Coordinated SINR Balancing Techniques for Multi-Cell Downlink Transmission
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Abstract—In this paper, we consider the network multiple-input multiple-output (MIMO) where the base stations (BS) exchange only channel state information (CSI) to jointly design their transmission strategy. We particularly focus on a signal-to-interference-plus-noise ratio (SINR) balancing problem. First, the achievable rate regions with symmetric complex (SC) and asymmetric complex (AC) signaling techniques are investigated. It is observed that the AC signaling shows a substantial gain over the SC signaling in terms of maximizing the worst-user rate as the system signal-to-noise-ratio (SNR) increases. After establishing the optimal SINR balancing algorithm with the SC signaling, we confirm the effectiveness of the AC signaling by proposing an efficient balancing scheme which outperforms the optimal SC signaling scheme in overall SNR regime.

I. INTRODUCTION

The network multiple-input multiple-output (MIMO) is a good candidate for achieving high spectral efficiency by actively overcoming the inter-cell interference. This paper considers the case where the base stations (BS) exchange the channel state information (CSI) only without users’ data coordination since the backhaul link between the BSs has the limited capacity in the coordinated multi-cell transmission. In this scenario, the BSs share global CSI to jointly design their transmission strategies such as power control, beamforming and rate control. The efforts to improve the network throughput of this kind of multi-cell systems have been made in [1] and [2]. Along with the sum-rate metric, other common criteria include the worst-user rate also referred to as signal-to-interference-plus-noise ratio (SINR) balancing [3] and fairness [4]. The SINR-balancing technique has an important meaning since its solution helps to provide quality-of-service (QoS) for users and characterize different achievable rate regions [5].

In this paper, we tackle the problem of optimizing the worst-user rate for multi-cell systems which can be modeled as single-input single-output (SISO) interference channels. First, we investigate the achievable rate regions with symmetric complex (SC) and asymmetric complex (AC) signaling techniques. The advantage of the AC signaling has been well investigated in terms of degree-of-freedom (DOF) for interference channels with constant channel coefficients in [6]. For the case of the SC signaling, an exact characterization of Pareto boundary is possible for an arbitrary number of cells as shown in [7]. Using the derived Pareto boundary, we observe that the AC signaling shows a substantial gain over the SC signaling in terms of maximizing the worst-user rate as the system signal-to-noise-ratio (SNR) increases. After establishing the optimal SINR balancing algorithm with the SC signaling, we confirm the effectiveness of the AC signaling by proposing an efficient balancing scheme which outperforms the optimal SC signaling scheme in overall SNR regime.

II. SYSTEM MODEL

We consider a multi-cell downlink transmission system where only the CSI is exchanged. Then, the overall system can be modeled as K-user interference channels where BS \( i \) has \( i = 1, \ldots, K \) transmits the message \( W_i \) intended for user \( i \). Adopting a frequency-flat fading model, the received signal at user \( i \) is written as

\[
y_i = h_{i,i}x_i + \sum_{j \neq i} h_{i,j}x_j + n_i,
\]

where \( x_j \) is the transmitted signal at BS \( j \), \( h_{i,j} \) indicates the channel coefficient from BS \( j \) to user \( i \), and \( n_i \) represents the additive Gaussian noise at user \( i \) as \( n_i \sim CN(0, N_0) \). For simplicity, every node is assumed to be equipped with single antenna so that all of \( y_j, x_j, h_{i,j} \) and \( n_i \) are complex scalars. Because the implementation of multi-user detection may be difficult, we assume that the interference terms are treated as noise [8]. Since each BS has its own power amplifier, we consider the per-BS power constraint as \( E[|x_i|^2] \leq P_{\text{max}} \).

