

SOLVING A SUPPLY CHAIN MANAGEMENT PROBLEM TO NEAR OPTIMALITY USING ANT COLONY OPTIMIZATION, IN AN INTERNATIONAL CONTEXT

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Abstract

The importance of achieving optimality or near optimality in supply routing is on the rise as globalization leads to scenarios in which multiple, heterogeneous and highly spatially distributed demands have to be satisfied under stringent constraints. However, there is no consensus concerning what constitutes an all-encompassing objective function for the supply planner, who faces what can easily constitute a problem requiring Non-deterministic Polynomial time for determining the solution even in its simplest formulations. The work presented in this article proposes a mathematically grounded approach that uses *Ant Colony Optimisation* to yield near optimal results across a large set of problem formulations and objective functions. The latter are designed to capture real-world goals such as cost reduction, optimal transportation management, flexibility and minimal lead-time. This study adds a new dimension to topics traditionally encountered in the literature, namely that of the cultural differences between partners engaged in international trade relations. Furthermore, the impact of the lag between determining and implementing the quasi-optimal strategy is forecast for an array of objective functions tailored to represent approaches encountered in international companies dealing with supply challenges in fields such as Information Technology. Finally, the framework thus established is employed to analyse the indirect relationship between Asian “white box” suppliers and a Romanian firm operating in Mobile Integrated Device space.

Keywords: International trade flows, supply chain management, control theory, ant colony optimisation, asymmetric travelling salesman problem.

JEL Classification: F17, F23, F47, J53, M11, M15, 032

Introduction

The concept of supply chain draws from systems theory (Boulding, 1956), finding its emergence in practice in the late 1980s (New, 1997). Recent studies consider that some of the main current challenges faced by supply planers include choosing the correct criteria for

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suppliers selection (Gheidari Kheljani, Ghodsypour and O'Brien, 2009; Yue, Xia and Tran, 2010), reducing the costs of transportation and managing transport capabilities (Berman and Wang, 2006; Zegordi, Abadi and Nia, 2010) and minimizing and controlling the lead time (Heydari, Baradaran Kazemzadeh and Chaharsooghi, 2009; He, Xu and Hayya, 2011). Other studies (Gogoneață, 2008; ElMaraghy and Majety, 2008; Zhang, Zhang, Cai and Huang, 2011) analyse the supply chain as an optimization problem, considering cost reduction, lead time or suppliers selection criteria as dependent variables.

The generalisation of international phenomena that can be broadly grouped under the globalisation moniker enlarges the complexity of the supply planner's problem (Morash and Clinton, 1997). Indeed, as underlined in (Neguț, 2011), international trade flows are one of the two main components of general international economic flows, and their coverage of the product space is comprehensive and complex, with both corporeal and intangible goods being freely shuffled across the world. It is worth noting that previous research acknowledges the international dimension of the supply chain, cultural differences between different suppliers or different actors involved in the process, but does not necessarily pursue a quantitative solution for the general problem when considering all of these factors at the same time. Moreover, highly abstract mathematical solutions, whilst appealing for theoretical economists and other specialist members of staff, are a hard sell in general for all those involved in running the supply chain.

Seeking to fill in these gaps, our study will approach the international planner's problem as a combinatorial optimization problem (Simeone, 1989), structured as the search for the optimal supply chain from the set of all feasible chains, where we define feasibility as the possibility for a company to implement a particular chain. We will consider the optimal solution to be a supply chain that includes the best possible suppliers (for some context dependent definition of best), has the lowest transportation costs and delivers the goods to the final place of consumption in the fastest way, being conscious of cultural differences between the actors involved. In a different vein from the existing body of work, we will employ a meta-heuristic algorithm with search customization in our quest for optimality. Furthermore, we do not fall into the trap of fixing the supply planner in a unique role, either that of being a demand generator or a supply provider – more often than not, real companies have to tackle both roles in order to be successful, and the optimization problem itself is the same. Therefore, we note that when we use the term *supply planner* throughout the text, the observations apply with no loss of generality to either the demand side or the supply side, unless otherwise noted. Our research will contribute to the literature on international trade relations and will provide a practical tool for companies that want to reduce the costs of their supply chain while optimizing the process.

1. Theoretical and methodological foundations

1.1 Theoretical foundations

International companies face numerous concerns in operating globally, including economic, political, logistical, competitive, cultural and infrastructural challenges (Manuj and Mentzer, 2008). It is widely acknowledged in the literature that the majority of these issues are important, and that customized supply chain management strategies should be developed for international contexts. In addition to the traditional requirements of a supply

chain, the international one demands highly coordinated flows of goods, services, and cash within and across national boundaries (Mentzer, 2000).

Whilst remaining apprehensive by rapport with the daunting complexity of the phenomenon, for the purposes of this paper we contend with focusing on a bounded parameter space. In effect, we will narrow our inclusion and analysis to the criteria used to choose suppliers, the challenges imposed by the transportation infrastructure, the requirements imposed by the delivery time and the significance of culture and cultural heterogeneity. We contend that this does not lead to a loss of generality, as we are factoring in elements that can be considered key parameters to be taken under consideration by any supply planner. Furthermore, we posit that our narrowing leads to a focusing on these key traits, an avoidance of the perils of over-determined models and, not in the least, a boost to the computational tractability of the model. The importance placed on the criteria used in the process of selecting suppliers rose once firms realised that suppliers play a critical role in a company's success, especially in an international context (Wagner and Johnson, 2004). Moreover, supplier selection and its related tasks are positioned at the front end of the supply chain process which could lead to difficulties being encountered by firms who engage in international distribution:

Real-world constraints may generate two strategic problems that the planner has to solve: the determination of the number of suppliers and the means to identify the most adequate ones among the alternatives on the market. The latter aspect was extensively analysed in literature. According to an empirical study (Dickson, 1966), the first three criteria identified by agents and managers in the United States of America and Canada to select/work with certain suppliers are the quality of the supplier's products, the competence of respecting a particular time of delivery and the history of the supplier's performance. A paper analysing the criteria approached in the main research articles published between 1966 and 1991 (Weber, Current and Benton, 1991) observes that the primary aspects discussed in the literature are price, delivery conditions, quality, production capacity and location of supplier. Studies that are more recent emphasize the importance of relations and connections with the suppliers (Dyer and Singh, 1998; Wagner and Johnson, 2004).

