SOMEWHAT FUZZY $\gamma$-IRRESOLUTE CONTINUOUS MAPPINGS

YOUNG BIN IM, JOO SUNG LEE* AND YUNG DUK CHO

Abstract. We define and characterize a somewhat fuzzy $\gamma$-irresolute continuous mapping and a somewhat fuzzy irresolute $\gamma$-open mapping on a fuzzy topological space.

AMS Mathematics Subject Classification : 54A40.
Key words and phrases : somewhat fuzzy $\gamma$-irresolute continuous mapping, somewhat fuzzy irresolute $\gamma$-open mapping.

1. Introduction

The concept of fuzzy $\gamma$-continuous mappings on a fuzzy topological space was introduced and studied by I. M. Hanafy in [2]. Also, the concept of fuzzy $\gamma$-irresolute continuous mappings on a fuzzy topological space were introduced and studied by Y. B. Im et al. in [8] and fuzzy irresolute $\gamma$-open mappings on a fuzzy topological space was introduced and studied by Y. B. Im in [3].

Recently, somewhat fuzzy $\gamma$-continuous mappings on a fuzzy topological space were introduced and studied by G. Thangaraj and V. Seenivasan in [9].

In this paper, we define and characterize a somewhat fuzzy $\gamma$-irresolute continuous mapping and a somewhat fuzzy irresolute $\gamma$-open mapping which are stronger than a somewhat fuzzy $\gamma$-continuous mapping and a somewhat fuzzy $\gamma$-open mapping respectively. Besides, some interesting properties of those mappings are also given.

2. Preliminaries

A fuzzy set $\mu$ on a fuzzy topological space $X$ is called fuzzy $\gamma$-open if $\mu \subseteq \text{ClInt} \mu \vee \text{IntCl} \mu$ and $\mu$ is called fuzzy $\gamma$-closed if $\mu^c$ is a fuzzy $\gamma$-open set on $X$.

A mapping $f : X \to Y$ is called fuzzy $\gamma$-continuous if $f^{-1}(\nu)$ is a fuzzy $\gamma$-open set on $X$ for any fuzzy open set $\nu$ on $Y$ and a mapping $f : X \to Y$ is...
called fuzzy $\gamma$-open if $f(\mu)$ is a fuzzy $\gamma$-open set on $Y$ for any fuzzy open set $\mu$ on $X$. It is clear that every fuzzy continuous mapping is a fuzzy $\gamma$-continuous mapping. And every fuzzy open mapping is a fuzzy $\gamma$-open mapping from the above definitions. But the converses are not true in general [2].

A mapping $f : X \to Y$ is called fuzzy $\gamma$-irresolute continuous if $f^{-1}(\nu)$ is a fuzzy $\gamma$-open set on $X$ for any fuzzy $\gamma$-open set $\nu$ on $Y$ and a mapping $f : X \to Y$ is called fuzzy irresolute $\gamma$-open if $f(\mu)$ is a fuzzy $\gamma$-open set on $Y$ for any fuzzy $\gamma$-open set $\mu$ on $X$. It is clear that every fuzzy $\gamma$-irresolute continuous mapping is a fuzzy $\gamma$-continuous mapping. And every fuzzy irresolute $\gamma$-open mapping is a fuzzy open mapping from the above definitions. But the converses are not true in general [8] and [3].

A mapping $f : X \to Y$ is called somewhat fuzzy $\gamma$-continuous if there exists a fuzzy $\gamma$-open set $\mu \neq 0_X$ on $X$ such that $\mu \leq f^{-1}(\nu) \neq 0_Y$ for any fuzzy open set $\nu$ on $Y$. It is clear that every fuzzy $\gamma$-continuous mapping is a somewhat fuzzy $\gamma$-continuous mapping. But the converse is not true in general.

A mapping $f : X \to Y$ is called somewhat fuzzy $\gamma$-open if there exists a fuzzy $\gamma$-open set $\nu \neq 0_Y$ on $Y$ such that $\nu \leq f(\mu) \neq 0_Y$ for any fuzzy open set $\mu$ on $X$. Every fuzzy open mapping is a somewhat fuzzy $\gamma$-open mapping but the converse is not true in general [9].

