Abstract

We present in this paper a novel approach dedicated to the measurement of velocity in fluid experimental flows. Such information, which is fundamental for specialists, is usually computed by correlation methods on a special kind of images (named PIV). We present here a motion estimation technique which is based on an optical-flow extension. Results are presented on an experimental flow and are compared to those computed with standard methods of the fluid mechanics community.

1. INTRODUCTION

Applications of fluid phenomena analysis from image sequences are numerous and concern various domains such as meteorology, climatology, oceanography, medical imaging or fluid mechanics experiments. In this latter case, dynamical informations about the evolution of an experimental fluid flow allows specialists to elaborate models or to have a better understanding on certain types of fluid flows. In that context, experimental imaging techniques now routinely produce various types of videos which constitute a unique source of information for both applied and theoretical studies [1].

Particle Image Velocimetry (PIV) methods are frequently used by fluid mechanical specialists to obtain measures of motion of fluid flows. These techniques, based on a correlation function measured on clouds of particles sowed in the fluid, are enable to obtain reliable sparse motion estimates. The fact that only sparse measurements can be obtained and other difficulties of PIV methods encourage researchers to devise more sophisticated techniques.

In this paper, we propose a method which permits to extract a dense motion field from fluid experimental image sequences. Unlike to most of motion estimation techniques used by specialists on PIV images, we introduce some physical laws within a variational optical-flow estimation. More precisely, we propose to introduce an observation constraint lying on the continuity equation and a smoothness term based on the divergence and the vorticity of the motion field. Experimental results are shown on PIV images and are analyzed with respect to fluid mechanics criteria.

2. A FLUID MOTION OPTICAL FLOW SCHEME

2.1. A new observation constraint

Classical optical flow schemes are based on Horn and Schunck formalism [8] and consist to minimize an energy function composed of two terms. The first one is derived from a brightness constancy assumption and assumes that a given point keeps the same intensity along its trajectory. It is expressed through the well known optical flow constraint equation (OFCE):

$$\Psi_1 \left[ \nabla E(x, t) \cdot \mathbf{v}(x, t) + \frac{\partial E(x, t)}{\partial t} \right] \, dx, \quad (1)$$

where $\mathbf{v}(x, t)$ is the unknown velocity field at time $t$ and location $x = (x, y)$ in the image plane $\Omega$, $E(x, t)$ being the image brightness. Function $\Psi_1$ is a penalty function that can be quadratic or issued from robust statistics [9], to limit the impact of locations where this assumption is violated.

In most image sequences and especially in fluid imagery, such a brightness constancy assumption is known to be violated in a number of locations in the image plane. Unlike rigid motion situations, the use of robust cost functions is far to be sufficient to cope with data model outliers. As a matter of fact, fluid imagery image sequences often exhibit dramatic temporal changes of brightness.

In the case of particle velocimetry experiments, the recorded irradiance often corresponds to a slice of the flow [1]. In that case, tri-dimensional motions non-parallel to the visualization plane cause fluid elements to enter or exit the imaged slice.

Based on this, and among others considerations, an alternative brightness constraint, which would be better suited to the physics of fluids, is proposed. A few authors have suggested the use of the continuity equation, as a more physically-grounded constraint, and they have demonstrated that it is indeed an appealing alternative [2, 7]. It turns out that the image irradiance for a fluid is usually related to the density of a physical quantity – e.g., particle concentration in particle image velocimetry...
where the divergence $\text{div} \ (\text{respectively the vorticity } \text{curl})$ is

The three-dimensional velocity field. This equation derives from a global conservation assumption by stating that the temporal variation of the quantity under consideration within an infinitesimal volume amounts exactly to the flux of this quantity through the boundary surface of the volume. One can then assume by analogy that the two-dimensional image brightness $E$ and apparent velocity $v$ satisfy:

$$\frac{\partial E}{\partial t} + \text{div}(Ev) = 0.$$  \tag{3}

For incompressible fluids such as water, the three-dimensional flow is divergence free. Assuming that the resulting apparent bi-dimensional flow is divergence free as well, the bi-dimensional continuity equation above amounts exactly to the brightness constancy constraint, since $\text{div}(Ev) = v \cdot \nabla E + E \text{div} v$. In other cases, i.e., when flows are compressible, the brightness constraint expressed by (3) differs from the standard one by the additional term $E \text{div} v$. The actual validity of this new brightness constraint depends on the type of images involved. See [3] for a discussion on this subject. In all cases, it is demonstrated that using this alternative is better, in our context, that the usual OFCE. The new observation constraint can be then expressed as:

$$\iint_{\Omega} \Psi_{E} \left( \frac{\partial E(x, t)}{\partial t} + \text{div}(E(x, t)v(x, t)) \right) \, dx.$$  \tag{4}

2.2. A new smoothness term

This single (scalar) observation term does not allow to estimate the velocity vectors. In order to solve this ill-posed problem, it is common to employ an additional smoothness constraint. Usually, this second term promotes the spatial coherence of the flow field. It relies on a contextual assumption which enforces a spatial continuity of the solution. This term usually reads:

$$\alpha \iint_{\Omega} [\nabla u(x, t)]^{2} + [\nabla v(x, t)]^{2},$$  \tag{5}

where $\alpha > 0$ is a parameter controlling the balance between the smoothness constraint and the global adequacy to the observation assumption.

