A Low Complexity VCS Method for *PAPR* Reduction in Multicarrier Code Division Multiple Access

Si-Si Liu, Yue Xiao, Qing-Song Wen, and Shao-Qian Li

***Abstract*—This paper investigates a peak to average power ratio (*PAPR*) reduction method in multicarrier code division multiple access (MC-CDMA) system. Variable code sets (VCS), a spreading codes selection scheme, can improve the *PAPR* property of the MC-CDMA signals, but this technique requires an exhaustive search over the combinations of spreading code sets. It is observed that when the number of active users increases, the search complexity will increase exponentially. Based on this fact, we propose a low complexity VCS (LC-VCS) method to reduce the computational complexity. The basic idea of LC-VCS is to derive new signals using the relationship between candidature signals. Simulation results show that the proposed approach can reduce *PAPR* with lower computational complexity. In addition, it can be blindly received without any side information.**

***Index Terms*—Low complexity variable code sets, multicarrier code division multiple access (MC-CDMA), peak to average power ratio, variable code sets.**

1. Introduction

In wireless communication systems, multicarrier code division multiple access (MC-CDMA) has attracted more and more attentions as a very promising modulation technique. The main idea behind MC-CDMA is to spread and convert input signals into parallel data streams, which are then transmitted over multiple carriers. MC-CDMA can realize the high bite rate and large capacity transmission. The intersymbol interference (ISI) and the influence of delayed waves are almost completely eliminated by introducing a guard time in MC-CDMA symbols.

However, one of the major disadvantages of MC-CDMA is its high peak to average power ratio (*PAPR*) which leads to a large nonlinear distortion at a high power amplifier (HPA), a significant power efficiency penalty, and the degradation of the bit error rate (BER). To overcome this problem, many methods have been proposed[1], such as clipping[2], multiple signal representation (MSR) which mainly includes partial transmit sequences (PTS)[3] and selected mapping (SLM)[4], and block coding[5]. Clipping is a conventional method to limit the *PAPR* at the end of the transmitter. However, it reduces signal power, degrades BER performance and causes out of band radiation. A lot of researches were done in the block coding algorithm, which is a method that reduces the *PAPR* by coding the input words into the code word with low *PAPR*.

In the PTS method, the information data are divided into disjoint subblocks and then phase rotated before combination to minimize the *PAPR*. In the SLM technique, the transmitter generates a set of sufficiently different candidate data blocks, all representing the same information as the original data block, and multiplies them with phase factors to choose the sequence with the lowest *PAPR*. Compared with other techniques for *PAPR* reduction, the main advantage of MSR method is that it is a distortionless technique that does not arise in-band distortion nor out-of-band emission, but it also increases the complexity of the system and loss of transmission efficiency by using side information.

Also some approaches emphasizing on the allocation strategies of the spreading and despreading sequences in MC-CDMA system have been proposed in [6]-[8].

In our work, a *PAPR* reduction scheme called low complexity variable code sets (LC-VCS) is proposed. The scheme needs little computational complexity comparing with variable code sets (VCS)[9]. VCS can reduce *PAPR* without sending side information; however, it needs large numbers of computations. The basic idea of LC-VCS is to use the relationship between signals to reduce the amount of computation; in addition, it can also be blindly received without any side information.

This paper is organized as follows. Section 1 introduces the conventional MC-CDMA system model and the definition of *PAPR*. In Section 3, we present the derivation and detailed algorithm of our proposed LC-VCS method. We give the simulation results in Section 4 and draw some conclusions in Section 5.

2. MC-CDMA System Model

The conventional MC-CDMA system model is shown in Fig. 1.

Manuscript received August 29, 2006; revised October 10, 2006.

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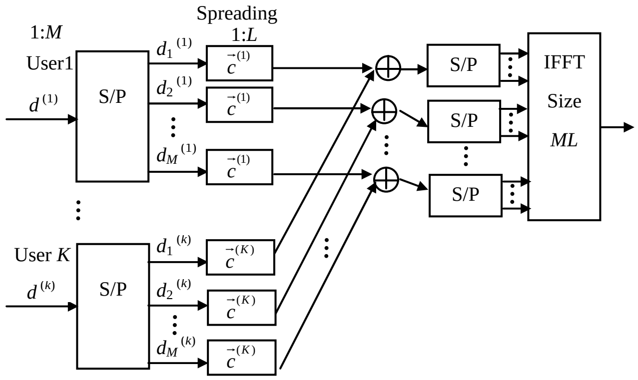

