Data-Driven Quality Monitoring and Fault Detection for Multimode Nonlinear Processes

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Abstract—This paper addresses the problem of quality monitoring and fault detection in nonlinear processes which are working in different operating points. For such processes the statistical model which is obtained from process data is different from one operating point to another, due to nonlinearities and set-point changes. Therefore the classical methods for quality monitoring and fault detection, e.g., partial least squares (PLS), may not be suitable. To this end, a new approach is proposed based on the modeling of nonlinear process as a piecewise linear parameter varying system, considering the behavior of the plant in each operating point as linear time invariant with different parameters in each operating point. The expectation-maximization (EM) algorithm is used to model the process as a finite mixture of Gaussian components and based on the identified model a Bayesian inference strategy is developed to detect the faults which influence the product quality. Finally, the usefulness of the proposed method is demonstrated on a laboratory continuous stirred tank heater (CSTH) setup.

I. INTRODUCTION

In many industrial manufacturing and production applications, early detection of faults and malfunctions which influence the product quality, is of great interest. Based on analytical process models which are obtained from first principles, model based schemes have been well established for monitoring and diagnosis purposes [1]. During the last decades the complexity of process plants has increased, which imposes great challenges on modeling and monitoring methods. Since the model-based techniques involve rigorous development of analytical process models, their application on complex large scale plants is sometimes not feasible and becomes unrealistic. On the other hand, in modern technical processes huge amount of process variables are measured and recorded in process historian which can be used to design monitoring systems. Using the historical process data, the well-established data-driven methods offer a powerful tool to extract useful information and underlying structure of process. For that several methods based on multivariate statistics [2], [3], [4], [5] and subspace based identification [6] have been proposed for linear systems.

Many industrial processes however are intrinsically nonlinear and operate in different operation regimes due to different product specifications, working environments and economic considerations. Due to the nonlinearity of processes, the performance of the classical multivariate statistical process monitoring methods, which are mainly based on the linearity assumption, becomes unsatisfactory, since the process model will change from one operating point to another. It leads to false alarms or missed detection of faults.

Recently, some research effort has been done for data-driven nonlinear process identification and monitoring based on the multiple model assumption. In [7], a new data-driven approach for multimode process monitoring based on Bayesian inference has been developed. The proposed approach is based on the identification of finite Gaussian mixture model (FGMM). A Bayesian inference strategy is further utilized to derive an index for fault detection purpose. In [8], a new probabilistic strategy has been proposed for result combination in different operating modes, to improve the performance of the probabilistic PCA approach [9] for process monitoring. In [10], the authors developed a method for nonlinear process identification based on multiple-model assumption, where the Auto Regressive eXogenous (ARX) local models are estimated using EM algorithm [11]. The approach is based on the assumption that the scheduling variable, which represents the operating points on which the nonlinear process is running, is available.

Motivated by the above observations, in this paper a new approach is proposed for monitoring the product quality in nonlinear systems and detection of the quality related faults. The nonlinear system is assumed to be linear around the operating points and considered as a piecewise linear system corresponding to each operating mode where the data in each operating condition follows multivariate normal distribution. Using EM algorithm, the regression model which describes the correlation structure of plant measurements and the quality variables is identified for each operating mode. Based on the regression coefficients the monitoring scheme is designed and using a Bayesian inference strategy a new index is proposed for fault detection purpose.

The rest of this paper is as follows: in Section II the preliminaries and problem formulation are described. Section III explains the identification of the regression model parameters and design of monitoring scheme. The paper follows with application of the proposed method on a CSTH process in Section IV and ends with concluding remarks in Section V.

II. PRELIMINARIES AND PROBLEM FORMULATION

Partial least squares (PLS) [4], [5] is recognized as a powerful tool to model processes and their underlying structures and used in numerous applications in chemometrics and industrial process monitoring. The goal of PLS regression is to predict the output data set (quality variables), \( \hat{Y} \) from an...
input data set (process measurements), $X$ and to describe their common structure.