Using the real-valued representation, the input-output equation of (1) can be equivalently given as

\[
y_i = H_{i,i}x_i + \sum_{j \neq i} H_{i,j}x_j + n_i,
\]

where \( y_i = [\Re\{y_i\} \ \Im\{y_i\}]^T \), \( x_i = [\Re\{x_i\} \ \Im\{x_i\}]^T \), \( n_i = [\Re\{n_i\} \ \Im\{n_i\}]^T \) and

\[
H_{i,j} = \begin{bmatrix}
\Re\{H_{i,j}\} & -\Im\{H_{i,j}\} \\
\Im\{H_{i,j}\} & \Re\{H_{i,j}\}
\end{bmatrix}.
\]

Assuming that the Gaussian codebook is used at each BS, the achievable rate of each link is determined by the covariance matrix of \( x_i \) denoted by \( Q_i \triangleq E[x_i x_i^H] \). Since \( Q_i \) is positive semi-definite, it can be expressed using the eigenvalue decomposition [9] without loss of generality as

\[
Q_i = J(\phi_i) \text{diag}(\lambda_i, P_i - \lambda_i) J(-\phi_i)
\]
where $0 \leq P_i \leq P_{\text{max}}$, $0 \leq \lambda_i \leq P_i$ and $\mathbf{J}(x)$ is a unitary rotation matrix defined as
\[
\mathbf{J}(x) = \begin{bmatrix}
\cos(x) & -\sin(x) \\
\sin(x) & \cos(x)
\end{bmatrix}.
\] (5)
Here, $P_i$ is the transmit power level at BS $i$ and $\lambda_i$ adjusts the degree of asymmetry, which means that the distribution of $x_i$ is circularly symmetric when $\lambda_i = \frac{P_i}{P_{\text{max}}}$ and the degree of asymmetry grows as $|\lambda_i - \frac{P_i}{P_{\text{max}}}|$ increases. Throughout the paper, we consider that the SC signaling is a subset of the AC signaling with $\lambda_i = \frac{P_i}{P_{\text{max}}}$. We notice that $\mathbf{H}_{i,j} \in \mathbf{P}$ can also be represented using the rotation matrix as $\mathbf{H}_{i,j} = \mathbf{A}_{i,j} \mathbf{J}(\theta_{i,j})$ where $A_{i,j} = |h_{i,j}|$ and $\theta_{i,j} = \angle h_{i,j}$.

### III. Achievable Rate Region with SC and AC Signaling

In this section, we study the rate regions which are achievable with the SC and AC signaling to observe the efficiency of the AC signaling in terms of SINR balancing.

#### A. SC Signaling

In the SC signaling, we can perform only the power control, i.e., $x_i = \sqrt{P_i} s_i$ where $s_i \sim CN(0,1)$ denotes the data symbol intended for user $i$ and $P_i \leq P_{\text{max}}$ is the transmit power at BS $i$. Then, the achievable rate of link $k$ is a function of the power levels of $P_i$ as $R_{\text{SC}}^k(\{P_i\}) = \log_2 (1 + \gamma_{\text{SC}}^k(\{P_i\}))$ where $\gamma_{\text{SC}}^k(\{P_i\})$ is the instantaneous channel power.

Theorem 1: The Pareto boundary set for $R_{\text{SC}}$ is exactly given as $S_{\text{SC}}$ where
\[
S_{\text{SC}} = \bigcup_{i=1,\ldots,K} \bigcup_{0 \leq P_i \leq P_{\text{max}},i} \{ R_{\text{SC}}^k(\{P_i\}), \ldots, R_{\text{SC}}^k(\{P_i\}) \}.
\]

Proof: To complete the proof, the following two statements should be shown:

1. The set of Pareto-optimal points $\subset S_{\text{SC}}$.
2. The set of Pareto-optimal points $\supset S_{\text{SC}}$.

Since the proof for (7) is straightforward, we illustrate how to show (8). Without loss of generality, it is sufficient to show that if $P_{k}^1 = P_{\text{max}}$, then $\{P_1^1, \ldots, P_K^1\}$ is Pareto-optimal for any $P_1^1, \ldots, P_{K-1}^1$. We do this by dividing all possible cases for $P_1^1, \ldots, P_{K-1}^1$ into the following two cases.

Case 1) $P_1^1 = \cdots = P_{K-1}^1 = 0$

If we want to make some of $R_1, \ldots, R_{K-1}$ nonzero, then some of $P_1^1, \ldots, P_{K-1}^1$ become nonzero and the rate of link $k$ decreases. Thus, $(P_1^1, \ldots, P_K^1)$ is Pareto-optimal for this case.