Directly connected with the decisions regarding the best suppliers is the one referring to the use of the appropriate means of transportation for the supplied merchandise. This aspect holds a major importance since primary transports costs often surpass inventory costs and facility costs (Okumura and Tsukai, 2003). However, the costs attached to a particular transportation means depends on the distribution strategy adopted by each company. The most common strategies (Berman and Wang, 2006) are the direct transportation (the merchandise is transported from the supplier to the plant, without any stop); the milk-run or the peddling transportation (goods are picked up from several suppliers and then delivered to one or several plants); cross-dock (merchandise is delivered from suppliers to a cross-dock and from the cross-dock to plants). These strategies played the role of a starting point for several studies. In fact, most papers dealing with transportation management can be classified according to the source and destination of the distributed goods, including single source/single destination (Zhao, Wang, Lai and Xia, 2004), single source/multiple destination (Chan et al., 2002), multiple source/single destination (Popken, 1994).

The type of transportation chosen by the planner will directly influence the delivery lead-time. Even though this might seem as a problem that concerns exclusively the supplier and his decisions regarding the maintenance of a high inventory level, lead-time becomes one

of the main concerns of the planner in the case of complex supply chains. In this context, we no longer discuss the lead-time guaranteed by the supplier, but the time to receipt, handling time from demand identification until the good is available to the customer, coupled with the supplier lead-time (Klapper et al., 1999).

An additional source of heterogeneity amongst the agents engaged in an international exchange context is culture. One of the seminal studies on this topic (Donaldson, 1990) includes a comprehensive exposition focused on the peculiarities of international sales, with a subsection being dedicated to the impact of culture on negotiating style. In a more recent treatise (Popescu and Chivu, 2008), some of the differences between states' cultural heritages are presented in a quantitative manner. Knitting together these disparate strands leads to the observation that the planner needs to factor in local peculiarities that go beyond measurable quantities like distance from point of supply to point of demand. It is clear that the objective functions we may construct for our optimization will have to be parameterized based on cultural aspects.

Juxtaposing all of the above observations, we arrive at the following conclusion: the optimization problem under consideration entails a multi-dimensional, rugged space of solutions. It is parameterized along three main conceptual directions: sales structures, cultural traits and spatial distribution. In the following sections, we will propose a model that captures these aspects, present an algorithm for solving to near optimality in the static case, sketch an approach for the dynamic case and, finally, correlate our findings with a real-world case.

1.2 Methodology

On a fundamental level, this is a Non-deterministic Polynomial-time-hard (henceforth NP-hard) combinatorial problem (Garey and Johnson, 1979), as there can truly be no convexity or compactness guarantees when dealing with the space of optimal supply allocation for a large, diverse number of customers. Therefore, the best one can hope for when dealing with real world situations is an approximate solution that is in the basin of attraction of some (preferably global) optimum. We do note, however, that approximate solution does not mean one based solely on intuition or prior experience or momentary inspiration.

We propose that we approach the planner's problem as an ATSP, with a customized cost function. In order to maintain self-containment of this article, we will now define a few key notions pertaining to the ATSP, including the canonical Travelling Salesman Problem (henceforth TSP) formulation. For a given set of n cities, the optimization goal in the TSP is to find the shortest possible tour that traverses all cities, passing through any single location only once and closing the tour loop by returning to the starting city. In a more rigorous manner, we take the representation of the TSP to be a complete weighted directed graph $G = (V, a, d)$ with n nodes and V the set of nodes, A the set of arcs and $d: A \rightarrow \mathbb{N}^+$ a mapping from the set of arcs into the set of positive integers representing the distance between nodes – in effect, this attaches $d_{ij} = \text{dist}(a_{ij})$, $a_{ij} \in A$ to each arc. In this context, the problem is to find the shortest Hamiltonian cycle (Hamilton, 1856) in the graph. In the general case of the ATSP, for at least one pair of nodes $d_{ij} \neq d_{ji}$ is true. There is ample literature on the topic of the TSP (Lawler, Kan, Lenstra and Shmoys, 1985), and it is known to be NP-hard in the general case. As evidenced in the introductory section, merely

minimising by rapport with distance is of little interest to the supply planner – his problem is of a significantly higher dimension. However, we can maintain the intuition that each arc, or, otherwise stated, each choice of a particular path to follow and, therefore, of a particular supply route, has a cost attached to it. We consider the following family of functions as being useful for representing costs in accordance with the planner's context:

$$\psi: \mathbb{R} \rightarrow \mathbb{R}^4 \tag{1}$$

As a simplification, we include only one parameter per direction which is to say that a parameter d is the distance from one node to another, another one δ represents the structural differential between the two communicating nodes and a third one λ represents the cultural dimension. The cultural dimension may require some additional clarification: we consider a continuous, bounded space of cultural traits Λ . We arbitrarily set the bounds $\liminf \Lambda = \{\text{collectivism, low uncertainty avoidance, long-term orientation}\}$ and $\limsup \Lambda = \{\text{individualism, high uncertainty avoidance, short-term orientation}\}$ where the concepts are defined in accordance with (Hofstede, 1983; Hofstede and McCrae, 2004). Additionally, we consider an extra parameter ξ that represents the cost of unmet demand, or, otherwise stated, the loss of profit associated with not choosing a particular node at a particular point. We posit the following about $\psi(X), X \in \mathbb{R}^4$:

$$\frac{\partial \psi}{\partial d} > 0 \tag{2}$$

$$\frac{\partial \psi}{\partial \delta} > 0 \tag{3}$$

$$\frac{\partial \psi}{\partial \lambda} > 0 \tag{4}$$

$$\frac{\partial \psi}{\partial \xi} > 0 \tag{5}$$

Otherwise stated, ψ is increasing in all of its arguments. The optimization problem thus becomes:

$$\min \sum_{i=1}^n \sum_{j=1}^n a_{ij} \psi(X_{ij}), \text{ where } a_{ij} \in \{0, 1\} \text{ a parameter showing if } ij \in \text{tour} \tag{6}$$

Solving the planner's problem to optimality, under these conditions, would ensure a minimization of his overall costs (be they monetary or otherwise) or, alternatively, a maximization of his profits (under caeteris paribus conditions). We note that the space of parametrizations for ψ is extensive. Without loss of generality, we assume a linear form: $\psi = A \times d_{X_i} + B \times \delta_{X_i} + C \times \lambda_{X_i} + D \times \xi_{X_i} + \epsilon, A, B, C, D \in \mathbb{R}, \epsilon \in \mathbb{N}(0, 1)$.