3. Somewhat fuzzy $\gamma$-irresolute continuous mappings

In this section, we introduce a somewhat fuzzy $\gamma$-irresolute continuous mapping and a somewhat fuzzy irresolute $\gamma$-open mapping which are stronger than a somewhat fuzzy $\gamma$-continuous mapping and a somewhat fuzzy $\gamma$-open mapping respectively. And we characterize a somewhat fuzzy $\gamma$-irresolute continuous mapping and a somewhat fuzzy irresolute $\gamma$-open mapping.

**Definition 3.1.** A mapping $f : X \to Y$ is called somewhat fuzzy $\gamma$-irresolute continuous if there exists a fuzzy $\gamma$-open set $\mu \neq 0_X$ on $X$ such that $\mu \leq f^{-1}(\nu)$ for any fuzzy $\gamma$-open set $\nu \neq 0_Y$ on $Y$.

It is clear that every fuzzy $\gamma$-irresolute continuous mapping is a somewhat fuzzy $\gamma$-irresolute continuous mapping. And every somewhat fuzzy $\gamma$-irresolute continuous mapping is a fuzzy $\gamma$-continuous mapping from the above definitions. But the converses are not true in general as the following examples show.

**Example 3.2.** Let $\mu_1$, $\mu_2$ and $\mu_3$ be fuzzy sets on $X = \{a, b, c\}$ and let $\nu_1$, $\nu_2$ and $\nu_3$ be fuzzy sets on $Y = \{x, y, z\}$ with
\[
\begin{align*}
\mu_1(a) &= 0.1, \mu_1(b) = 0.1, \mu_1(c) = 0.1, \\
\mu_2(a) &= 0.2, \mu_2(b) = 0.2, \mu_2(c) = 0.2, \\
\mu_3(a) &= 0.5, \mu_3(b) = 0.5, \mu_3(c) = 0.5 \quad \text{and} \\
\nu_1(x) &= 0.3, \nu_1(y) = 0.2, \nu_1(z) = 0.3, \\
\nu_2(x) &= 0.5, \nu_2(y) = 0.5, \nu_2(z) = 0.5, \\
\nu_3(x) &= 0.5, \nu_3(y) = 0.2, \nu_3(z) = 0.5.
\end{align*}
\]
Let $\tau = \{0_X, \mu_1, \mu'_1, \mu_2, 1_X\}$ be fuzzy topologies on $X$ and let $\tau^* = \{0_Y, \nu_1, \nu_2, 1_Y\}$ be fuzzy topologies on $Y$. Consider the mapping $f : (X, \tau) \to (Y, \tau^*)$ defined by $f(a) = y$, $f(b) = y$ and $f(c) = y$. Then we have $\mu_1 \leq f^{-1}(\nu_1) = \mu_2$, $f^{-1}(\nu_2) = \mu_3$ and $\mu_1 \leq f^{-1}(\nu_3) = \mu_2$. Since $\mu_1$ is a fuzzy $\gamma$-open set on $(X, \tau)$, $f$ is somewhat fuzzy $\gamma$-irresolute continuous. But $f^{-1}(\nu_1) = \mu_2$ and $f^{-1}(\nu_3) = \mu_2$ are not fuzzy $\gamma$-open sets on $(X, \tau)$. Hence $f$ is not a fuzzy $\gamma$-irresolute continuous mapping.