Fig.1. Conventional MC-CDMA system model

Assume user k has M data symbols which are presented as

$$\vec{\mathbf{d}}^{(k)} = (d_1^{(k)}, d_2^{(k)}, \cdots, d_M^{(k)}),\quad k = 1, 2, \cdots, K$$

where K is the number of active users. After serial-to-parallel (S/P) conversion, each symbol is spread by the orthogonal spreading code

$$\vec{\mathbf{c}}^{(k)} = (c_1^{(k)}, c_2^{(k)}, \ldots, c_L^{(k)})$$

and L is the length of the spreading sequence. Each user must select unique code to guarantee the accurate reception. After the spreading process, all users' symbols are added together. Taking another S/P conversion, these $M \times L$ parallel data are sent into the inverse fast Fourier transform (IFFT) modulation, whose size is also $M \times L$. The baseband representation of the MC-CDMA signal is give by

$$s(t) = \sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{k=1}^{K} d_m^{(k)} c_l^{(k)} e^{j2\pi\{M(l-1)+(m-1)\}t/T_s},\quad 0 \le t \le T_s \qquad (1)$$

where T_s is the symbol period of a MC-CDMA symbol.

In the following of the paper, only discrete-time representation of MC-CDMA signal will be used, which is expressed as

$$s(i) = \sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{k=1}^{K} d_m^{(k)} c_l^{(k)} e^{j2\pi\{M(l-1)+(m-1)\}i/NT_s} \qquad (2)$$

For the MC-CDMA downlink transmitter, the cyclic prefix (CP) is inserted in the symbols for avoiding ISI which is caused by multipath fading.

Since MC-CDMA is a multicarrier modulaton technique containing many subcarriers as can be seen from the above equation, it can give a high *PAPR* when all subcarriers added up coherently. Using the discrete-time definition, the corresponding *PAPR* is defined as

$$PAPR = \frac{\max\limits_{0 \le i \le N-1} |s(i)|^2}{E[|s(i)|^2]} \qquad (3)$$

where $E[|s(t)|^2]$ denotes the average power and $\max\limits_{0 \le i \le N-1} |s(i)|^2$ the peak power.

The more convenient way to express the *PAPR* of multicarrier signals is to utilize the probability characteristic that the *PAPR* is larger than a certain level, we call it the complementary cumulative distribution function (*CCDF*), which is expressed as

$$CCDF(PAPR(s(i)) = \Pr(PAPR(s(i) > \varsigma) \qquad (4)$$

The reason for using *CCDF* lies in the fact that when the amount of subcarriers grows large, the signal amplitude can be approximately thought as Rayleigh distribution, so the high peaks actually happen rarely. Therefore the statistical distribution property of *PAPR* always becomes more meaningful comparing with the absolute value of *PAPR*.

3. Proposed Technology

In this section, we first introduce the conventional VCS method, then emphasis on our proposed LC-VCS scheme and its detailed algorithm.

3.1 Conventional VCS Method

In [9], the authors proposed a scheme called VCS, the block diagram of downlink MC-CDMA system using VCS is shown in Fig. 2. The main idea of VCS is to try to allocate more than one spreading codes to each user, while in the conventional MC-CDMA system the spreading code per user is one. After calculating all the corresponding *PAPR* of the signals, we choose the code set which has the smallest *PAPR* and transmit the correspond signal.

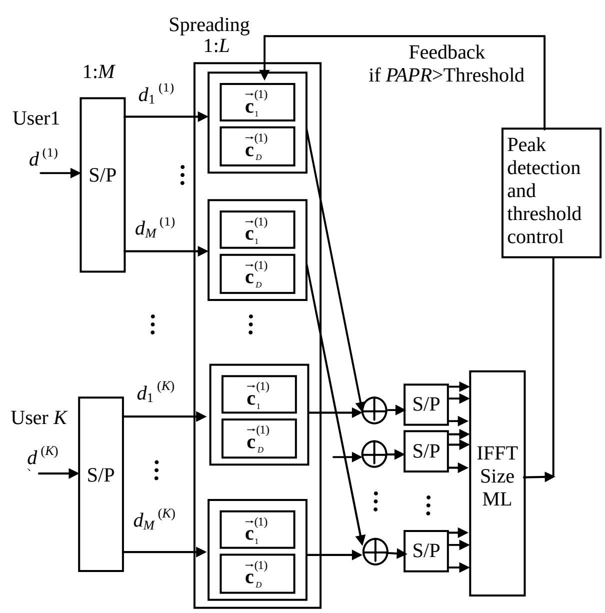

Fig. 2. VCS system model.