1.3 Ant Colony Optimization as Solving Algorithm

Ant Colony Optimization (henceforth ACO) is a population based meta-heuristic, part of the larger set of evolutionary algorithms. (Dorigo, 1992; Dorigo, Maniezzo and Colorni, 1996) represent fundamental references on the topic and serve as a medium for introduction

of the Ant System (henceforth AS), the ACO approach we employ. The key underlying metaphor associated with ACO and by extension AS is that of a cooperative search emulating, to a certain extent, the behaviour of real, foraging ants. In AS, a pheromone trail strength is associated with each arc in the graph. An ant positioned at a particular node has knowledge that includes the strength of this trail and the distance to all other accessible nodes, and probabilistically chooses its next step based on this information. Upon tour completion, pheromone is deposited along traversed edges – this has the conceptual role of collective memory, as the intensity of the trails serves as a pointer to other ants, eventually driving them along better quality paths. Below, in (fig. no. 1) we present the schematic structure of the AS algorithm as it can be found in (Stützle & Hoos, 1996):

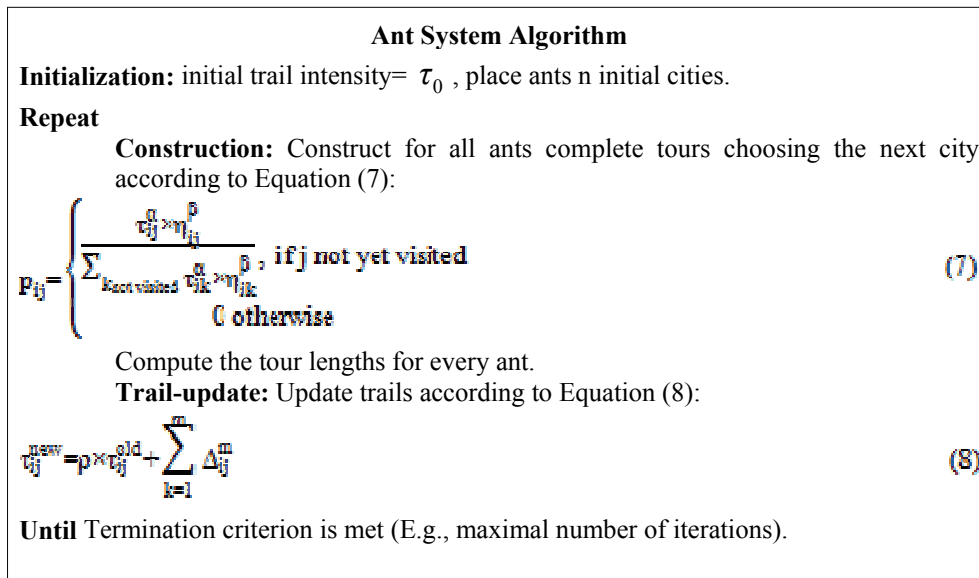


Figure no. 1: Schematic presentation of the Ant System algorithm

Source: Stützle and Hoos, 1996, p. 3.

Equations 7 and 8 shall be fleshed out below. We place emphasis on the term “metaphor” used to describe the algorithm. Indeed, our ants and their behaviour is only loosely related to the real-world counterpart and, in effect, each can simply be understood as an abstract explorer of some search space, employing collective memory – the pheromone trails – to communicate with other similar explorers and accelerate solution finding. In our case the ants map to supply planners exploring the optimal supply chain, which communicate through some common repository of knowledge – the pheromone trails. We now detail each of the above steps:

- **Construction** – each ant in a set of m ants independently constructs a solution to the problem, leveraging a 3-tuple $(\phi_{ij}, \eta_{ij}, \tau_{ij})$, where ϕ_{ij} identifies the edge under consideration η_{ij} is a heuristic function and τ_{ij} is the trail strength; in our case $\eta_{ij} = \frac{1}{\psi(x_{ij})}$ the propensity for

choosing destination j from base i , which is clearly inversely proportional with the cost attached with the choice. An ant chooses the next city to visit based on a probability distribution:

$$P_{ij} = \begin{cases} \frac{\tau_{ij}^\alpha \times \eta_{ij}^\beta}{\sum_{k \in \text{not visited}} \tau_{ik}^\alpha \times \eta_{ik}^\beta}, & \text{if } j \text{ not yet visited} \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

α, β are parameters that can be adjusted in accordance with the problem under consideration and which bias the probability distribution by adjusting the relative influence of the trail strength and heuristic function respectively; each ant starts from a randomly chosen city, maintains a list of visited cities so as not to pass through the same node twice and calculates the length L of its tour upon completion.

• **Trail updating** – post tour construction pheromone trails undergo updating based on the newly acquired information; all ants are allowed to deposit a constant pheromone quantity Q along the edges they have traversed and, moreover, trails are affected by decay represented as a fixed evaporation factor reducing pheromone levels along all edges; the formula used for trail updates is:

$$\tau_{ij}^{new} = \rho \times \tau_{ij}^{old} + \sum_{k=1}^m \Delta_{ij}^k \quad (8)$$

with $\rho \in (0, 1)$ a parameter representing the persistence of the trail and $\Delta_{ij}^k = \frac{Q}{L_k}$ the amount of pheromone added to an arc if ant k has visited it; as already mentioned, in the general sense the trail strength can be interpreted as a form of indirect long-term, collective memory, and in our particular use case it represents the supply planner's accumulation of knowledge with regards to the landscape.

We would like to place emphasis on the fact that there have been numerous refinements of the ACO published after its introduction, with notable examples being (Stützle and Hoos, 1996; Dorigo and Gambardella, 1997; Blum and Dorigo, 2004). We chose not to pursue any of these updated approaches in the present treatment to avoid drawing attention from the core topic of solving the supply planner's problem.

Table no.1: Mappings from ACO metaphor to the real-world supply planner's problem

ACO element	Real-world mapping	Interaction with other ACO elements
Ant	Planner exploring potential solutions for the optimal supply problem.	Traverses nodes, levers the pheromone trail in order to choose the best candidate node at each turn.
Node	Point of demand or point of supply (city, production / sales facility etc.)	Is traversed by Ants; is characterized by quantities associated with the main traits we factor in the model.
Pheromone trail	Commonly accessible memory repository (file, computer database) that gets updated as the search continues.	Is read by Ants; evolves towards a probability distribution that describes the best step from each node in the graph.

1.3 Using the Graphics Processing Unit to accelerate AS

Whilst the AS meta-heuristic is reasonably easy to implement, one of the main critiques raised against it was that of the high computational cost (Stützle and Hoos, 1996) – in effect, its asymptotic complexity is $O(mn^2)$ with m being the number of ants used during the solve step. As more often than not m is chosen to equal n , the number of nodes in the graph, this pushes complexity to $O(n^3)$, which is high. A redeeming property of the model is that it lends itself well to being implemented in a parallel fashion, at least for the construction step, as each ant represent an independent execution path in that stage of the algorithm. Given this trait, we have considered the opportunity of using the Graphics Processing Unit (henceforth GPU) to accelerate the computation.