The PLS algorithm involves projecting $X \in \mathbb{R}^{N \times n}$ and $Y \in \mathbb{R}^{N \times m}$ onto the latent variables $T \in \mathbb{R}^{N \times \gamma}$ to build the correlation model of $X$ and $Y$

$$X = TP^T + \tilde{X} = \hat{X} + \tilde{X}$$

$$Y = TQ^T + E = XM + E_y$$ \hspace{1cm} (1)

where $\gamma$ is the number of latent variables and $P \in \mathbb{R}^{p \times \gamma}$ and $Q \in \mathbb{R}^{m \times \gamma}$ are loading matrices of $X$ and $Y$. The matrix $M \in \mathbb{R}^{p \times m}$ is known as regression coefficient. Typically the number of latent variables $\gamma$ is determined by using cross-validation test [12]. In other words, PLS decomposes the subspace of regressor $X$, into two subspaces $\hat{X}$ and $\tilde{X}$, with respect to their correlation to dependant variables $Y$. For process monitoring purpose, the Hotelling’s $T^2$ and squared prediction error (SPE) indices are used for monitoring these two subspaces.

The standard PLS approach for modeling the industrial process is formulated as follows:

- Collect $N$ samples of $x$ and $y$ and normalize them to zero mean and unit variance to build $X = [x_1 \cdots x_p] \in \mathbb{R}^{N \times p}$ and $Y = [y_1 \cdots y_m] \in \mathbb{R}^{N \times m}$
- Perform $\gamma$ times following iterative computation for $k = 1, \cdots, \gamma$:
  $$(w_k^T, q_k^T) = \arg \max_{|w_k|=1, |q_k|=1} w_k^T X_k^T Y_{qk}, \ X_1 = X$$
  $$t_k = X_1 w_k, \ p_k = \frac{X_k}{|t_k|^2}, \ X_{k+1} = X_k - t_k p_k^T$$
  $$r_1 = w_1^T, \ r_k = \prod_{i=1}^{k-1} (I_{p \times p} - w_i^T p_i^T) w_k^T, \ k > 1$$
- Calculate $P$, $Q$, $R$, $T$ and $M$ using
  $$P = [p_1 \cdots p_T], \ T = [t_1 \cdots t_T]$$
  $$Q = [q_1 \cdots q_T], \ R = [r_1 \cdots r_T]$$
  $$M = RQ^T$$

Recently, in [13] it has been revealed that the classical PLS approach may result in variation in $\hat{X}$ orthogonal to $Y$ and $\tilde{X}$ may contain large variability of $X$ and is therefore not suitable to monitor the faults which affect the quality variables and leads to miss-classification of faults. Moreover the iterative procedure involved in computation of the regression model leads to difficulties to interpret the PLS model. To solve the above mentioned problems a new modified approach has been proposed in [14], which avoids the above mentioned drawbacks and is computationally more simple. Using this approach, when $N \gg \max(n, m)$, (1) can be written as follows

$$\frac{1}{N-1} Y^T X = \frac{1}{N-1} M^T X^T X + \frac{1}{N-1} E^T X \approx M^T X^T X \frac{N}{N-1}$$ \hspace{1cm} (2)

This is due to the fact that $\text{cov}(e_y, x) = 0$. Thus, the regression coefficient can be calculated as

$$M = (X^T X)^{-1} X^T Y$$ \hspace{1cm} (3)

