Comparative analysis of strategies for a semi-active suspension of a ¼ vehicle

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Abstract. The vehicle suspension isolates the chassis from road irregularities, reacting to forces produced by the tires and the braking torques, always keeping the road tire contact, providing stability and safety. Stability and safety are two antagonistic characteristics in suspension design, when improving one the other is impaired and vice versa. The semi-active suspension is a type of vehicle suspension that can change its stiffness and/or damping in real time depending on the vehicle response to the actual road profile. The On-Off semi-active suspension changes its damping coefficient between two fixed limit values. This work proposes an On-Off semi-active suspension model, in which the damping coefficient changes its values considering the road profile function frequency. A control strategy is proposed in a way to improve performance keeping the same simplicity, without any structural change of the semi-active suspension. On the proposed control strategy one of the damping coefficients is obtained through the linear quadratic regulator (LQR) algorithm, with the aim to set the coefficient from the gain matrix associated to the velocity of the suspended mass. This model is compared to anothers found in literature.

1 Introduction

The vehicle suspension is intended to support the weight of the vehicle over its axles, keep the wheels in proper steering and angulation, react to external excitations produced by the tires, braking and acceleration torques, lateral loads, attenuating vibrations from the excitation of the course and ensuring the contact of the tires with the ground. According to [1] it is increasingly common to find vehicles with various electronic systems distributed by the vehicle, making the automobile an electromechanical system. Modern automobiles are controlled by one or more electronic power stations, making driving these vehicles increasingly safe and comfortable. The active suspensions, characterized by the installation of an actuator between the sprung and unsprung masses, are in theory very efficient, capable of satisfactorily isolating the chassis from the vibrations arising from the terrain profile. However, its practical application presents a number of disadvantages due to the complexity of the system, which makes its acquisition and maintenance costly. Another factor that avoids its use in commercial automobiles is the need to supply energy from an external source, associated with the great weight of the system, active suspension increases fuel consumption considerably [2]. In order to reduce the complexity and cost while maintaining the good performance of an active suspension, the semi-active suspension arises as an alternative

between the active and passive systems, combining the simplicity of the passive suspensions with the efficiency of the active ones. It uses less components when compared to the active suspensions, being more robust. It works even off, acting as a passive suspension, this makes this type of suspension more applicable and more reliable [3]. In semi-active suspensions, the damper works as an actuator itself. There are several types of controllers involved in the control process, which can be in two states (On-Off), multiple states and continuous variation. On-Off controllers are usually represented by open-loop block diagram and work as an on-off system. Dampers with more than one stage or even continuous regulation with countless stages within a boundary are used in closed loop systems [2]. The change in damping can be attributed to the use of electromagnetic valves and / or magneto-rheological fluids [4]. Automotive suspension systems have served as basis for analytical and experimental studies, always aiming for comfort and safety, that are conflicting concepts. Because of this it is generally defined an intermediate level between these concepts so that the suspension meets the design requirements in the best way possible, but not always it is possible to deliver excellent levels of comfort and security only with passive elements, alternative options are presented over the last few years as a solution to this impasse. Rahman et al [5], evaluates the performance of two control algorithms, Linear Quadratic Regulator or Linear Quadratic Regulator (LQR) and Proportional Integral Derivative (PID), using a nonlinear 1/4 vehicle nonlinear system. Comparing four different mathematical models that describe the suspension of a vehicle, two of them with the Skyhook, Galanti [6] proposal, compares them with the efficiency of the open-loop and closed-loop models. Rao [7] investigates the performance of a quarter-car semiautomatic suspension system using the PID controller under the MATLAB Simulink model. In this work it is presented a comparison between semiautomatic and passive suspension system and its dynamic characteristics. A new approach to the On-Off semi-active suspension model is proposed: a new control method no longer dependent on the displacement of the sprung mass and unsprung mass, but rather depends on the frequency of the road profile in order to reduce the sudden damping changes, characteristics of an On-Off semi-active model, that cause the passengers feeling discomfort. While maintaining the same simplicity of the traditional On-Off semi-active model, no changes are required in the physical structure of the suspension for its implementation in a real vehicle.

2 Mathematical formulation

The 1/4 vehicle model was chosen for its simplicity and its commitment to accurately describe the dynamic behaviour of the suspension [1]. Figure (1) shows the simplified 1/4 vehicle model, where the components that influence the vertical dynamics of the vehicle are represented.

Based on Figure (1), it is possible to list the main parameters used for the suspension modeling: m_s: sprung mass; m_u: unsprung mass; k_s: suspension stiffness;· c_s: suspension damping;·k_u: tire stiffness;·z: input excitation;·x_s: displacement of the sprung mass; x_u: unsprung mass displacement.