An in-depth analysis of GPU architecture is beyond the scope of this article, but we will direct the interested reader to (Hennessy and Patterson, 2011, chap.4) for a comprehensive treatment. We will restrain ourselves to noting that the GPU is a widely parallel processor that relies on the Single Instruction Multiple Data (henceforth SIMD) paradigm (Hennessy and Patterson, 2011) to provide high arithmetic throughput in a size efficient package. Due to their widespread availability and high theoretical potential, GPUs have been researched intensively in recent years, with many scientists trying to ascertain whether they can be used as general-purpose accelerators (Brodtkorb et al., 2010; Dongarra and van der Steen, 2012).

Concerning ACO in general and AS in particular, the most notable development that we were aware of at the time of writing was (Cecilia et al., 2012), wherein speed-ups of more than 20x were achieved versus a sequential implementation of AS. We considered the potential for acceleration too notable to ignore, therefore we proceeded to implement AS in C++ AMP (Gregory and Miller, 2012), drawing from the work quoted in the first phrase of this paragraph. Upon finalization, we could efficiently evaluate scenarios that included more than one thousand five hundred nodes (cities / sources of demand), which is an adequate magnitude given the problem at hand. In what follows we shall endeavour to detail the effective implementation that we have developed for our research. The source-code is written integrally in C++, and runs in a Windows environment. We will divide the discussion in two halves, one focusing on the use of data-structures and one which fleshes out algorithmic flow.

The data-structure lying at the core of a (A)TSP is a two-dimensional array, a square matrix to be exact, which contains the costs associated with each edge in the graph. We tag it as the cost (distance) matrix. Its size equals n^2 for a naïve implementation of the symmetrical case (symmetry can be taken advantage of so as to approximately halve this cost, by treating the matrix as upper / lower triangular) and for all straightforward implementations of the asymmetrical. In our solution we treat costs as integers (type `int` in C++), and we pack them in an adequately sized `std::vector` (Josuttis, 2012, chap.7), a linear container. We use the `concurrency::array_view` (Gregory and Miller, 2012, chap.3) class template to wrap the data and transfer it to the GPU. It should be noted that this is our approach in general, so the presence of a `(std::vector, concurrency::array_view)` pair is to be assumed. Given that costs are constant quantities for the entire run, we decorate the data with the `const` modifier.

The following data-structures are also needed in the context of SF:

- a two-dimensional matrix which holds the pre-calculated η_{ij} heuristic values, having the same size as the cost matrix, but holding floating-point values (type `float` in C++), which we call the heuristic matrix; this data is also constant, being a mere reciprocation of the edge costs;
- a two-dimensional matrix which holds the τ_{ij} pheromone trail intensity values, with the same size and characteristics as the heuristic matrix, and which we dub the pheromone matrix; unlike the data-structures introduced up to now, the pheromone matrix is updated at the end of each iteration in accordance with equation 8;
- a two-dimensional matrix acting as a scratch-pad where ants can write out the tours they generate in each iteration, retaining the n^2 size, but using the `concurrency::index<2>` (Gregory and Miller, 2012, chap.3) class template as the stored type, given that we choose to represent tours as edge-lists; this structure gets overwritten within each iteration, and needs to be valid only before the trail updating stage;
- a vector of size n , which stores the best tour (found by the algorithm in prior iterations) represented as an edge-list; this structure is only updated if within a given iteration i one of the constructed tours is of better quality than the already stored one.

All of the above mentioned data containers reside in the GPU's global memory, which represents the base level in the memory hierarchy and is characterised by large storage capacity coupled with high latency. Additionally, we also employ fast `tile_static` memory (Gregory and Miller, 2012, chap.4), which is only discretely owned by a particular ant. The following two data-structures reside in this fast, programmer controlled cache:

- an integer vector of size n (in effect, node indices), which can be used by the owning ant to write-out the nodes already transferred – since `std::vector` is not directly usable in C++ AMP, we use a class template that we have developed on the basis of the `std::array` container (Josuttis, 2012, chap.7); at the end of the tour construction phase, this vector will hold the tour an ant constructed within an iteration, which will be exported to the scratch-pad matrix;
- a floating-point vector of size m , where the latter is the tile size currently in use (the best results were achieved when using 64-thread tiles); within each iteration, the vector acts as a local scratch-pad for intermediate computations;
- On the level of each thread, we use a container of bits template class, similar to the one presented in (Capper, 2001, pp.315–320), but adapted to be usable in a C++ AMP context. We make use of this final construct so as to maintain a quasi-immediately accessible “tabu-list” of cities that have already been visited. We are now properly equipped to describe the algorithmic flow of an iteration of the SF approach (we also list data set-up steps, which run only once, as opposed to per-iteration):
- we fill out the cost matrix with the values of the function ψ for all pairs of nodes in the graph, using input data for the parameters values from external databases (this is a set-up step);

- we fill out the heuristic matrix with the reciprocal of the values in the cost matrix (this is a set-up step);
- we launch a number of tiles equal to the count of nodes in the graph, each of former including a given count of threads; each of the tiles (the conceptual equivalent of an ant) concerns itself with constructing a tour, in the following manner:
 - a selection value is associated with each node, in accordance with the Iterative Roulette rule (Cecilia et al., 2012):
 - first, we check if the value of the bit in the tabu-list associated with the node is set, case in which the node is tabu and the selection value is 0;
 - if the node is not tabu, the heuristic value associated with the edge between the last visited node and the node under consideration is read from the heuristic matrix;
 - the heuristic value is multiplied with the value of the pheromone trail for the edge, and with a pseudo-random number;
 - we employ a parallel reduction (Gregory and Miller, 2012, chap.8) in order to determine the maximum of the set of selection values, choosing as the next step the node which it characterises;
 - the thus chosen node is marked as tabu;
 - we repeat from step 1 as long as a complete tour hasn't been constructed.
- each ant exports the tour it generated to the scratch-pad matrix;
- based on the scratch-pad matrix we update the pheromone matrix, using tile_static memory to reduce global memory accesses, in accordance with the non-atomic updating algorithm proposed in (Cecilia et al., 2012);
- the algorithm is repeated from the first non-set-up step, until convergence is achieved;
- after convergence, we have an (at least locally, but potentially globally) optimal tour, which in our case translated into an optimal allocation of supply to points of consumption or, if the analysis is conducted from the standpoint of a consumer, an optimal allocation of supply demands; in the same vein, based on the intensity of the pheromone trail, we can generate a hierarchy of tours, being thus able to evaluate an ordered set of potential allocations.