Case 2) $l$ elements of $P_1^1, \ldots, P_{K-1}^1$ are nonzero

Without loss of generality, suppose that $P_1^1, \ldots, P_l^1$ are nonzero where $1 \leq l \leq K-1$. If we consider any other power tuple $(P_l^1, \ldots, P_{K-1}^1)$ such that $(P_1^1 \neq P_1^1)$ or $\cdots$ or $(P_l^1 \neq P_l^1)$, then the proof is completed by showing that there exists user $i$ such that $R_{\text{SC}}^k(\{P_l^1\}) < R_{\text{SC}}^k(\{P_l^1\})$. To this end, all possible cases are classified into the following two subcases.

Case 2-1) $P_l^1 > P_{K-1}^1$

Since $P_{K-1}^1 = P_{\text{max}}$, we have $R_{\text{SC}}^k(\{P_l^1\}) < R_{\text{SC}}^k(\{P_l^1\})$.

Case 2-2) $P_l^1 < P_{K-1}^1$

Define two partition sets $S_{\text{down}}$ and $S_{\text{up}}$ for the whole index set $\{1, \ldots, K\}$ as
\[
S_{\text{down}} = \{i | P_i^1 < P_i^1\},
S_{\text{up}} = \{1, \ldots, K\} - \{i_{\min}\} - S_{\text{down}}.
\]

Then, we can show that $R_{\text{SC}}^k(\{P_l^1\}) < R_{\text{SC}}^k(\{P_l^1\})$ where $i_{\min} = \arg \min_{i \in S_{\text{down}}} \{P_i^1 - P_i^1\}$. First, $R_{\text{SC}}^k(\{P_l^1\})$ is computed as
\[
R_{\text{SC}}^k(\{P_l^1\}) = \log_2 \left( 1 + \frac{g_{i_{\min},j} P_l^1}{N_0 + \sum_{j \in S_{\text{up}}} g_{i_{\min},j} P_j^1 + \sum_{j \in S_{\text{down}}} g_{i_{\min},j} P_j^1} \right).
\]

Defining $\alpha = \frac{P_l^1}{P_j^1}$, the upperbound on $R_{\text{SC}}^k(\{P_l^1\})$ is derived as
\[
R_{\text{SC}}^k(\{P_l^1\}) = \log_2 \left( 1 + \frac{g_{i_{\min},j} P_l^1}{N_0 + \sum_{j \in S_{\text{up}}} g_{i_{\min},j} P_j^1 + \sum_{j \in S_{\text{down}}} g_{i_{\min},j} P_j^1} \right)
\]

where the inequality comes from $P_j^1 \geq P_j^1$ if $j \in S_{\text{up}}$ and $P_j^1 \geq \alpha P_j^1$ if $j \in S_{\text{down}}$. Consider a function $f(\eta)$ defined as
\[
f(\eta) = \log_2 \left( 1 + \frac{\eta g_{i_{\min},j} P_l^1}{N_0 + \sum_{j \in S_{\text{up}}} g_{i_{\min},j} P_j^1 + \sum_{j \in S_{\text{down}}} g_{i_{\min},j} P_j^1} \right)
\]

which is monotonically increasing with respect to (w.r.t) $\eta$. For positive $\eta$, we get the following inequality as
\[
R_{\text{SC}}^k(\{P_l^1\}) = f(\eta)|_{\eta=1} > f(\eta)|_{\eta=\alpha} \geq R_{\text{SC}}^k(\{P_l^1\})
\]

In conclusion, it is shown that $R_{\text{SC}}^k(\{P_l^1\}) > R_{\text{SC}}^k(\{P_l^1\})$.

#### B. AC Signaling

Compared to the SC signaling case, the achievable rate characterization with the AC signaling needs more parameters. Thus, we focus on the case of $K = 2$ for simplicity. Then,
the achievable rate of each user $k$ is given as (10) where we define $1 = 2$ and $2 = 1$.