In the VCS scheme, every time when computing *PAPR*, we need to do IFFT whose size is $M \times L$ after spreading; assume each user has D different spreading codes for choosing, we have to do D^K times of IFFT. If the number of active users increases, the computational complexity increases exponentially. This will cost a large amount of computations to achieve the aim of reducing *PAPR*.

Therefore, finding a method to decrease the computational complexity of VCS is very necessary and significant.

3.2 The Basic Idea of LC-VC

Here we define user k has the code sets of $\vec{\mathbf{c}}_1^k, \vec{\mathbf{c}}_2^k, \cdots, \vec{\mathbf{c}}_D^k$, $\vec{\mathbf{c}}_x^k = (c_{x,1}^{(k)}, c_{x,2}^{(k)}, \cdots c_{x,L}^{(k)})$, $x = 1, 2, \cdots, D$. The data after spreading is

$$\begin{aligned} x &= \sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{k=1}^{K} d_m^{(k)} c_{x,l}^{(k)} \\ &= \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(1)} c_{x,l}^{(1)} + \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(2)} c_{x,l}^{(2)} + \cdots + \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(K)} c_{x,l}^{(K)} \end{aligned} \quad (5)$$

We assume each user uses the first code ($\vec{\mathbf{c}}_1^{(1)}$ for user 1, $\vec{\mathbf{c}}_1^{(2)}$ for user 2, and so on, we choose $\vec{\mathbf{c}}_1^{(1)}, \vec{\mathbf{c}}_1^{(2)}, \ldots, \vec{\mathbf{c}}_1^{(K)}$ as first code set randomly) to generate x_1

$$\begin{aligned} x_1 &= \sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{k=1}^{K} d_m^{(k)} c_{x,l}^{(k)} \\ &= \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(1)} c_{1,l}^{(1)} + \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(2)} c_{1,l}^{(2)} + \cdots + \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(K)} c_{1,l}^{(K)} \end{aligned} \quad (6)$$

When user 1 chooses $\vec{\mathbf{c}}_2^{(1)}$ and other user's codes remain the same, the new data is

$$\begin{aligned} x_2 &= \sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{k=1}^{K} d_m^{(k)} c_{x,l}^{(k)} \\ &= \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(1)} c_{2,l}^{(1)} + \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(2)} c_{1,l}^{(2)} + \cdots + \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(K)} c_{1,l}^{(K)} \end{aligned} \quad (7)$$

We can make use of the relationship between x_1 and x_2 to get x_2 from x_1 without calculating x_2, the relationship is

$$\begin{aligned} x_2 &= \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(1)} c_{2,l}^{(1)} + \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(2)} c_{1,l}^{(2)} + \cdots + \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(K)} c_{1,l}^{(K)} \\ &= \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(1)} c_{1,l}^{(1)} + \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(2)} c_{1,l}^{(2)} + \cdots + \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(K)} c_{1,l}^{(K)} \\ &\quad + \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(1)} c_{2,l}^{(1)} - \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(1)} c_{1,l}^{(1)} \\ &= x_1 - \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(1)} (c_{1,l}^{(1)} - c_{2,l}^{(1)}) \end{aligned} \quad (8)$$

Because IFFT is a linear transform, equation (8) can be adapted to

$$\begin{aligned} X_2 &= IFFT(x_2) \\ &= X_1 - IFFT(\sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(1)} (c_{1,l}^{(1)} - c_{2,l}^{(1)})) \end{aligned} \quad (9)$$

By the same way, when user 1 chooses $\vec{\mathbf{c}}_2^{(1)}$ and user 2 chooses $\vec{\mathbf{c}}_2^{(2)}$, other user's codes don not change, the data is

$$\begin{aligned} X_2' &= IFFT(x_2) \\ &= IFFT(\sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(1)} c_{2,l}^{(1)} \\ &\quad + \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(2)} c_{2,l}^{(2)} + \cdots + \sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(K)} c_{1,l}^{(K)}) \\ &= X_2 - IFFT(\sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(2)} (c_{1,l}^{(2)} - c_{2,l}^{(2)})) \end{aligned} \quad (10)$$

We can use this method to compute all the D^K code sets. As for the last half part in (9) and (10), we can calculate and store them in a table beforehand. When they are in need, we will find them by the indices of the table.