Based on it $X$ can be decomposed into two parts, $\hat{X}$ and $\tilde{X}$ such that $\hat{X}$ is fully correlated to $Y$ and $\tilde{X}$ is orthogonal to $\hat{X}$ and has no contribution in predicting $Y$. To detect the faults which may affect the product quality $Y$, an index is proposed based on monitoring the $\hat{X}$ subspace

$$T_i^2 = x^T P_M (P_M^T X P_M)^{-1} P_M^T x$$ \hspace{1cm} (4)

where $P_M \in \mathbb{R}^{m \times m}$ is obtained by performing SVD on $M M^T$

$$M M^T = [P_M \ \tilde{P}_M] \begin{bmatrix} \Lambda_m & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_M^T \\ \tilde{P}_M^T \end{bmatrix}$$ \hspace{1cm} (5)

and the threshold for fault detection follows

$$J_{th,i}^2 = \frac{m(N^2 - 1)}{N(N - m)} F_{\alpha}(m, N - m)$$ \hspace{1cm} (6)

where $F_{\alpha}(m, N - m)$ is $F$-distribution with parameters $m$ and $N - m$ and $\alpha$ is the confidence level. If the number of training samples is large enough then

$$J_{th,i}^2 = \chi_{\alpha,m}^2$$ \hspace{1cm} (7)

where $\chi_{\alpha,m}^2$ is $\chi^2$ distribution with $m$ degrees of freedom and confidence level $\alpha$. The application of standard PLS algorithm is based on the uni-model multivariate normal distribution of $x$ and $y$. However in some industrial applications, the process has nonlinear structure and standard PLS approach cannot be used to model the underlying structure due to operating point changes.

To cope with this problem, in this paper it is assumed that the model of nonlinear process under consideration can be represented with finite number of linear models corresponding to each operating point. Moreover the data for each linear model follows multivariate Gaussian distribution. Thus, the training data available for design of monitoring scheme is a mixture of finite Gaussian components [15].

III. MULTIMODE PROCESS MONITORING

Consider a nonlinear process working in $K$ different operating modes $M_1, M_2, \cdots, M_K$, in which, each mode is characterized by (1) with different model parameters $M_i$. The purpose is to design a monitoring scheme for the process using the modified PLS approach in (2)-(6) assuming that the model parameters for each operating mode are unknown.

A. Estimation of model parameters

To design the monitoring scheme, first the mixture model should be identified, using historical data. The historical data set contains measurements for all normal process operating modes. Assume that the available historical data $D$ is collected from $N$ different samples, each sample contains the measurements of process and quality variables

$$D = \left\{ [y(1) \ x(1) \ \cdots \ y(N) \ x(N)] \right\}$$

where $d_i \in \mathbb{R}^{m+n}$ is a sample from a multimode process and its probability density function can be represented as a finite Gaussian mixture model