The achievable rate region with the AC signaling is determined as

$$R_k^A(\{P_i, \lambda_i, \phi_i\}) = \frac{1}{2} \log_2 \left( \frac{\det \left( \sum_j H_{k,j} \mathbf{J}(\phi_j) \text{diag}(\lambda_j^2, (P_j - \lambda_j)^2) \mathbf{J}(-\phi_j) H_{k,j}^T + \frac{N_0}{2} \mathbf{I} \right)}{\det \left( H_{k,k} \mathbf{J}(\phi_k) \text{diag}(\lambda_k^2, (P_k - \lambda_k)^2) \mathbf{J}(-\phi_k) H_{k,k}^T + \frac{N_0}{2} \mathbf{I} \right)} \right).$$

(10)

In order to plot the rate region $R_k^A$, the exhaustive search is required over 6 parameters. In the following theorem, we show that the reduced 4-dimensional search can be carried out without loss of optimality.

**Theorem 2:** A set $S_k^A$ includes all the Pareto-boundary points of $R_k^A$ where $S_k^A$ is defined as

$$S_k^A = \bigcup_{k=1,2} \left\{ \left\{ R_k^A(\{P_i, \lambda_i, \phi_i\}), R_k^A(\{P_i, \lambda_i, \phi_i\}) \right\} \right\} .$$

Here, $[\phi_1^{\text{min}}, \phi_1^{\text{max}}]$ are the reduced range of $\phi_1$ for given parameters $\{P_i, \lambda_i\}$ defined as

$$\phi_1^{\text{min}} = \min \{ \phi_1^R_i(\{P_k, \lambda_k\}), \hat{n} \pi + \phi_1^R_i(\{P_k, \lambda_k\}) \},$$

$$\phi_1^{\text{max}} = \max \{ \phi_1^R_i(\{P_k, \lambda_k\}), \hat{n} \pi + \phi_1^R_i(\{P_k, \lambda_k\}) \},$$

where $\phi_1^R_i(\{P_k, \lambda_k\})$ and $\hat{n}$ are given as

$$\phi_1^R_i(\{P_k, \lambda_k\}) = \begin{cases} -\beta_i, & \text{if } (P_k - 2\lambda_k)(2\lambda_i - P_i) \geq 0, \\ -\beta_i + \frac{\pi}{2}, & \text{else,} \end{cases}$$

$$\hat{n} = \arg \min_{n \in \mathbb{Z}} \left| \phi_1^R_i(\{P_k, \lambda_k\}) - \phi_1^R_i(\{P_k, \lambda_k\}) - n\pi \right|$$

and $\beta_i$ is defined as $\beta_i = \theta_{i,i} - \theta_{i,i}$.

**Proof:** Following the same steps as in the proof of Theorem 1, we can show that at least one of $P_1$ and $P_2$ should be $P_{\text{max}}$ for any Pareto boundary points. Also, it is easy to see that for given other parameters, the above rate depends only on $\phi_1 - \phi_2$. Thus, we can set $\phi_2 = 0$ without loss of generality. Using some trigonometric formula, we can obtain a simplified rate expression (11). For given other parameters, both $R_1^A(\{P_i, \lambda_i\}, \phi_1)$ and $R_2^A(\{P_i, \lambda_i\}, \phi_2)$ are periodic functions w.r.t. $\phi_1$ with period $\pi$. Also, $R_k^A(\{P_i, \lambda_i\}, \phi_1)$ hits its maximum at $\phi_1 = \phi_1^R_i(\{P_i, \lambda_i\}).$ Thus, for given $\{P_i, \lambda_i\}$, it is sufficient to search within the range between $\phi_1^R_i(\{P_i, \lambda_i\})$ and $n\pi + \phi_1^R_i(\{P_i, \lambda_i\})$ for any $n \in \mathbb{Z}$. The integer $\hat{n}$ is chosen to minimize the length of the search range.

It is interesting to see that in the rate expression (11), the achievable rate does not depend on $\phi_1$ if at least one of two BSs uses the SC signaling (i.e., $\lambda_i = \frac{P_i}{2}$). This tells us that if either the desired signal or the interference-plus-noise term is circularly symmetric, then the dominant direction of the distribution of the asymmetric complex term has no influence on the achievable rate. In Figure 1, we present the achievable rate region for a sample channel realization at low and high SNR. The optimal point in terms of the worst-user rate is the intersection between the line $R_1 = R_2$ and the Pareto boundary of the rate region. It is shown in the plot that the AC signaling can achieve the highest worst-user rate than the SC signaling and a performance gain increases as SNR grows.

