulation are shown in Fig. 10. Due to the symmetrical structure of our proposed WHR, only one arm is analyzed here. Comparing the simulation results shown in Figs. 10 and 9 from ADAMS, it is easy to find that our proposed method has a good performance from the simulations. Moreover, observing the errors as shown in Fig. 11, they are located in a satisfactory range.

Fig. 8. Joints torques of left arm solved by ADAMS software.

Fig. 9. Joints torques of right arm solved by ADAMS software.
6. Conclusion

This paper proposes a dynamic modeling method for a wheeled humanoid robot based on Lie groups and screw theory. Using the d’Alembert’s principle to solve the nonholonomic constraints, we exploited second variation form called Jourdain principle to deal with the problem of the nonholonomic velocity constraints. The canonical dynamics model has a concise form and is intuitive in reflecting the dynamic relationship between the input and the output. Its concise forms will have such advantages as the computational efficiency and easy solvable form, which can also be approached to model the dynamics of similar tree-like topological multi-body
systems. Some simulations are made to perform the cooperative work by dual-arm, the computational results are compared with the results derived by ADAMS software, the proposed method is validated at last.

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References

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