$$p(d|\theta) = \sum_{k=1}^{K} w_k g(d|\theta_k).$$ \hspace{1cm} (9)
$K$ is the number of mixture components, $w_k$ is the weight of $k$th component, $\mathcal{M}_k$, $\theta_k$ are sets of parameters of the $k$th Gaussian component, $\theta = \{\theta_1, \ldots, \theta_K\}$ and
\[
g(d|\theta_k) = \frac{1}{(2\pi)^{m/2}|\Sigma_k|^{1/2}} \exp \left[ -\frac{1}{2} (d - \mu_k)^T \Sigma_k^{-1} (d - \mu_k) \right].
\]
To construct the monitoring scheme, the following parameters should be identified
\[
\Theta_k = \{w_k, \mu_{x,k}, \mu_{y,k}, x_{xx,k}, x_{yy,k}, y_{yy,k}\}
\]  
for $k = 1, 2, \ldots, K$. This can be done by assignment of log-likelihood of mixture components as
\[
\log p(D|\Theta) = \log \prod_{i=1}^{N} p(d_i|\Theta) = \sum_{i=1}^{N} \log \sum_{k=1}^{K} w_k p(d_i|\Theta_k). 
\]
The maximum likelihood estimate (MLE) can be achieved by
\[
\hat{\Theta}_{MLE} = \arg \max_{\Theta} \{\log p(D|\Theta)\}, 
\]  
or alternatively the solution can be achieved by maximum a posteriori (MAP) criterion
\[
\hat{\Theta}_{MAP} = \arg \max_{\Theta} \{\log p(D|\Theta) + \log p(\Theta)\}. 
\]
The solution of (12) and (13) can not be found analytically. The EM algorithm can be used for this purpose. EM is an iterative algorithm which finds the local maxima of log-likelihood functions in (12) or (13). The EM algorithm is based on the assumption that $D$ is an incomplete data set in which the missing part is filled in finite mixture modeling can be understood as $N$ tags, $\mathcal{D} = \{1, 2, \ldots, N\}$ represents each sample generated in the respective operating mode. In EM algorithm the conditional expectation of the log-likelihood for complete data $\mathcal{C} = \{D, \mathcal{D}\}$ is calculated in E-step as follows
\[
\mathcal{D}(\Theta|\Theta^{old}) = E\{\log p(D, \mathcal{D}|\Theta)| D, \Theta^{old}\}
\]
and updates the estimation of parameters in M-step using
\[
\Theta = \arg \max_{\Theta} \mathcal{D}(\Theta|\Theta^{old})
\]
for MLE or
\[
\Theta = \arg \max_{\Theta} \mathcal{D}(\Theta|\Theta^{old} + \log p(\Theta))
\]
Based on MAP estimation. The conditional expectation in (14) can be written as shown in (17). In derivation of (17) it is assumed that the quality variable at $i$th sampling time instant is independent of the variable $z$ and process variables $x$ and only depends on their current values. The same assumption is made for process variables $x$. Thus $p(y_N, \ldots, y_1 | x_N, \ldots, x_1, z_N, \ldots, z_1, \Theta)$ and $p(x_N, \ldots, x_1 | z_N, \ldots, z_1, \Theta)$ are simplified to $\prod_{i=1}^{N} p(y_i | x_i, z_i, \Theta)$ and $\prod_{i=1}^{N} p(x_i | z_i, \Theta)$ respectively. Moreover
\[
\begin{align*}
x_i | z_i, \Theta & \sim \mathcal{N}(\mu_{x,k}, \Sigma_{x,k}) \\
y_i | x_i, z_i, \Theta & \sim \mathcal{N}(\mu_{y,k}, \Sigma_{y,k})
\end{align*}
\]
where
\[
\begin{align*}
\mu_{y,k} &= \mu_{y,k} + \Sigma_{y,k}^{-1} \Sigma_{x,k}^{-1} (x_i - \mu_{x,k}) \\
\Sigma_{y,k} &= \Sigma_{y,k} - \Sigma_{y,k}^{-1} \Sigma_{x,k}^{-1} \Sigma_{x,k} 
\end{align*}
\]
The M-step in EM algorithm is carried out by derivation of $\mathcal{D}(\Theta|\Theta^{old})$ with respect to the relevant unknown parameters. After performing the derivation and equating it to zero, the update of parameters in M-step is shown in (20).
\[
\begin{align*}
\mu_{x,k} &= \frac{\sum_{i=1}^{N} N \delta_k(d_i, x_i)}{\sum_{i=1}^{N} N \delta_k(d_i)} \\
\mu_{y,k} &= \frac{\sum_{i=1}^{N} N \delta_k(d_i, y_i)}{\sum_{i=1}^{N} N \delta_k(d_i)} \\
\Sigma_{x,k} &= \frac{\sum_{i=1}^{N} N \delta_k(d_i, x_i - \mu_{x,k}) (x_i - \mu_{x,k})^T}{\sum_{i=1}^{N} N \delta_k(d_i)} \\
\Sigma_{y,k} &= \frac{\sum_{i=1}^{N} N \delta_k(d_i, y_i - \mu_{y,k}) (y_i - \mu_{y,k})^T}{\sum_{i=1}^{N} N \delta_k(d_i)} \\
w_k &= \frac{\sum_{i=1}^{N} N \delta_k(d_i)}{N}
\end{align*}
\]
where $p(\mathcal{M}_k|d_i)$ is calculated in E-step as
\[
\begin{align*}
p(\mathcal{M}_k|d_i) &= \frac{w_k g(d_i|\theta_k)}{\sum_{k=1}^{K} w_k g(d_i|\theta_k)}
\end{align*}
\]
After the parameter estimation using EM algorithm the regression coefficients for modes $\mathcal{M}_1, \ldots, \mathcal{M}_K$ in PLS model can be calculated as shown in (3) using
\[
M_k = \Sigma_{x,k}^{-1} \Sigma_{y,k}
\]
or in the case that $\Sigma_{x,k}$ is not a full rank matrix
\[
M_k = \Sigma_{x,k}^{-1} \Sigma_{y,k}
\]
for $k = 1, \ldots, K$. In the next step, the estimated parameters $M_k, \mu_{x,k}, \mu_{y,k}, x_{xx,k}, x_{yy,k}$ will be used to design the monitoring scheme. The focus will be on the detection of faults, which influence the quality of products.