**IV. COORDINATED SINR BALANCING WITH SC SIGNALING**

In this section, we establish an SINR balancing scheme with the SC signaling. Since each user’s rate $R_k^SC(\{P_i\})$ is
monotonically increasing w.r.t. $\gamma_k^{SC}(\{P_i\})$, the optimization problem for the SINR balancing can be formulated as

$$\max_{0 \leq P_i \leq P_{\text{max}}, \forall i} \min_k \gamma_k^{SC}(\{P_i\}).$$

For the remainder of this section, we focus on the two-cell environment for simplicity.

Thanks to Theorem 1, the optimal solution for (12) can be obtained as one of the solutions of the following two problems:

$$\max_{0 \leq P_i \leq P_{\text{max}}, P_2 = P_{\text{max}}, \forall i} \min_k \gamma_k^{SC}(\{P_i\}),$$

$$\max_{0 \leq P_i \leq P_{\text{max}}, P_1 = P_{\text{max}}, \forall i} \min_k \gamma_k^{SC}(\{P_i\}).$$

Let $\hat{P}_1$ and $\hat{P}_2$ denote the solutions for the problems (13) and (14), respectively. To illustrate the key idea of the proposed algorithm, we describe how to find $\hat{P}_1$, i.e., the optimization of $P_1$ for fixed $P_2 = P_{\text{max}}$. To this end, we define two functions

$$\gamma_1^{P_1}(P_1) = \gamma_1(\{P_1\})|_{P_2 = P_{\text{max}}} \text{ and } \gamma_2^{P_2}(P_1) = \gamma_2(\{P_1\})|_{P_2 = P_{\text{max}}}.$$

Then, finding $\hat{P}_1$ can be formulated as

$$\hat{P}_1 = \arg \max_{0 \leq P_1 \leq P_{\text{max}}} \min\{\gamma_1^{P_1}(P_1), \gamma_2^{P_2}(P_1)\}. \quad (15)$$

To solve the above problem efficiently, the following properties are useful. First, $\gamma_1^{P_2}(0) > \gamma_2^{P_2}(0) = 0$. Second, $\gamma_1^{P_2}(P_1)$ and $\gamma_2^{P_2}(P_1)$ are monotonically increasing and decreasing w.r.t. $P_1$, respectively. From these properties, we see that if $\gamma_1^{P_2}(P_{\text{max}}) > \gamma_2^{P_2}(P_{\text{max}})$, there exists a crossover point within $0 \leq P_1 \leq P_{\text{max}}$, and the minimum of $\gamma_1^{P_2}(P_1)$ and $\gamma_2^{P_2}(P_1)$ is maximized when $\gamma_1^{P_2}(P_1) = \gamma_2^{P_2}(P_1)$. Otherwise, the solution for (13) is just $P_1 = P_{\text{max}}$. As a result, $\hat{P}_1$ and $\hat{P}_2$ are obtained as (16). In summary, the optimal solution $(P_{\text{opt}}^{P_1}, P_{\text{opt}}^{P_2})$ for the problem (12) is given by

$$(P_{\text{opt}}^{P_1}, P_{\text{opt}}^{P_2}) = \arg \max_{(P_1, P_2) \in \pi} \min_k \{R_k^{SC}(\{P_i\})\}$$

where $\pi = \{(\hat{P}_1, P_{\text{max}}), (P_{\text{max}}, \hat{P}_2)\}$. Note that this optimal solution requires a search over only 2 candidates.

V. IMPROVED SINR BALANCING WITH AC SIGNALING

In this section, we propose an efficient SINR balancing scheme based on the AC signaling for 2x2 downlink where the system equation with the real-valued representation is rewritten from (2) as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} H_{1,1} & H_{1,2} \\ H_{2,1} & H_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}.$$ 

In this 2x2 MIMO interference channels with real channel coefficients, we highlight the performance gain of the AC signaling at high SNR by assuming a rank-1 transmission for each BS, since the total DOF of two-user 2x2 MIMO interference channels exactly equals 2 [10]. This choice corresponds to $\lambda_0 = 0$ or $\lambda_1 = P_1$ which leads to an extremely asymmetric distribution of $x_i$.