The motion equations of the ¼ vehicle model are described below:

$$m_s \ddot{x}_s = -k_s(x_s - x_u) - c_s(t)(\dot{x}_s - \dot{x}_u) \tag{1}$$

$$m_u \ddot{x}_u = k_s(x_s - x_u) + c_s(t)(\dot{x}_s - \dot{x}_u) - k_u(x_u - z) \tag{2}$$

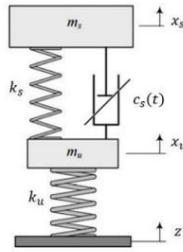$$c_{max} < c_s(t) < c_{min} \tag{3}$$

Figure 1. Semi-active quarter car model (adapted from [8])

3 Semi-active control strategy

The Bang Bang control, also called control On-Off control, is a feedback controller that suddenly changes between two limit values. This device compares the input with a target value, so that if the output exceeds the input, the actuator is switched off, otherwise, the actuator is now on. Figure (2) shows an example of a bang bang controller block diagram. This is a low cost controller, further its simplicity and convenience.
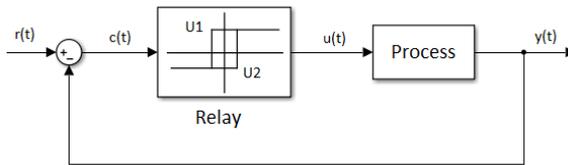

Figure 2. Bang bang control

According to [2] the two-stage semi-active suspension is very simple, characterizing its greatest advantage. The damper, generally of the magnetorheological type, alternates its stages between the highest and the lowest damping coefficient according to the speed applied to the suspension. When the velocity acting on the damper is in the same direction as the mass velocity m_s the maximum stage is employed. However when the opposite situation is verified the minimum stage is employed.

The control strategy is to set the suspension damping parameter using on-off control, varying suspension damping c_s, switching from one extreme set of values to the other. The optimal parameter value of c_s is obtained based on linear optimal control algorithm (linear quadratic regulator – LQR) [9]. First a LQR controller is designed assuming an active suspension system and neglecting the actuator dynamics. The optimal actuator force $\boldsymbol{u}(t)$ is defined by the gain matrix $\boldsymbol{G}$. The actuator force is not really applied on the suspension, this force is applied through a semi-active damper.

The linear optimal control problem consist in finding the control vector $\boldsymbol{u}(t)$ that minimizes the performance index J. In structural control, the performance index is usually chosen as a quadratic function in $\boldsymbol{z}(t)$ and $\boldsymbol{u}(t)$, as follows

$$J = \int_{t_0}^{t_f} \left[\mathbf{z}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{z}(t) + \mathbf{u}^{\mathrm{T}}(t)\mathbf{R}\mathbf{u}(t) \right] dt \qquad (4)$$

Where $\boldsymbol{Q}$ is a $2n$ x $2n$ positive semi-definite matrix and $\boldsymbol{R}$ is a m x m positive definite matrix. The matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ are referred to as weighting matrices, whose magnitudes are assigned according to the relative importance attached to the state variables and to the control forces in the minimization procedure. The minimization problem leads to a Riccati differential

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equation system [9], where $P(t)$ is the Ricatti matrix. The control vector $u(t)$ is linear in z(t). In this case, the linear optimal control law is

$$\mathbf{u}(t) = \mathbf{G}(t)\mathbf{z}(t) = -\frac{1}{2}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}(t)\mathbf{z}(t) \tag{5}$$

where $G(t)$ is the control gain. In most cases, numerical analysis show that Riccati matrix $P(t)$ stays constant during the control range, converging fast to zero on the neighborhood of t_f [9]. Thus $P(t)$, in most cases, can be approximated by a constant matrix P and and the constant control gain is given by

$$\mathbf{G} = -\frac{1}{2}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} \tag{6}$$

As it was stated above the control vector $u(t)$ is proportional to state vector $z(t)$ which contains system displacements and velocities. The element of control gain matrix G that multiplies $\dot{x}_s$ is considered to set optimum value for semi-active suspension parameter c_s.

3 Results and discussion

For the numerical simulations of the present work the properties considered for the models of ¼ vehicle analyzed were: m_s = 800 Kg; m_u = 50 Kg; k_s = 10500 N/m; k_u = 100000 N/m; c_s = 1200 Ns/m; c_{max} = 3000 Ns/m; c_{min} = 1000 Ns/m. Initially, it is necessary to obtain an optimized value of c_s that improves the performance of the semi-active On-Off. The proposed model is based on the use of the Linear Quadratic Regulator (LQR) algorithm, it is used to calculate a new damping coefficient c_{lqr} that is used in the On-Off control, it is expected that the LQR will return an optimum value for the damping leading to a better performance than traditional On-Off. It is worth mentioning that in this case the LQR algorithm is used to return an optimum damping value, and not to calculate the active force. The new coefficient is calculated by obtaining the values of the gain matrix, which in turn is obtained with the resolution of the Riccati matrix, as described previously.