**B. Design of monitoring scheme**

For the fault detection purpose an index is further defined to represent the probability that the monitored sample belongs to a fault
\[
J_g(i) = p(x(i) \in f).
\]
\[ \mathcal{D}(\Theta | \Theta^{\text{old}}) = E \{ \log p(y_N, \cdots, y_1, x_N, \cdots, x_1, z_N, \cdots, z_1 | \Theta) | \mathcal{D}, \Theta^{\text{old}} \} \\
= E \{ \log p(y_N, \cdots, y_1 | x_N, \cdots, x_1, z_N, \cdots, z_1, \Theta) p(x_N, \cdots, x_1 | z_N, \cdots, z_1, \Theta) p(z_N, \cdots, z_1 | \Theta) | \mathcal{D}, \Theta^{\text{old}} \} \\
= E \{ \sum_{i=1}^{N} \log p(y_i | x_i, z_i, \Theta) + \log p(x_i | z_i, \Theta) + \log p(z_i | \Theta) | \mathcal{D}, \Theta^{\text{old}} \} \\
= \sum_{i=1}^{N} \sum_{k=1}^{K} p(z_i = k | \mathcal{D}, \Theta^{\text{old}}) \log p(y_i | x_i, \Theta_k) + \sum_{i=1}^{N} \sum_{k=1}^{K} p(z_i = k | \mathcal{D}, \Theta^{\text{old}}) \log p(x_i | \Theta_k) + \\
\sum_{i=1}^{N} \sum_{k=1}^{K} p(z_i = k | \mathcal{D}, \Theta^{\text{old}}) \log p(z_i = k | \Theta_k) \tag{17} \]

This index can be obtained through marginalization as

\[ J_g(i) = \frac{1}{K} \sum_{k=1}^{K} p(x(i) \in f | x(i) \in \mathcal{M}_k) p(x(i) \in \mathcal{M}_k). \tag{25} \]

The second term on the right side of (25) represents the probability that the given sample belongs to component \( \mathcal{M}_k \) which can be calculated from the probability density function (PDF) of multivariate normal distribution given the estimated parameters for each mode. The first term represents the probability of fault provided that the sample belongs to component \( \mathcal{M}_k \). To calculate \( p(x(i) \in f | x(i) \in \mathcal{M}_k) \), the \( T_k^2 \) index introduced in (4) will be utilized:

\[ p(x(i) \in f | x(i) \in \mathcal{M}_k) = p(T_k^2(x, k) \leq T_k^2(x, k)) \tag{26} \]

which can be calculated by integrating the \( F \) probability density function in (6) or \( \chi^2 \) distribution in (7) with appropriate degrees of freedom. Since

\[ 0 \leq J_g(i) \leq 1 \tag{27} \]

a confidence level \( (1 - \alpha) \) can be specified for fault detection purpose with the hypothesis as follows.

\[ \begin{align*}
J_g & \leq 1 - \alpha \quad \text{fault free} \\
J_g & > 1 - \alpha \quad \text{faulty} \tag{28}
\end{align*} \]

where \( \alpha \) represents the false alarm rate. The procedure for design of the proposed monitoring scheme is summarized in Algorithm 1.