Then, the transmit beamforming is performed at BS $i$ for $i = 1, 2$ as $x_i = v_i u_i$, where $u_i \sim \mathcal{N}(0, 1)$ is the data symbol intended for user $i$ and $v_i \in \mathbb{R}^{2 \times 1}$ denotes the transmit beamformers. Also, defining $w_i \in \mathbb{R}^{1 \times 2}$ as the receive combiner at user $i$, the receive filter output $\hat{u}_i$ is given as $\hat{u}_i = w_i y_i = w_i H_i v_i u_i + w_i H_i v_i u_i + \hat{n}_i$, where $\hat{n}_i = w_i n_i \sim \mathcal{N}(0, \frac{N_0}{2} ||w_i||^2)$ is the filtered noise.

Thus, the individual rate of user $i$ is computed as $R_i = \frac{1}{2} \log_2 \left( 1 + \frac{|w_i H_i v_i|^2}{||w_i||^2} \right)$. Then, the problem for maximizing the worst-user rate is formulated as

$$\max_{v_i, w_i, \forall i} \min_{1 \leq i \leq 2} \{R_i^{AC}, R_2^{AC}\},$$

subject to $||v_i||^2 \leq P_{\text{max}}, \forall i$. (17)

Since identifying the optimal solution for (17) is somewhat complicated, we propose an one-shot suboptimal algorithm by assuming zero-forcing (ZF) receivers, i.e., $w_i$’s are designed such that $w_i H_{1,i} v_2 = w_i H_{2,i} v_1 = 0$. It will be shown in the simulation section that the loss from this assumption becomes minor at high SNR of our interest. Our problem is then simplified as

$$\max_{v_i, w_i, \forall i} \min_{1 \leq i \leq 2} \frac{|w_i H_i v_i|^2}{||w_i||^2},$$

subject to $||v_i||^2 \leq P_{\text{max}}, w_i \perp H_i v_i, \forall i$ where the interference terms are eliminated due to the ZF constraint.

In this formulation, we should optimize the power levels and directions of the beamforming vectors $\{v_i\}$. Since we have no inter-cell interference, the full power transmission is optimal, i.e., $||v_i||^2 = P_{\text{max}}$. Then, the transmit beamforming vectors have a form of $v_i = \sqrt{P_{\text{max}}} j(\phi_i)$, for $i = 1, 2$ where $j(x)$ is defined as $j(x) = [\cos(x) \sin(x)]^T$ and $\phi_i$ determines the direction of $v_i$. Note that $j(x)$ and $j^T(y)$ have the useful properties as $J(x)j(y) = j(x+y)$, and $j(x)^T j(y) = \cos(-x+y)$. Using the above property, the received signal vectors can be rewritten as

$$y_k = \sqrt{P_{\text{max}}} A_{k,k} j(\theta_k + \phi_k) u_k + \sqrt{P_{\text{max}}} A_{k,k} j(\theta_k + \phi_k) u_k + n_k.$$ 

For given $\{\phi_i\}$, the ZF receiver is computed as $w_k = j(\theta_k + \phi_k - \frac{\pi}{2})$. Thus, the ZF filter output is given as

$$\hat{u}_k = \sqrt{P_{\text{max}}} A_{k,k} \cos(\frac{\pi}{2} + \theta_k - \theta_k + \phi_k - \phi_k) u_k + \hat{n}_k,$$

where $\hat{n}_k \sim \mathcal{N}(0, \frac{N_0}{2})$ since $||w_k||^2 = 1$.