**Example 3.3.** Let $\mu_1, \mu_2$ and $\mu_3$ be fuzzy sets on $X = \{a, b, c\}$ and let $\nu_1, \nu_2$ and $\nu_3$ be fuzzy sets on $Y = \{x, y, z\}$ with

$\mu_1(a) = 0.2, \mu_1(b) = 0.2, \mu_1(c) = 0.2,$
$\mu_2(a) = 0.5, \mu_2(b) = 0.5, \mu_2(c) = 0.5,$ and
$\nu_1(x) = 0.3, \nu_1(y) = 0.2, \nu_1(z) = 0.3,$
$\nu_2(x) = 0.5, \nu_2(y) = 0.5, \nu_2(z) = 0.5.$

Let $\tau = \{0_X, \mu'_1, 1_X\}$ be fuzzy topologies on $X$ and let $\tau^* = \{0_Y, \nu_1, \nu_2, 1_Y\}$ be fuzzy topologies on $Y$. Consider the mapping $f : (X, \tau) \to (Y, \tau^*)$ defined by $f(a) = y$, $f(b) = y$ and $f(c) = y$. Since $f^{-1}(\nu_2) = \mu_2$ is fuzzy $\gamma$-open sets on $(X, \tau)$, $f$ is fuzzy $\gamma$-continuous. But the inverse images $0_X \leq f^{-1}(\nu_1) = \mu_1$ of a fuzzy $\gamma$-open set $\nu_1$ on $(Y, \tau^*)$ is not fuzzy $\gamma$-open on $(X, \tau)$. Hence $f$ is not a fuzzy somewhat $\gamma$-irresolute continuous mapping.

**Definition 3.4** ([9]). A fuzzy set $\mu$ on a fuzzy topological space $X$ is called fuzzy $\gamma$-dense if there exists no fuzzy $\gamma$-closed set $\nu$ such that $\mu < \nu < 1$.

**Theorem 3.5.** Let $f : X \to Y$ be a mapping. Then the following are equivalent:

1. $f$ is somewhat fuzzy $\gamma$-irresolute continuous.
2. If $\nu$ is a fuzzy $\gamma$-closed set of $Y$ such that $f^{-1}(\nu) \neq 1_X$, then there exists a fuzzy $\gamma$-closed set $\mu \neq 1_X$ of $X$ such that $f^{-1}(\nu) \leq \mu$.
3. If $\mu$ is a fuzzy $\gamma$-dense set on $X$, then $f(\mu)$ is a fuzzy $\gamma$-dense set on $Y$.

**Proof.** (1) implies (2): Let $\nu$ be a fuzzy $\gamma$-closed set on $Y$ such that $f^{-1}(\nu) \neq 1_X$. Then $\nu^c$ is a fuzzy $\gamma$-open set on $Y$ and $f^{-1}(\nu^c) = (f^{-1}(\nu))^c \neq 0_X$. Since $f$ is somewhat fuzzy $\gamma$-irresolute continuous, there exists a fuzzy $\gamma$-open set $\lambda \neq 0_X$ on $X$ such that $\lambda \leq f^{-1}(\nu^c)$. Let $\mu = \lambda^c$. Then $\mu \neq 1_X$ is fuzzy $\gamma$-closed such that $f^{-1}(\nu) = 1 - f^{-1}(\nu^c) \leq 1 - \lambda = \lambda^c = \mu$.

(2) implies (3): Let $\mu$ be a fuzzy $\gamma$-dense set on $X$ and suppose $f(\mu)$ is not fuzzy $\gamma$-dense on $Y$. Then there exists a fuzzy $\gamma$-closed set $\nu$ on $Y$ such that $f(\mu) < \nu < 1$. Since $\nu < 1$ and $f^{-1}(\nu) \neq 1_X$, there exists a fuzzy $\gamma$-closed set $\delta \neq 1_X$ such that $\mu \leq f^{-1}(f(\mu)) < f^{-1}(\nu) \leq \delta$. This contradicts the assumption that $\mu$ is a fuzzy $\gamma$-dense set on $X$. Hence $f(\mu)$ is a fuzzy $\gamma$-dense set on $Y$.