2. Analysis, results and discussion

2.1 Studying the static case

In the static case, we have to solve one single optimization problem, given a fixed setting. In this context, static is associated with the time dimension or, otherwise stated, the landscape does not evolve throughout time, but rather is kept fixed. This translates into non-varying parameter values associated with each node (city, point of consumption) in the problem's graph. One can therefore study two important areas:

- the importance of organizational memory / accumulated experience, as captured by the parameter α , which influences the weight given to pheromone trails in the probability distribution associated with choosing the next supply target; by varying it across a given range we were able to evaluate under which circumstances it is preferable to allow for extensive learning and when one should merely rely on current information;
- the impact of context heterogeneity on the speed of convergence and on the achievable optimal results – by keeping the distance between nodes constant and varying the loadings placed on Δ, λ, ξ we were able to evaluate a number of scenarios, from a typical, tightly clustered one, representative of local / regional distribution, to a highly heterogeneous one, which would map to a firm that has production, stocking and distribution facilities in a different socio-cultural space and which opts for a truly internationalized model of business. Pursuing these objectives, we proceeded to explore the parameter space:

Table no. 2: Parameter space characteristics for static analysis

Characteristic	Value	Explanation
α	0	Only current information.
α	2	Historical data used.
α	5	Historical data important.
Δ, λ, ξ	Sampled from a normal distribution.	Low heterogeneity.
Δ, λ, ξ	Each follows a random-walk process.	High heterogeneity.

Conducting a series of computer-assisted experiments with simulation conditions varying in accordance with the outline put forth in (table no. 2), we were able to derive the following observations:

- in a highly heterogeneous setting, the importance of learning, as embodied in the use of the pheromone trail, is high – this can be explained through the higher “ruggedness” of the optimization landscape, and thus the increased risk associated with merely myopically choosing the next best step; it is also worth noting that in this case it is preferable to choose a high pheromone persistence, so as to reduce the rate of learning and encourage exploration;
- conversely, in a highly regular landscape, a greedy approach that focuses on making the best local, myopic choice, yields more than adequate results, in lower cost conditions – what in computation translates into evaluating less parameters, in practice could translate to either having a smaller team handling supply, or affording to have high turnover in that department in order to minimise wage-associated costs, given that organizational memory is of low importance;
- the quality of the optima obtained in the more regular case is constantly better, in the sense that there's always a positive distance from them to the heterogeneous optima, which can be equated with less savings being possible / higher costs being attached to a highly variant business landscape.

Practically, if the methodology we propose would be employed in a supply-planning department for static analysis, the planning team would be in a position to evaluate a number of scenarios based on their current knowledge of the setting they evolve in, and then set their long-term strategy, assuming no shocks or discontinuities. Obviously, this

exercise can be repeated with some given frequency, to correct variations recorded between the forecast evolution and the actually recorded one.

2.2 Studying the dynamic case

We commence by noting that the dynamic case is notably more complex than the static case. Varying the time dimension as well in effect means that the landscape is no longer constant, and that our optimization problem needs to be solved repeatedly across multiple states. Moreover, our experience with it and the AS based approach being limited, we will defer a full experimental exploration to future works, and constrain ourselves to sketching the framework we have sought to apply in order to analyse supply planning as a time varying process. Using notation from (Rust, 1996), a generic agent's optimization problem becomes choosing some optimal decision rule φ , to achieve the following maximization:

$$\max E_{\varphi} \{U_T(\mathbf{s}, \mathbf{d})\} \equiv \int_{\mathbf{s}_0} \dots \int_{\mathbf{s}_T} \left[\sum_{t=0}^T \gamma^t u(\mathbf{s}_t, \phi_t(\mathbf{s}_t)) \right] \prod_{t=1}^T p(\mathbf{d}_{s_t} | \mathbf{s}_{t-1}, \phi_{t-1}(\mathbf{s}_{t-1})) P_0 \mathbf{d}_{s_0} \quad (9)$$

where P_0 represents a probability distribution over the initial state \mathbf{s}_0 . Obviously, this is a rather complicated if not intractable in the general case, problem. The canonical approach is to use dynamic programming, and we recommend the comprehensive treatments in (Kendrick, 1981; Rust, 1996) to the interested reader. In what follows, we will sketch a simplified approach tailored to our case and leveraging AS. First, note that, with all else held constant, the maximization problem above becomes, from the supply planner's perspective, a minimization problem:

$$\min E_{\varphi} \{\psi_T(\mathbf{s}, \mathbf{d})\} = \int_{\mathbf{s}_0} \dots \int_{\mathbf{s}_T} \left[\sum_{t=0}^T \gamma^t \psi(\mathbf{s}_t, \phi_t(\mathbf{s}_t)) \right] \prod_{t=1}^T p(\mathbf{d}_{s_t} | \mathbf{s}_{t-1}, \phi_{t-1}(\mathbf{s}_{t-1})) P_0 \mathbf{d}_{s_0} \quad (10)$$

where ψ has the meaning of a cost / expenditure function detailed in prior sections. Otherwise stated, the problem is to minimize the expected value of expenditure, starting from an initial state – in our case the distribution of partners / places of consumption and their traits – in the context of some transition probability associated with the state space. The minimization is achieved when an optimal vector of control φ is implemented – in our case, if for every time-step and every associated state space, the planner makes the optimal choice with regards to allocating supply to places of demand. The simplified approach that we propose, aimed at rapid analysis, entails the following steps:

- use AS to solve to optimality or near optimality the minimization problem at each time-step, for the associated state space and some finite number of time-steps;
- at the end, the integration step becomes a simple summation over partial results;
- from one time-step to another, state changes follow a Markov chain endowed with the Markov property, but within a time-step, we regard state as a deterministic rather than stochastic variable.

We note that this is a significant simplification of the initial problem, and therefore it should not be directly compared. However, it can allow for rapid analysis of a successful state dependent time varying strategy, since one can verify how aspects such as general

context volatility (transition probabilities between states are high) or organizational directions (different parameters used in AS) affect the supply problem.