**Algorithm 1: Design of fault detection scheme**

Step 1 Collect the normal operation data from different operating modes.

Step 2 Apply the EM algorithm to estimate the parameters shown in (10) using (20) and (21).

Step 3 Obtain \( M_k \) using (22) or (23) for \( k = 1, \cdots K \).

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**Fig. 1.** Piping and instrumentation diagram of CSTH plant

- \( T_1 \): Water temperature inside the stirred tank
- \( T_2 \): Water temperature inside the heating jacket
- \( T_3 \): Inflow water temperature
- \( T_4 \): Reserved water temperature
- \( L_1 \): Water level inside the stirred tank
- \( F_1 \): Cold water flow
- TIC: Temperature controller
- LIC: Level controller
- P: Pumps
- V: Valves
- M: Motors
Step 4 In on-line step when a new sample of measurements is available, for $k = 1, \cdots, K$:

4.1 Compute $T_2^\hat{x}(x_i,k)$ using (4) and the identified parameters of $\mathcal{M}_k$.

4.2 Compute $p(x(i) \in \mathcal{M}_k)$ using the multivariate Gaussian PDF.

4.3 Compute $p(x(i) \in f(x) \in \mathcal{M}_k)$ using $T_2^x(x_i,k)$ and (26).

Step 5 Calculate the fault detection index in (25).

Step 6 Use the fault detection hypothesis shown in (28) to detect fault and go to step 4.

IV. APPLICATION ON CSTH PROCESS

To study the performance and effectiveness of the proposed technique, it has been applied on a continuous stirred tank heater (CSTH) plant. CSTH plants are widely used in chemical industry to ensure optimal conditions for several chemical reactions to take place. Inside the CSTH a certain temperature and level of reactants are being held, that define the operating point under which an optimal reaction is possible. The laboratory sized CSTH considered here is a RT 682 CSTH demonstrator plant manufactured by G.U.N.T. Geraetebau GmbH Hamburg, available at the institute of Automatic Control and Complex Systems, University of Duisburg-Essen. It uses water as reactants and is structurally depicted in Fig. 1. The plant mainly consists of the tank in which a certain amount of the preheated reactant is mixed, heated further and held at a certain temperature. The plant has a steady through flow of reactants which is controlled via a hand valve on the inflow side. An actuated valve controls the outflow and can thus be used as an actuator for level control. The temperature inside the tank is raised via a heater in the surrounding, water filled heating jacket, whose power can be controlled. By heating the jacket, a steady heat flow through the tank’s walls and into the reactant will increase the temperature inside the stirred tank. The inflowing reactant is pre-heated from the out flowing product via a heat exchanger, so that it is only $5-10^\circ\text{C}$ colder than the desired tank temperature.

The main dynamics of the plant are the change of level $h_{\text{tank}}$ as a function of in- and outflowing water masses $\dot{m}_{\text{in}}$; $\dot{m}_{\text{out}}$, and the change of temperature $T_{\text{tank}}$ as a function of the heat flows in and out of the tank $\dot{Q}_{\text{in}}$; $\dot{Q}_{\text{out}}$

$$h_{\text{tank}}(t_1) = \int_{t_0}^{t_1} \frac{1}{A \cdot \rho \cdot (T_{\text{tank}}(t_1))} (\dot{m}_{\text{in}}(t) - \dot{m}_{\text{out}}(t)) dt + h_{\text{tank}}(t_0),$$

(29)

and

$$T_{\text{tank}}(t_1) = \int_{t_0}^{t_1} \frac{1}{c_p \cdot m(t)} (\dot{Q}_{\text{in}}(t) - \dot{Q}_{\text{out}}(t)) dt + T_{\text{tank}}(t_0),$$

(30)

where $A, \rho, c_p, m$ are the base area of the cylindrical tank, density of water, specific heat capacity of water and the mass of water in the tank, respectively.