(3) implies (1): Let $\nu \neq 0_Y$ be a fuzzy $\gamma$-open set on $Y$ and $f^{-1}(\nu) \neq 0_X$. Suppose there exists no fuzzy $\gamma$-open $\mu \neq 0_X$ on $X$ such that $\mu \leq f^{-1}(\nu)$. Then $(f^{-1}(\nu))^c$ is a fuzzy set on $X$ such that there is no fuzzy $\gamma$-closed set $\delta$ on $X$. This contradicts the assumption that $\mu$ is a fuzzy $\gamma$-dense set on $X$. Hence $f(\mu)$ is a fuzzy $\gamma$-dense set on $Y$.
with \((f^{-1}(\nu))^c < \delta < 1\). In fact, if there exists a fuzzy \(\gamma\)-open set \(\delta\) such that \(\delta^c \leq f^{-1}(\nu)\), then it is a contradiction. So \((f^{-1}(\nu))^c\) is a fuzzy \(\gamma\)-dense set on \(X\). Then \(f((f^{-1}(\nu))^c)\) is a fuzzy \(\gamma\)-dense set on \(Y\). But \(f((f^{-1}(\nu))^c) = f(f^{-1}(\nu)) \neq \nu^c < 1\). This is a contradiction to the fact that \(f((f^{-1}(\nu))^c)\) is fuzzy \(\gamma\)-dense on \(Y\). Hence there exists a \(\gamma\)-open set \(\mu \neq 0_X\) on \(X\) such that \(\mu \leq f^{-1}(\nu)\). Consequently, \(f\) is somewhat fuzzy \(\gamma\)-irresolute continuous. □

A fuzzy topological space \(X\) is \textit{product related} to a fuzzy topological space \(Y\) if for fuzzy sets \(\mu\) on \(X\) and \(\nu\) on \(Y\) whenever \(\gamma^c \geq \mu\) and \(\delta^c \geq \nu\) (in which case \((\gamma^c \times 1) \lor (1 \times \delta^c) \geq (\mu \times \nu))\) where \(\gamma\) is a fuzzy open set on \(X\) and \(\delta\) is a fuzzy open set on \(Y\), there exists a fuzzy open set \(\gamma_1\) on \(X\) and a fuzzy open set \(\delta_1\) on \(Y\) such that \(\gamma_1^c \geq \mu\) or \(\delta_1^c \geq \nu\) and \((\gamma_1^c \times 1) \lor (1 \times \delta_1^c) = (\gamma^c \times 1) \lor (1 \times \delta^c) [1]\).

**Theorem 3.6.** Let \(X_1\) be product related to \(X_2\) and \(Y_1\) be product related to \(Y_2\). Then the product \(f_1 \times f_2 : X_1 \times X_2 \rightarrow Y_1 \times Y_2\) of somewhat fuzzy \(\gamma\)-irresolute continuous mappings \(f_1 : X_1 \rightarrow Y_1\) and \(f_2 : X_2 \rightarrow Y_2\) is also somewhat fuzzy \(\gamma\)-irresolute continuous.

**Proof.** Let \(\lambda = \bigvee_{i,j}(\mu_i \times \nu_j)\) be a fuzzy \(\gamma\)-open set on \(Y_1 \times Y_2\) where \(\mu_i \neq 0_{Y_1}\) and \(\nu_j \neq 0_{Y_2}\) are fuzzy \(\gamma\)-open sets on \(Y_1\) and \(Y_2\) respectively. Then \((f_1 \times f_2)^{-1}(\lambda) = \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j))\). Since \(f_1\) is somewhat fuzzy \(\gamma\)-irresolute continuous, there exists a fuzzy \(\gamma\)-open set \(\delta_i \neq 0_{X_1}\) such that \(\delta_i \leq f_1^{-1}(\mu_i) \neq 0_{X_1}\). And, since \(f_2\) is somewhat fuzzy \(\gamma\)-irresolute continuous, there exists a fuzzy \(\gamma\)-open set \(\eta_j \neq 0_{X_2}\) such that \(\eta_j \leq f_2^{-1}(\nu_j) \neq 0_{X_2}\). Now \(\delta_i \times \eta_j \leq f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j) = (f_1 \times f_2)^{-1}(\mu_i \times \nu_j)\) and \(\delta_i \times \eta_j \neq 0_{X_1 \times X_2}\) is a fuzzy \(\gamma\)-open set on \(X_1 \times X_2\). Hence \(\bigvee_{i,j}(\delta_i \times \eta_j) \neq 0_{X_1 \times X_2}\) is a fuzzy \(\gamma\)-open set on \(X_1 \times X_2\) such that \(\bigvee_{i,j}(\delta_i \times \eta_j) \leq (f_1 \times f_2)^{-1}(\bigwedge_{i,j}(\mu_i \times \nu_j))\). Therefore, \(f_1 \times f_2\) is somewhat fuzzy \(\gamma\)-irresolute continuous. □