Now, the remaining problem is to optimize $\phi_1$ and $\phi_2$ as

$$\max_{\phi_1, \phi_2} \min_{i=1,2} A_{i,i} \cos^2(\phi_1 - \phi_2 + \alpha_i)$$
where \( \alpha_k = (-1)^{k-1} \left( \frac{\pi}{2} + \theta_{k,k} - \theta_{k,k} \right) \). Since the objective function depends only on \( \phi_1 - \phi_2 \), we set \( \phi_2 = 0 \) without loss of generality and we obtain the modified problem as 
\[
\phi_1^{\text{opt}} = \arg \max \phi_1 f(\phi_1) \quad \text{where} \quad f(\phi_1) = \min_{i=1,2} |A_{i,i} \cos(\phi_1 + \alpha_i)|.
\]

It is straightforward to see that the maximum value of \( f(\phi_1) \) occurs only when \( |A_{\min,i} \cos(\phi_1 + \alpha_{\min})| \) is maximized or \( |A_{1,1} \cos(\phi_1 + \alpha_1)| = |A_{2,2} \cos(\phi_1 + \alpha_2)| \) where we define \( i_{\min} = \arg \min_i A_{i,i} \). In other words, the objective function is maximized only when two envelopes have the same absolute value or the absolute value of the envelope with the small amplitude is maximized. According to the ratio \( \frac{A_{1,1}}{A_{2,2}} \) and the phase offset \( \alpha_1 - \alpha_2 \), the point satisfying the above conditions is optimal for maximizing the objective function. Now, we illustrate how to find the optimal \( \phi_1 \). First, two envelopes have the same absolute value when \( A_{1,1} \cos(\phi_1 + \alpha_1) = \pm A_{2,2} \cos(\phi_1 + \alpha_2) \). After some manipulations, we see that this is satisfied when \( \phi_1 = \pm \frac{\pi}{2} \left( \frac{A_{1,1} \cos \alpha_1 \pm A_{2,2} \cos \alpha_2}{A_{1,1} \sin \alpha_1 \pm A_{2,2} \sin \alpha_2} \right) \).

Second, it is easy to see that the envelope with the small amplitude has its maximum absolute value at \( \phi_1 = -\alpha_{i_{\min}} \).

As a result, the transmit beamformers \( \mathbf{v}_k \) and the receiver combiners \( \mathbf{w}_k \) of our proposed scheme for \( k = 1, 2 \) are computed as
\[
\mathbf{v}_k = \sqrt{P_{\text{max}}} (\phi_1^{\text{opt}}), \quad \text{and} \quad \mathbf{w}_k = \mathbf{j} \left( \theta_{k,k} + \phi_1^{\text{opt}} - \frac{\pi}{2} \right)^T,
\]
where \( \phi_1^{\text{opt}} = \arg \max \phi_1 f(\phi_1) \) and \( \phi_2^{\text{opt}} = 0 \). Here the search candidate \( \chi \) of size 3 is given as
\[
\chi = \left\{ \tan^{-1} \left( \frac{A_{1,1} \cos \alpha_1 \pm A_{2,2} \cos \alpha_2}{A_{1,1} \sin \alpha_1 \pm A_{2,2} \sin \alpha_2} \right), -\alpha_{i_{\min}} \right\}.
\]

In the following section, we show that this simple proposed scheme achieves much improved performance over the power control schemes with the SC signaling in terms of the worst-user rate.

VI. SIMULATION RESULTS

In this section, we present simulation results to confirm the effectiveness of the AC signaling in terms of the worst-user rate. Figure 2 plots the average worst-user rate of various schemes as a function of \( \frac{P_{\text{max}}}{N_0} \). As expected, the proposed power control scheme exhibits the optimal performance for the SC signaling. However, the SINR balancing schemes with the SC signaling cannot achieve the linear increase w.r.t. SNR as they operate with the number of data streams large than the total DOF of two-user SISO interference channels which is equal to 1 [10]. On the other hand, the transmission schemes based on the AC signaling provide the worst-user rate which increases linearly with the SNR. The proposed choice of \( \phi_1 \) and \( \phi_2 \) achieves the same performance as the exhaustive search algorithm and obtains a performance gain of about 7 dB compared to the choice of \( \phi_1 = \phi_2 = 0 \).

VII. CONCLUSION

In this paper, we have addressed the problem of SINR balancing for multicell downlink systems. We have first studied the achievable rate regions with the SC and AC signaling. Meanwhile, the AC signaling is shown to have a potential to outperform the SC signaling as SNR increases. Thus, we have realized a performance gain of the AC signaling by proposing an efficient SINR balancing which outperforms the optimal SC signaling for overall SNR regime.

REFERENCES