The mass of water in the tank $m(t)$, that defines the relation between the enthalpy and the temperature of the water in (30), is supposed to be constant in an operating point. The heat flow into the tank $\dot{Q}_{\text{in}}$ is a nonlinear function of the controllable power of the heater, the water level in the heating jacket and the level of the tank. Furthermore the preheated inflowing and the outflowing reactant provide an additional heat flow in and out of the tank which are functions of the mass flows and the temperature of those masses. These are considered to be constant in an operating point and so are the heat flows. To describe heat losses due to non-optimal insulation, the heat flow out of the system, $\dot{Q}_{\text{out}}$, was introduced in which all non-optimal effects can be lumped. Taking all this into consideration the equations (29) and (30) become

$$h_{\text{tank}}(t_1) = \frac{1}{A \cdot \rho \cdot (OP)} \int_{t_0}^{t_1} (\dot{m}_{\text{in}}(OP) - \dot{m}_{\text{out}}(t)) dt + h_{\text{tank}}(t_0)$$

(31)

$$T_{\text{tank}}(t_1) = \frac{1}{c_p \cdot m(\text{OP})} \int_{t_0}^{t_1} (\dot{Q}_{\text{in}}(\text{heater}(t), h_{\text{tank}}(\text{OP}), h_{\text{tank}}(t)))$$

$$- \dot{Q}_{\text{out}}(\text{OP}) + \dot{Q}_{\text{in}}(\text{OP}) - \dot{Q}_{\text{out}}(\text{OP}) dt + T_{\text{tank}}(t_0)$$

(32)

From this, the overall integrating behavior of the two subsystems for level and temperature can be seen as well as the nonlinearities and operating point dependencies of the system model. Three different operating points, as shown in Table I, were chosen for the measurements presented here. For each operating point 500 samples were used to train the model using Algorithm 1. For validation purpose, faulty operation is induced by scaling the signal given to level controlling valve V1 behind the controller output. To simulate a faulty actuator for fault cases 1 and 2 scaling by 0.5 was done and scaling by 0.8 for fault case 3. Fig. 2 shows the run of all measurable plant variables for normal and faulty operation in OP1, OP2 and OP3 each consisting of 100 samples. The level of the reactant inside the stirred tank, $L_1$, is considered as quality variable, $Y$ and remaining variables are considered as input variables $X$. In on-line implementation all other process variables are measured and the proposed method described in Algorithm 1 is implemented for monitoring. Furthermore the fault detection index $p(x_i \in f)$ is depicted in Fig. 3 over the same faulty and normal operation intervals. The horizontal dashed line represents the threshold with 97% confidence level for $\alpha = 0.03$. It can be seen that all three faults are successfully detected within the confidence level. Moreover

$$\begin{array}{|c|c|c|c|}
\hline
\text{values} & \text{OP 1} & \text{OP 2} & \text{OP 3} \\
\hline
\text{level } L_1 / [\text{cm}] & 10 & 12 & 18 \\
\hline
\text{level heating jacket} / [\text{cm}] & 20 & 20 & 20 \\
\hline
\text{temperature } T_1 / [\text{C}] & 50 & 45 & 50 \\
\hline
\text{through flow } P_1 / [\text{L/h}] & 105 & 105 & 105 \\
\hline
\end{array}
$$

\textbf{TABLE I}

VALUES DEFINING PLANT OPERATING POINTS

1 see www.gunt.de for details.
the false alarm rate in this case is 2% which is less than the permitted false alarm rate $\alpha$.

V. CONCLUSION

In this paper an approach has been developed for data-driven fault detection and product quality monitoring in nonlinear processes. The method is based on identification of the regression model for multimode process using EM algorithm in off-line design step. In on-line step, an index has been introduced for fault detection purpose which shows the probability of fault occurrence and is a combination of the hypotheses that a sample belongs to a component and it is faulty provided that it belongs to that component. For validation purpose, the method has been applied on a CSTH process, with 3 different operating points and 3 different fault scenarios, where all faults have been detected successfully.

REFERENCES