**Theorem 3.7.** Let \(f : X \rightarrow Y\) be a mapping. If the graph \(g : X \times X \times Y\) of \(f\) is a somewhat fuzzy \(\gamma\)-irresolute continuous mapping, then \(f\) is also somewhat fuzzy \(\gamma\)-irresolute continuous.

**Proof.** Let \(\nu\) be a fuzzy \(\gamma\)-open set on \(Y\). Then \(f^{-1}(\nu) = 1 \land f^{-1}(\nu) = g^{-1}(1 \times \nu)\). Since \(g\) is somewhat fuzzy \(\gamma\)-irresolute continuous and \(1 \times \nu\) is a fuzzy \(\gamma\)-open set on \(X \times Y\), there exists a fuzzy \(\gamma\)-open set \(\mu \neq 0_X\) on \(X\) such that \(\mu \leq g^{-1}(1 \times \nu) = f^{-1}(\nu) \neq 0_X\). Therefore, \(f\) is somewhat fuzzy \(\gamma\)-irresolute continuous. □

**Definition 3.8.** A mapping \(f : X \rightarrow Y\) is called somewhat fuzzy irresolute \(\gamma\)-open if there exists a fuzzy \(\gamma\)-open set \(\nu \neq 0_Y\) on \(Y\) such that \(\nu \leq f(\mu)\) for any fuzzy \(\gamma\)-open set \(\mu \neq 0_X\) on \(X\).

It is clear that every fuzzy irresolute \(\gamma\)-open mapping is a somewhat fuzzy irresolute \(\gamma\)-open mapping. And every somewhat fuzzy irresolute \(\gamma\)-open mapping is a fuzzy \(\gamma\)-open mapping. Also, every fuzzy \(\gamma\)-open mapping is a somewhat fuzzy \(\gamma\)-open mapping from the above definitions. But the converses are not true in general as the following examples show.
Example 3.9. Let $\mu_1$ and $\mu_2$ be fuzzy sets on $X = \{a, b, c\}$ and let $\nu_1$ and $\nu_2$ be fuzzy sets on $Y = \{x, y, z\}$ with

$$
\begin{align*}
\mu_1(a) &= 0.1, \mu_1(b) = 0.1, \mu_1(c) = 0.1, \\
\mu_2(a) &= 0.2, \mu_2(b) = 0.2, \mu_2(c) = 0.2 \\
\nu_1(x) &= 0.0, \nu_1(y) = 0.1, \nu_1(z) = 0.0, \\
\nu_2(x) &= 0.0, \nu_2(y) = 0.2, \nu_2(z) = 0.0, \\
\nu_3(x) &= 0.0, \nu_3(y) = 0.8, \nu_3(z) = 0.0, \\
\nu_4(x) &= 0.0, \nu_4(y) = 0.9, \nu_4(z) = 0.0.
\end{align*}
$$

Let $\tau = \{0_X, \mu_1, \mu_2, 1_X\}$ be fuzzy topologies on $X$ and let $\tau^* = \{0_Y, \nu_1, \nu_2, 1_Y\}$ be fuzzy topologies on $Y$. Consider the mapping $f : (X, \tau) \to (Y, \tau^*)$ defined by $f(a) = y, f(b) = y$ and $f(c) = y$. Since $f(\mu_1) = \nu_1, f(\mu_2) = \nu_2, f(\mu_3^*) = \nu_3$ and $f(\mu_4^*) = \nu_4$, $f$ is somewhat fuzzy irresolute $\gamma$-open. But $f(\mu_2) = \nu_2$ is not a fuzzy $\gamma$-open set on $(Y, \tau^*)$. Hence $f$ is not a fuzzy irresolute $\gamma$-open mapping.