2.3 Studying a real-world scenario

Prior studies (Crişan, Ilieş and Salanță, 2010) convincingly argue in favour of the use of optimisation to tackle logistic issues faced by Romanian firms. Given this issue, we consider the case of a company operating in Romania, whose object of activity is constituted by the commercialisation of Mobile Integrated Devices (henceforth MID). This is currently a rather dynamic space, with a very appealing Totally Addressable Market (henceforth TAM). To be exact, the bulk of the product portfolio is made up by: classical mobile phones as well as smart-phones; tablets; laptops; car navigation systems.

It must be clarified that the subject company does not have production facilities – it acquires so-called “white-box” elements from foreign partners and merely brands them, e.g. in the case of a mobile phone, they are constrained to picking one of the unbranded models offered by an Asian manufacturer, attach its logotype and brand related aspects, and resell it. This applies to all devices being sold, which is to say tablets and laptops as well. Moreover, it does not yet hold the capability to develop software for its devices and therefore must contend with using whatever the producer can ship, or it can look at other specialized companies. It does not maintain a distribution network, and relies on third parties to re-sell its products. A competitive advantage based on differentiation is difficult to achieve since all competitors end up relying on the same set of producers with low possibilities for customization, therefore a cost-based advantage is sought. From the supply planner for such a firm, three important questions have to be answered: how to ensure early supply of new products, and afterwards maintain it throughout their life-cycle, achieving optimal costs in both cases; how to handle software development outsourcing; how to direct available supply to points of demand / partners.

All of the above aspects are directly expressible in terms of the parameters that we have chosen to include in our model. For example, all of the partners are spatially distributed, therefore making delivery times intensely reliant on the choice of means of transportation, and making the company’s planner more sensitive to lead-times proposed by the suppliers and, respectively, the time of receipt. Furthermore, as is evidenced by many studies, there are broad cultural differences between Europe and Asia, in general, and between China and India in particular, therefore forcing the planner to pay heed to heterogeneity in this area. Our cost function approximation conveniently subsumes all of these aspects, as well as the others outlined in the literature review section of this study.

Given this context, the supply planner will have to choose between multiple Chinese sellers (China is the overwhelmingly dominant source of such products, so we constrain analysis of producers to its national space), each with a different spatial position, and potentially different traits. It must also choose where to direct its demand for customized software, if it were to exist, which means either working with the same Chinese partner from above, some other Chinese company, an Indian company or, at the extreme, a Romanian software company. Whilst in the case of the MID provider we are dealing with a physical deliverable, i.e. a good that is packaged in containers and physically shipped, for the software provider there is no such provision, since software is not a physical good. It therefore becomes obvious that we should regard the distance coordinate in a different key,

as a factor that leads to increased receipt delays due to more lax control in the case of a partner further away. In effect, delivery of software goods is instantaneous (or almost instantaneous), given modern communication means like the Internet, however the physical distance between the customer and the provider frequently has a direct impact on the time it takes for the software to be shipped in a form that matches the design parameters.

The distance between nodes need not be strictly a spatial distance, but rather we can fix it as capturing the time it takes from demand being recorded to it being met, as we have already hinted to in the context of the software supplier. Given the above, we can set up a large array of simulations to explore the space of potential decisions, varying the importance bestowed upon organizational learning or supplier / software provider homogeneity. (Table no. 3) synthesizes all of the parameter pairings we have considered in our computer-assisted experimentation:

Table no. 3: Parameter space for a real-world company

*	Value	Explanation
α	3	Emphasis on pheromone trails / history.
β	2	Do not ignore cost-based heuristic info.
ρ	0.99	Favour exploration by having the pheromone trails decay slowly/making memory persistent.
δ	Customized based on literature about structural organization in each country (Miron, 2003).	Construct a structural difference function based on the literature, attach $\delta_i = A \times \Delta_{\text{structural}} + B + \varepsilon, \varepsilon \in N(0, 1)$.
λ	Customized based on the literature about cultural characteristics (Hofstede, 1983; Hofstede and McCrae, 2004)	Construct a cultural difference function, attach $\lambda_i = A \times \Delta_{\text{cultural}} + B + \varepsilon, \varepsilon \in N(0, 1)$.
ξ	Customized based on simple linear forecasting of demand based on the firm's prior performances.	Construct an estimated demand function $v_i = A \times v_{i-1} + B + \varepsilon, \varepsilon \in N(0, 1)$, associate $\xi_i = \max(0, (v_i - \text{delivered}) \times p)$.

Given these parameter values, we proceed to refine the decision space into a 512-element potential action choice space. We do this by considering a continuity of possible choices attached with each partner and discretize them with a given granularity (we chose to refine up to a 512 element as this is a power of two value that has desirable properties in the context of GPU accelerated computing). Whilst this may seem odd at first – after all the count of firms is hardly that high – the potential impact of negotiation needs to be considered. Indeed, advantageous supply and distribution conditions can be obtained via able negotiation, and we consider that the only way to capture this large space of possibility is to assume that choices by rapport with a given partner are continuous and therefore require a high refinement granularity in order to be accurately represented. For clarity we detail the process of calculating parameter values for the (firm, Chinese supplier) pair:

- conducting an investigation based on the literature (Miron, 2003) so as to determine their presence / ubiquitousness in Romania and, respectively, China, we arbitrarily attach a value t to each of them, with $t_{\text{China}} = 512 \times t_{\text{Romania}}$, after which we employ a strictly increasing, upper bounded by the value $t_{\text{China}} - t_{\text{Romania}} = 511 \times t_{\text{Romania}}$, logarithmic function for interpolating intermediary $\Delta_{\text{structural}}$ values;

- starting from the three cultural dimensions under consideration, we extract the values attached to China and Romania from (Countries - Geert Hofstede, 2012), after which we proceed analogously to interpolate intermediary ~~Δ~~_{cultural} values;

- finally, we estimate future levels of demand for the firm's products using historical results as input data and employing a simple linear regression; assuming sales price stability, we can therefore forecast values for ξ .

Holding the time dimension static, and varying the values associated with the distance parameter, d (which, as detailed above, captures more than geographical distance), by running several batches of 1024 experiments, we could derive the following:

- it pays dividends to allow for more exploration once a certain break-even point is reached, but before that rapid convergence to some optimum, even if not global, is desired, so as to achieve low time to market;

- employing a software providing entity different from the producer that is strongly culturally different leads to lower realised profits / supply-chain induced cost savings (all achieved optima in such cases were of lower quality) – for the case of our firm, who has no resources for in-house software development, it is preferable to stick with what is shipped by the producer, and not involve a 3rd party;

- a more exploratory approach, as embodied by a high pheromone trail persistence / low loss of accrued knowledge, is desirable if the firm has the resources for survival; the optima achieved after longer explorations were of high quality, whereas using a lower pheromone trail persistence and thus encouraging rapid convergence led to being trapped in the basin of attraction of lower quality local optima - this reinforces the frequently uttered idea that in business a long-term approach is preferable to a short-term focused one.