The intention here is to propose a damping constant from the gain matrix coefficient related to the velocity of the suspended mass. Since this portion of the active force necessarily corresponds to the model of a linear viscous damping force. The gain matrix obtained was as follows: G = [-3336,5   1559,4   7335,5   2828,4]. Where G [1], G [2], G [3] and G [4] are associated respectively with displacement of the sprung mass, velocity of the sprung mass, displacement of the unsprung mass and velocity of the unsprung mass for the calculation of the components of an active force vector. In this case we are interested only in the gain related to the velocity of the sprung mass G [2]. This value is added to the damping coefficient c_{min}, (the minimum damping generated by a magnetorheological damper), thus proposing the following damping coefficient c_{lqr} = 2759,4 Ns/m.

It is worth mentioning that this value is within the range of operation of the damper, considered in this work that is between 1000 [Ns / m] and 3000 [Ns / m]. The control technique of this model also differs from the traditional OnOff model, because in this model the criterion for the variation between the damping coefficient, c_{min} or c_{lqr} is given by the frequency of the wheel-chassis set response. From a bode diagram which reports the amplitude of the transfer function of the set in (dB) by the frequency in (Hz) of the passive models with minimum damping and damping calculated by LQR it is verified the frequency where the suspension model with the coefficient calculated via LQR no longer has the smallest amplitude. For frequencies from approximately 4,5 rad/s the proposed control will alternate the damping coefficient calculated via LQR, c_{lqr}, for the minimum damping coefficient, c_{min}.

In this work a comparative analysis was performed between five suspension models excited to different terrain profiles: Passive Suspension; Traditional On-Off Semi-active; Skyhook

Continuous Semi-active Suspension; Semi-active Suspension On-Off via LQR; Active Suspension via LQR. The models were excited by road profiles: step and white noise.

The results of the simulations for the systems when submitted to an excitation are presented below. The analyzed models were excited by a step function, $z(t)$ with an amplitude of 2 cm. Figure (3) shows the behavior in the form of displacements of the five models analyzed
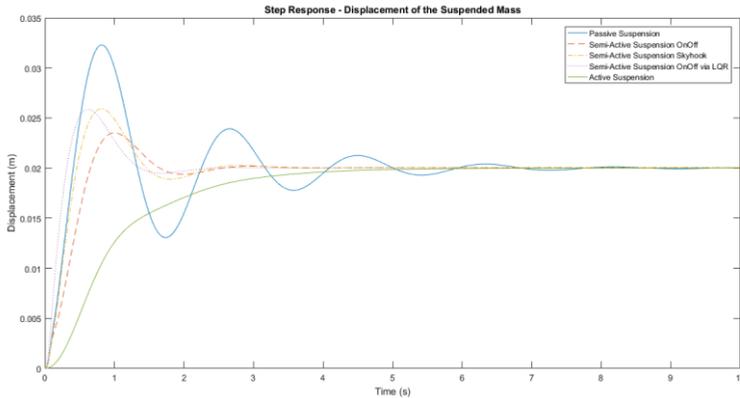

Figure 3. Displacement time evolution of the sprung mass.

Analyzing Figure (3), it is possible to see the characteristics of the system of semi-active Suspension On-Off via LQR, with overshoot equivalent to the one of skyhook semi-continuous suspension system and with the best accommodation time, ts, the model obtained a good performance. Only the On-Off semi-active suspension models and active, obtained better results for overshoot.

In the following simulations the analyzed models were excited to the white noise, $z(t)$. As in previous simulation (t) try to simulate a hypothetical road profile with an amplitude of 2 cm. Table 1 shows the system response for the 5 suspension models analyzed. It is verified in this case that for acceleration levels (comfort) the On-Off semi-active system via LQR obtained the best results except for the performance of the active suspension.

Table 1. System responses to white noise road profile

Suspension Model	Max amplitude $\ddot{X}_{max}$ (m/s^2)	RMS amplitude $\ddot{X}_{rms}$ (m/s^2)
Passive	2,532	0,683
Semi-active On-Off	3,938	0,851
Semi-active continuous skyhook	2,272	0,622
Semi-active On-Off via LQR	2,270	0,622
Active	1,637	0,357

4 Conclusions

A semi-active suspension model was proposed, the focus of the study was kept the ride quality or cabin comfort. For comparison parameters, four other models were simulated: one passive model, one active model and two models already discussed and known in the literature, all were excited by two different road profiles (step and white noise).

When submitted to the road step, the proposed model (On-Off model via LQR) presented an excellent displacement of the sprung mass, having the same overshoot value of the Skyhook model and the shorter accommodation time between the semi-active models. In the white noise excitation case, the active suspension system presented the lower acceleration levels.

But semi-active On-Off LQR had a better performance compared to the other three systems analyzed. It can be concluded that semi-active On-Off system presented good performance and the model can for future works be improved with further studies.

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