In a word, once we have the signals of a certain code set, we can use the equations above to derive all the other signals. This will greatly reduce the computational complexity. The VCS scheme needs to compute D^K times to search all the possible combinations, it means to do D^K times of IFFT; but using our method, we need only to search one set of combination and generate the other D^K-1 sets recursively. Let X and Y denote two different signals, their only differences are the ith user's spreading code, so the recursive equation can be generally defined as

$$Y = X - IFFT(\sum_{m=1}^{M}\sum_{l=1}^{L} d_m^{(i)} (c_{x,l}^{(i)} - c_{y,l}^{(i)})) \quad (11)$$

3.3 Algorithm and Complexity Comparison

Firstly, we compute X_1 which uses the first code set ($\vec{c}_1^{(1)}, \vec{c}_1^{(2)}, \cdots, \vec{c}_1^{(K)}$, we choose them as first code set randomly) as spreading codes; then we set *Stage*=1 and derive $X_2, X_3, \cdots, X_D$ (whose only differences with X_1 are the first user's spreading code) from X_1 using (9). By the same token, we set *Stage*=2 and generate $X_{22}, X_{23}, \cdots, X_{2D}$ from X_2, $X_{32}, X_{33}, \cdots, X_{3D}$ from X_3, and $X_{D2}, X_{D3}, \cdots, X_{DD}$ from X_D respectively. While $X_{i2}, X_{i3}, \cdots, X_{iD}$ distinguish from X_i by the second user's spreading code, $i = 1, 2, \cdots, D$. In the same way, we compute all the candidature signals until *Stage*=*K*. By this manner, all the D^K sets can be searched.

Here we will summarize the algorithm of LC-VCS

1) For each user, choose one spreading code from the D candidature codes randomly and generate X_1 by (6). Compute and store the last half part of (11). Set *Stage* =0.

2) Set *Stage* = *Stage* +1. From each signal currently in storage, generate new signals by changing the *Stage-th* user's spreading code. Compute the *PAPR* of the new signals and store them.

3) If *Stage*<*K*, go to Step 2); else transmit the signal with the lowest *PAPR*.

Table 1 gives the comparison of VCS and our LC-VCS method in the aspect of computational complexity.

Table 1
The computational complexity of the two methods

Complexity	VCS	LC-VCS
Addition	$D^K(K-1)ML+D^K v$	$[K(D-1)+1]v+L(D-1)\times K+(D^K+K-2)ML$
Multiplication	$D^K v/2+D^K MLK$	$[K(D-1)+1]v/2+MLD^K$

4. Simulation Results

The simulation parameters used in the MC-CDMA system are set as follows: QPSK modulation, the number of data symbols per user *M* is 8, Walsh code is used as spreading code with length *L*=16 and *L*=64; the number of active users is 4, and the number of candidature spreading codes per user *D* is 2, 3 and 4 respectively. The IFFT size is $M \times L$=128 in Fig. 3 and 512 in Fig. 4.

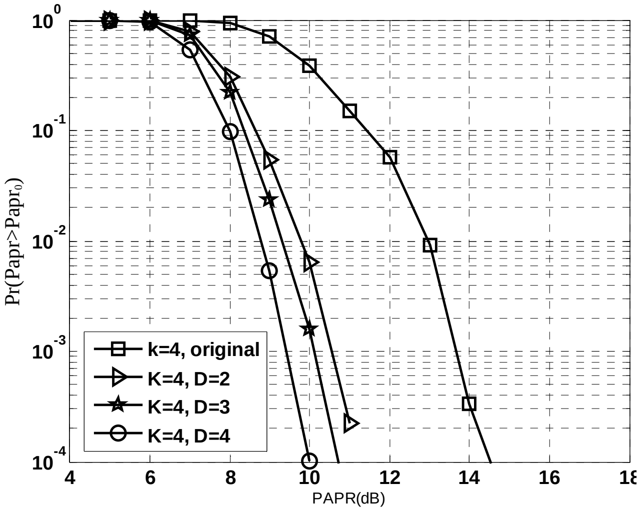

Fig. 3. *CCDF* of MC-CDMA signals using LC-VCS with *L*=16 and different *D*.