Conclusions

In this paper, we have investigated the applicability of the AS meta-heuristic as solver for the supply planner's problem. Factoring in the modern context, this constitutes a highly complex problem, characterized by high heterogeneity in the relevant spaces associated with the decision making process. Whilst our solution does not solve to full optimality, it satisfactorily achieves near optimality across a number of scenarios. Throughout the text, we have listed parameterizations for real-world traits influencing supply departments' decisions.

Our experiments based on real-world conditions lead us to surmise that the approach can prove to be quite useful for planners who have to explore optimal solutions in a complicated competitive environment. Using intuitive parameters adjusted in accordance with publicly available literature, we could tailor the method to match the context of a Romanian firm engaged in a novel and dynamic market, deriving insight into the best strategies and tactics the supply-planning department in such a company could employ. We deem the synthesising a comparative analysis, centred on our approach by comparison with two other potential solutions, most opportune. On one hand, we will consider inspirational planning (the planned makes no effort to mathematically ground his choice, but relies exclusively on experience) and, on the other, we will choose a diametrically opposed

approach in which a branch-and-bound algorithm is used to solve the ATSP exactly (e.g. (Little, Murty, Sweeney and Karel, 1963)):

Table no. 4: A comparison between solvers for the planner's problem

Characteristics	Our method	Inspirational planning	Planning based on exact ATSP solving
Solution quality	At least locally optimal.	Impossible to anticipate, can be either globally optimal or not optimal at all.	Globally optimal.
Difficulty of understanding for non-specialists.	Low, the and / explorer metaphor is easy to comprehend.	Very low, the "algorithm" can be summed up in a few words.	High, the mathematical tool-set is complex.
Opportunities for experimenting with / simulating alternative scenarios.	Numerous, interventions are possible in the space of parameterizations and function use.	Impossible to evaluate, but the quality of the experiments is likely to be rather low.	Numerous, but less so than in the case of our method, because the impact of organizational learning can not be taken into account.
Contributions to the organisation's accrual of knowledge.	Consistent, the algorithm is independent from the individual who introduces it (it can be used after the latter has departed from the organisation).	Low (limited to tacit knowledge), this method is bound to the planner's presence in the organisation.	Consistent, but lower than in the case of our method, because it is intellectually accessible solely to employees with adequate training.
Computational complexity.	$> = O(N^3)$, GPU use feasible.	Does not apply.	$> O(N^3)$, GPU use is difficult.

The table shown above encloses an objective analysis of the approaches that can be employed in solving the supply planner's problem in an international context. This brief enumeration outlines at least three notable advantages of the solution developed throughout the present body of work: high applicability and comprehensibility, a medium implementation challenge and the feasibility of alternative scenario exploration. In our future work, we will seek to explore refinements to the AS algorithm that would reduce running times further and enhance the quality of the produced solutions, so as to allow for the efficient evaluation of even more complex functional forms attached with the cost of choosing a particular allocation of supply to points of demand. Furthermore, we aim to fully flesh out the methodology we have sketched for dynamic scenarios, as we believe it holds potential for the analysis of time varying cases.

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References

- Anon, 2012. *Countries - Geert Hofstede*, [online] Available at: <<http://geert-hofstede.com/countries.html>> [Accessed 12 Dec. 2012].
- Berman, O. and Wang, Q., 2006. Inbound Logistic Planning: Minimizing Transportation and Inventory Cost. *Transportation science*, 40(3), pp.287–299.
- Boulding, K.E., 1956. General Systems Theory—The Skeleton of Science. *Management Science*, 2(3), pp.197–208.
- Brodtkorb, A.R., Dyken, C., Hagen, T.R., Hjelmervik, J.M. and Storaasli, O.O., 2010. State-Of-The-Art in Heterogeneous Computing. *Scientific Programming*, 18(1), pp.1–33.
- Capper, D., 2001. *Introducing C++ for Scientists, Engineers and Mathematicians*, 2nd ed. Springer.
- Cecilia, J.M., Garcia, J.M., Nisbet, A., Amos, M and Ujaldon, M., 2012. Enhancing Data Parallelism for Ant Colony Optimization on Gpus. *Journal of Parallel and Distributed Computing*, [online] Available at: <<http://www.sciencedirect.com/science/article/pii/S0743731512000032>> [Accessed 13 Sep. 2012].
- Chan, L.M.A., Muriel, A., Shen, Z.M., Simchi-Levi, D. and Teo, C., 2002. Effective Zero-Inventory-Ordering Policies for the Single-Warehouse Multiretailer Problem with Piecewise Linear Cost Structures. *Management Science*, 48(11), pp.1446–1460.
- Crișan, E., Ilieș, L. and Salanță, I., 2010. Management Best Practices Used in Romanian Logistics Customer Service Planning. *Amfiteatru Economic*, XI(27), pp. 215-227.
- Dickson, G.W., 1966. An Analysis of Vendor Selection Systems and Decisions. *Journal of Purchasing*, 2(1), pp.5–17.
- Donaldson, B., 1990. *Sales Management: Theory and Practice*, [online] Macmillan. Available at: <<http://library.wur.nl/WebQuery/clc/572594>> [Accessed 12 Sep. 2012].
- Dongarra, J.J. and Van der Steen, A.J., 2012. High-Performance Computing Systems: Status and Outlook. *Acta Numerica*, 21, pp.379–474.
- Dorigo, M., 1992. Optimization, Learning and Natural Algorithms. *Ph. D. Thesis, Politecnico di Milano, Italy*, [online] Available at: <<http://ci.nii.ac.jp/naid/10016599043/>> [Accessed 13 Sep. 2012].
- Dorigo, M. and Gambardella, L.M., 1997. Ant Colony System: A Cooperative Learning Approach to the Traveling Salesman Problem. *Evolutionary Computation, IEEE Transactions on*, 1(1), pp.53–66.
- Dorigo, M., Maniezzo, V. and Colomi, A., 1996. Ant System: Optimization by a Colony of Cooperating Agents. *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on*, 26(1), pp.29–41.
- Dyer, J.H. and Singh, H., 1998. The Relational View: Cooperative Strategy and Sources of Interorganizational Competitive Advantage. *Academy of Management Review*, 23(4), pp.660–679.
- ElMaraghy, H.A. and Majety, R., 2008. Integrated Supply Chain Design Using Multi-Criteria Optimization. *The International Journal of Advanced Manufacturing Technology*, 37(3), pp.371–399.