One can demonstrate, by using Euler-Lagrange conditions of optimality, that the minimization of such a term is equivalent to the minimization of $\iint_{\Omega} \text{div}^{2}(v) + \text{curl}^{2}(v)$, where the divergence div (respectively the vorticity curl) is expressed as $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ (resp. $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$). A first order regularization therefore penalizes both the divergence and the vorticity of the minimizer. It encourages laminar velocity fields (div = curl = 0).

For fluid motion estimation, such a first-order smoothing does not seem appropriate since the flows of interest are expected to exhibit large “concentrations” of vorticity and divergence. In fluid mechanics experiments, for example, turbulent flows develop vortices which are interacting rotational structures characterized by large vorticity. The key information present in these concentrations of divergence and vorticity will then be partly ignored by standard optical flow estimation techniques based on a first-order regularization.

Hence, we propose to use divergence and curl preserving regularization. To that end, the second-order div-curl regularizer introduced by Suter [10]:

$$\alpha \iint_{\Omega} \left| \nabla \text{div} \right|^{2} + \left| \nabla \text{curl} \right|^{2},$$  \tag{6}

is particularly appealing. As its direct numerical implementation is difficult, we considered two important modifications of this smoothing term. First, we substitute the quadratic penalty function by a robust one. Second, we lower the order of this regularization term through the introduction of two auxiliary scalar functions, $\xi$ and $\zeta$, which will respectively constitute direct estimates of the divergence and vorticity of the unknown velocity $v$. The new regularizer is then given by:

$$\alpha \iint_{\Omega} \left| \nabla \text{div} - \xi \right|^{2} + \lambda \Psi_{E}(|\nabla \xi|) + \alpha \iint_{\Omega} \left| \nabla \text{curl} - \zeta \right|^{2} + \lambda \Psi_{E}(|\nabla \zeta|).$$  \tag{7}

The first part of each integral encourages the displacement to comply with the current divergence and vorticity estimates $\xi$ and $\zeta$, through a quadratic goodness-of-fit. The second part equips the divergence and the vorticity estimates with a robust first-order regularization favoring piece-wise smooth configurations.

Although it is not investigated in this paper, the introduction of the two scalar fields $\xi$ and $\zeta$ permits even more sophisticated priors on the divergence and the vorticity of the imaged flow. In the same vein, the explicit manipulation of div and curl estimates should simplify the assimilation of physical measurements provided by dedicated probes, such as sparse vorticity measurements obtained by thermal anemometry [11].

The augmented formulation also provides computational advantages. The second-order regularization is replaced by two interleaved first-order regularizations. From the Euler-Lagrange point-of-view, this amounts to replacing the fourth-order PDE associated with (6), by a system of two coupled PDEs of order two.

2.3. Large displacements

A motion estimation relying on (4) presents a major flaw: due to its differential nature, this expression is no longer valid when large displacements occur between consecutive frames of the sequence. In contrast to previous studies, we thus choose to express the continuity equation constraint in an integrated way. Noting that $dE / dt = \partial E / \partial t + \nabla E \cdot \nu$,
the integration of the resulting first-order ODE leads to the minimization of:

$$\int_{\Omega} E(\mathbf{x} + \mathbf{d}(\mathbf{x}), t + \Delta t) \exp(\text{div}(\mathbf{d}(\mathbf{x}))) - E(\mathbf{x}, t), \quad (8)$$

where $\Delta t$ is the temporal sampling rate and $\mathbf{d}(\mathbf{x}) = \Delta t \mathbf{n}(\mathbf{x}, t)$ represents the displacement from time $t$ to $t + \Delta t$ of the point located at position $\mathbf{x}$ at time $t$. According to this constraint, the brightness is scaled by the factor $\exp(\text{div}(\mathbf{d}(\mathbf{x})))$. It decreases (resp. increases) for motions with negative (resp. positive) divergence. When the divergence is zero, this constraint amounts exactly to the brightness constancy constraint. This new integrated continuity equation has the advantage of dealing explicitly with displacements instead of velocities, but, as a consequence, the dependency on the unknown vector field $\mathbf{d}$ is now highly nonlinear. To cope with that non-linearity, we resort to a previous estimate of the displacement field obtained at a coarser resolution, a first-order expansion of (8) is performed around $(\mathbf{x} + \mathbf{d}(\mathbf{x}), t + \Delta t)$. Dropping the time indices of the intensity function for sake of clarity, we end up with the following data term for our dedicated cost function, at each resolution level:

$$\int_{\Omega} \Psi_1 \left[ \exp(\text{div}(\mathbf{d}(\mathbf{x}))) \right] \left( (\tilde{E}(\mathbf{x}) \nabla \text{div}(\mathbf{d}(\mathbf{x})) + \nabla \tilde{E}(\mathbf{x})) \cdot \mathbf{h}(\mathbf{x}) + \tilde{E}(\mathbf{x}) - E(\mathbf{x}) \right) \, d\mathbf{x}, \quad (9)$$

where $\mathbf{h} = \mathbf{d} - \bar{\mathbf{d}}$, and $\tilde{E}(\mathbf{x})$ tends for the backward registered image $E(\mathbf{x} + \mathbf{d}(\mathbf{x}), t + \Delta t)$ at time $t + \Delta t$.

To embed the regularization term (7) in such an incremental hierarchical framework, we must express it in terms of the displacement increment $\mathbf{h} = \mathbf{d} - \bar{\mathbf{d}}$, where $\bar{\mathbf{d}}$ is a crude estimate of the displacement field. The second term of our global cost function, to be combined with (9), is then:

$$\alpha \sum_{i=1,2} \int_{\Omega} \left| \mathbf{s}_{i} (\mathbf{d} + \mathbf{h}) - s_{i} \right|^2 + \lambda \Psi_2( |\nabla s_{i}|), \quad (10)$$

with $\bullet = \text{div}, \text{curl}$ and $\bullet = (\xi, \zeta)$. The minimization of the global energy function is done alternatively with respect to $\mathbf{h}, \xi$ and $\zeta$ until convergence. Let us now analyze performances of the method on an experimental flow.

3. EXPERIMENTAL RESULTS

We have tested the proposed method on a sequence of 180 images corresponding to a free turbulent shear layer flow, whose main characteristics are known. This kind of flow appears when the velocity of two fluids that slide along each other have the same directions but within different amplitude. A photograph of such a mixing can be seen on Fig. 1. Many studies have been done on this flow and some of its fundamental characteristics can be extracted from its motion field [6]. For an instantaneous velocity field, the two most important ones are the spatial distribution of the longitudinal velocity (whose aspect is represented by an arctan function) and the vorticity map. With respect to the image sequence, one can extract more statistical parameters that characterize the flow. Those parameters, that may be easily recovered from the set of motion fields [6], are:

1. the blooming parameter $B$, which is the derivative of the vorticity depth ($d_{\nu}$) and which is an indicator of the evolution of turbulent structures, or its expansion parameter $\sigma$ which is defined by $\sigma = \sqrt{\pi}/B$;

2. the spatial derivative of the depth of the absolute motion (named $\Theta$) which is an indicator of the speed of the flow.

On Fig. 2 (a,b), we present two consequent images obtained by PIV techniques. On this pair of images, we applied our motion estimation method and a commercial PIV approach based on correlation. Let us note that this latter approach is frequently used by fluid mechanics specialists, since the relevance of its results has been proven. Instantaneous velocity fields, spatial distributions of the longitudinal velocity and vorticity maps are respectively represented on figures 2 (c,e,g) for the proposed method and on figures 2 (d,f,h) for the PIV one. One can immediately observe that results are globally similar for both methods (instantaneous velocity fields, vorticity maps as well as longitudinal velocities –which are, as expected, similar to an arctan function–). The main and major difference comes from the quantity of informations that our method provides (a dense motion field of $1024 \times 1024$ vectors versus $64 \times 64$ vectors with the PIV method). To estimate the two statistical parameters, we applied the two approaches on the whole sequence. From each instantaneous velocity field, the vorticity depth ($d_{\nu}$) and the absolute motion $\Theta$ have been extracted. The different results are gathered in table 1. They can be compared to the theoretical values of these parameters. We can observe that for both methods, the results have the same order of magnitude, and are in accordance with the ground truth. From those results, it is not possible to assess which one is the most accurate method, even if our dense estimator seems to perform slightly better. However, it is important to note that the proposed approach produces motion information at much higher resolution level. This fact allows to recover less noisy vorticity maps, and enables without any interpolation process to extract streamlines trajectories and other integrated quantities [4].

Figure 1: Example of a free shear layer.
4. CONCLUSION

In this paper, we presented an approach based on a deviation of standard optical flow methods that is dedicated to the estimation of motion in experimental fluid flows. We introduced physical constraints, on the observation term as well as the regularization constraint. We tested the developed approach on an experimental flow (where main characteristics are known) and compared it with a common approach employed by fluid mechanical specialists. Both results have the same order of magnitude but the proposed approach gives a number of informations highly superior. One then can hope to extract measurements on smaller scales of the flow that are not available with PIV approaches.

5. REFERENCES


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**Table 1: Comparative results on main characteristics.**

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**Figure 2:** A pair of PIV images, the instantaneous velocity field, its longitudinal velocity distribution and the vorticity map obtained by our approach (left) and usual ones (right).