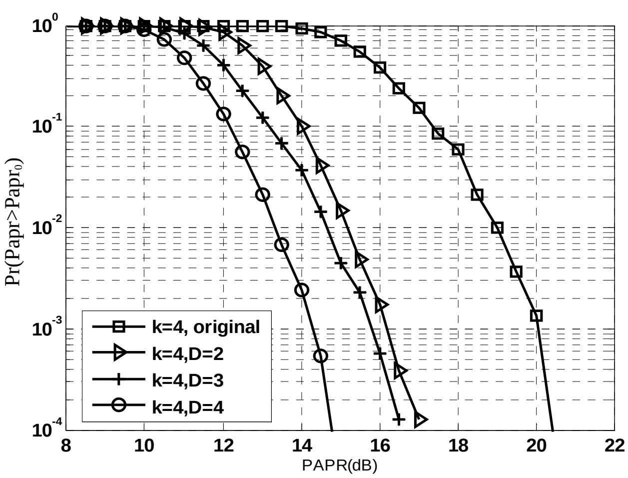

Fig. 4. *CCDF* of MC-CDMA signals using LC-VCS with *L*=64 and different *D*.

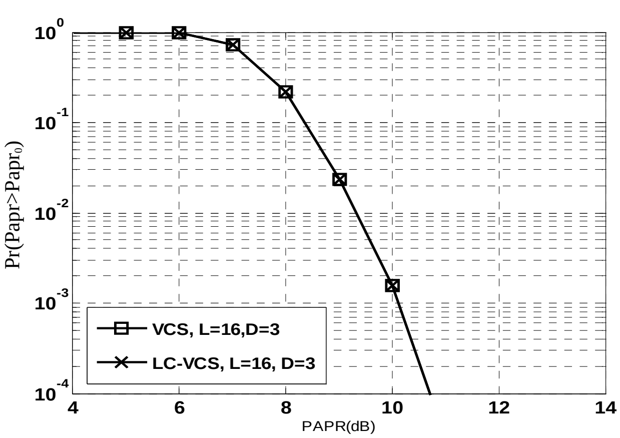

Fig. 5. Comparison of the *CCDF* of MC-CDMA signals using LC-VCS and VCS with *L*=16.

Because our LC-VCS can reduce the computational complexity of VCS without degrading its *PAPR* reduction performance, the *CCDF* of these two kinds of methods are exactly the same. As can be seen in Fig. 5, which shows the *CCDF* of MC-CDMA signals using LC-VCS and VCS respectively in the case of *L*=16 and *D*=3, the two kinds of lines superpose.

Fig. 3 is the *CCDF* of our approach comparing with that of original MC-CDMA signals in the case of *L*=16 and *D*=2, 3, 4 respectively. For instance, when *CCDF* is 10^{-3}, the *PAPR* of our proposed scheme with *D*=4 can be about 4 dB smaller than that of the conventional MC-CDMA system. In this case, VCS needs to do 3.28×10^5 times of addition and 2.46×10^5 times of multiplication. While the computational complexity of our method is only 4.49×10^4 times of addition and 7.87×10^3 times of multiplication respectively.

Fig. 4 is the *CCDF* with the length of Walsh code is 64 and different *D*. When $CCDF = 10^{-3}$, the *PAPR* of our method with *D*=4 can be approximately reduced by 5.5 dB comparing with the original MC-CDMA signals.

5. Conclusions

In this paper, we propose an efficient *PAPR* reduction and low complexity method called LC-VCS. The basic idea is to allocate each user with more than one spreading code and choose the code set which results in the least *PAPR* for transmission. But unlike VCS, we do not compute all the combinations of code sets, instead, we only need to compute one combination of code sets and derive all the other candidature signals from it recursively. The computational complexity reduces exponentially with the decrease of the number of active users. The simulation result shows LC-VCS can also reduce *PAPR* efficiently. Furthermore, it can also be blindly received without any side information due to the orthogonality of spreading codes.

The proposed scheme can substantially reduce *PAPR* and simplify the computations of MC-CDMA system considering the tradeoff between computational feasibility and system performance.

Furthermore, we can constrain the number of searched signals at each stage; that means the signals with the large *PAPR* will be discarded when the number of currently stored signals equals the defined number. Using this method combined with the threshold controlling, the complexity of the method is rapidly reduced with only slight performance degradation.

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