- Garey, M.R. and Johnson, D.S., 1979. *Computers and Intractability: A Guide to the Theory of NP-Completeness, First Edition*, W. H. Freeman.
- Gheidari Kheljani, J., Ghodsypour, S.H. and O'Brien, C., 2009. Optimizing Whole Supply Chain Benefit versus Buyer's Benefit through Supplier Selection. *International Journal of Production Economics*, 121(2), pp.482–493.
- Gogoneață, B., 2008. An analysis of explanatory factors of logistics performance of a country. *Amfiteatru Economic*, X(24), pp. 143-156.
- Gregory, K. and Miller, A., 2012. *C++ AMP: Accelerated Massive Parallelism with Microsoft Visual C++*, Microsoft Press.
- Hamilton, W.R., 1856. Memorandum Respecting a New System of Roots of Unity. *Philosophical Magazine*, 12, p.446.
- He, X.J., Xu, X. and Hayya, J.C., 2011. The Effect of Lead-Time on the Supply Chain: the Mean versus the Variance. *International Journal of Information Technology & Decision Making*, 10(01), pp.175–185.
- Hennessy, J.L. and Patterson, D.A., 2011. *Computer Architecture: a Quantitative Approach*, Elsevier.
- Heydari, J., Baradaran Kazemzadeh, R. and Chaharsooghi, S.K., 2009. A Study of Lead Time Variation Impact on Supply Chain Performance. *The International Journal of Advanced Manufacturing Technology*, 40(11), pp.1206–1215.
- Hofstede, G., 1983. National Cultures in Four Dimensions: a Research-Based Theory of Cultural Differences Among Nations. *International Studies of Management & Organization*, 13(1/2), pp.46–74.
- Hofstede, G. and McCrae, R.R., 2004. Personality and Culture Revisited: Linking Traits and Dimensions of Culture. *Cross-cultural Research*, 38(1), pp.52–88.
- Istocescu, A., 2005. *Management comparat international*, Bucuresti: Editura ASE.
- Josuttis, N.M., 2012. *The C++ Standard Library: A Tutorial and Reference*, 2nd ed. Addison-Wesley Professional.
- Kendrick, D., 1981. Control Theory with Applications to Economics. *Handbook of Mathematical Economics*, 1, pp.111–158.
- Klapper, L.S., Hamblin, N., Hutchison, L, Novak, L. and Vivar, J., 1999. *Supply Chain Management: a Recommended Performance Measurement Scorecard*, [online] DTIC Document. Available at <<http://oai.dtic.mil/oai:0ai?verb=getRecord &metadataPrefix=html&identifier=ADA366787>> [Accessed 19 Sep. 2012].
- Lawler, E.L., Kan, A.H.G.R., Lenstra, J.K. and Shmoys, D.B., 1985. *The Traveling Salesman Problem*, Wiley.
- Little, J.D.C., Murty, K.G., Sweeney, D.W. and Karel, C., 1963. An Algorithm for the Traveling Salesman Problem. *Operations Research*, 11(6), pp.972–989.
- Manuj, I. and Mentzer, J.T., 2008. Global Supply Chain Risk Management. *Journal of Business Logistics*, 29(1), pp.133–155.
- Mentzer, J.T. (Thomas) ed., 2000. *Supply Chain Management*, 1st ed. Sage Publications, Inc.
- Miron, D., 2003. *Comerț internațional*. București: Editura ASE.

- Morash, E.A. and Clinton, S.R., 1997. The Role of Transportation Capabilities in International Supply Chain Management. *Transportation Journal*, 36(3), pp.5–17.
- Neguț, S., 2011. Rapid Changes of International Trade Flows Geography. An Approach Grounded on the Knowledge-Based Economy Concept. *Amfiteatru Economic*, XIII(30), pp.632–645.
- New, S.J., 1997. The Scope of Supply Chain Management Research. *Supply Chain Management: an International Journal*, 2(1), pp.15–22.
- Okumura, M. and Tsukai, M., 2003. Distribution Network Configuration Considering Inventory Cost. In: *43rd Congress of the European Regional Science Association, University of Jyväskylä, Finland*, [online] Available at <<http://www.jyu.fi/ersa2003/cdrom/papers/343.pdf>> [Accessed 19 Sep. 2012].
- Popescu, D. and Chivu, I., 2008. *Dezvoltarea abilităților de comunicare și negociere*. București: Editura Lucafărul.
- Popken, D.A., 1994. An Algorithm for the Multiattribute, Multicommodity Flow Problem with Freight Consolidation and Inventory Costs. *Operations Research*, 42(2), pp.274–286.
- Ristea, A.-L., Purcărea, T. and Tudose, C., 1996. *Distribuția mărfurilor*. București: Editura Didactică și Pedagogică.
- Rust, J., 1996. Numerical Dynamic Programming in Economics. *Handbook of Computational Economics*, 1, pp.619–729.
- Stützle, T. and Hoos, H., 1996. Improving the Ant System: a Detailed Report on the MAX-MIN Ant System. [online] Available at: <<http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.35.9895>> [Accessed 13 Sep. 2012].
- Tordjman, A., 1994. *Le commerce en Europe*. HEC.
- Wagner, S.M. and Johnson, J.L., 2004. Configuring and Managing Strategic Supplier Portfolios. *Industrial Marketing Management*, 33(8), pp.717–730.
- Weber, C.A., Current, J.R. and Benton, W.C., 1991. Vendor Selection Criteria and Methods. *European Journal of Operational Research*, 50(1), pp.2–18.
- Yue, J., Xia, Y. and Tran, T., 2010. Selecting Sourcing Partners for a Make-To-Order Supply Chain. *Omega*, 38(3), pp.136–144.
- Zegordi, S.H., Abadi, I.N. and Nia, M.A., 2010. A Novel Genetic Algorithm for Solving Production and Transportation Scheduling in a Two-Stage Supply Chain. *Computers & Industrial Engineering*, 58(3), pp.373–381.
- Zhang, W.Y., Zhang, S., Cai, M. and Huang, J.X., 2011. A New Manufacturing Resource Allocation Method for Supply Chain Optimization Using Extended Genetic Algorithm. *The International Journal of Advanced Manufacturing Technology*, 53(9), pp.1247–1260.
- Zhao, Q.H., Wang, S.Y., Lai, K.K. and Xia, G.P., 2004. Model and Algorithm of an Inventory Problem with the Consideration of Transportation Cost. *Computers & Industrial Engineering*, 46(2), pp.389–397.