Example 3.10. Let $\mu_1, \mu_2$ and $\mu_3$ be fuzzy sets on $X = \{a, b, c\}$ and let $\nu_1$ and $\nu_2$ be fuzzy sets on $Y = \{x, y, z\}$ with

$$
\begin{align*}
\mu_1(a) &= 0.4, \mu_1(b) = 0.1, \mu_1(c) = 0.4, \\
\mu_2(a) &= 0.5, \mu_2(b) = 0.5, \mu_2(c) = 0.5, \\
\mu_3(a) &= 0.1, \mu_3(b) = 0.0, \mu_3(c) = 0.1 \\
\nu_1(x) &= 0.0, \nu_1(y) = 0.1, \nu_1(z) = 0.0, \\
\nu_2(x) &= 0.0, \nu_2(y) = 0.5, \nu_2(z) = 0.0.
\end{align*}
$$

Let $\tau = \{0_X, \mu_1, \mu_2, 1_X\}$ be fuzzy topologies on $X$ and let $\tau^* = \{0_Y, \nu_1, \nu_2, 1_Y\}$ be fuzzy topologies on $Y$. Consider the mapping $f : (X, \tau) \to (Y, \tau^*)$ defined by $f(a) = y, f(\mu_1) = \nu_1$ and $f(\mu_2) = \nu_2$ are fuzzy $\gamma$-open sets on $(Y, \tau^*)$, $f$ is fuzzy $\gamma$-open. But $\mu_3 \neq 0_X$ is a fuzzy $\gamma$-open set on $(X, \tau)$ and $f(\mu_3) = 0_Y$. Hence $f$ is not a fuzzy somewhat irresolute $\gamma$-open mapping.

Example 3.11. Let $\mu_1, \mu_2$ and $\mu_3$ be fuzzy sets on $X = \{a, b, c\}$ with

$$
\begin{align*}
\mu_1(a) &= 0.1, \mu_1(b) = 0.1, \mu_1(c) = 0.1, \\
\mu_2(a) &= 0.2, \mu_2(b) = 0.2, \mu_2(c) = 0.2 \\
\mu_3(a) &= 0.3, \mu_3(b) = 0.3, \mu_3(c) = 0.3.
\end{align*}
$$

Let $\tau = \{0_X, \mu_2^*, 1_X\}$ and $\tau^* = \{0_Y, \mu_1, \mu_3, 1_X\}$ be fuzzy topologies on $X$. Consider the identity mapping $i_X : (X, \tau) \to (X, \tau^*)$. We have $\mu_3 \leq i_X(\mu_2^*) = \mu_2^*$. Since $\mu_3$ is a fuzzy $\gamma$-open set on $(X, \tau)$, $i_X$ is somewhat fuzzy $\gamma$-open. But $i_X(\mu_2^*) = \mu_2^*$ is not a fuzzy $\gamma$-open set on $(X, \tau^*)$. Hence $i_X$ is not a fuzzy $\gamma$-open mapping.
Theorem 3.12. Let \( f : X \to Y \) be a bijection. Then the following are equivalent:

1. \( f \) is somewhat fuzzy irresolute \( \gamma \)-open.
2. If \( \mu \) is a fuzzy \( \gamma \)-closed set on \( X \) such that \( f(\mu) \neq 1_Y \), then there exists a fuzzy \( \gamma \)-closed set \( \nu \neq 1_Y \) on \( Y \) such that \( f(\mu) < \nu \).

Proof. (1) implies (2): Let \( \mu \) be a fuzzy \( \gamma \)-closed set on \( X \) such that \( f(\mu) \neq 1_Y \). Since \( f \) is bijective and \( \mu^c \) is a fuzzy \( \gamma \)-open set on \( X \), \( f(\mu^c) = (f(\mu))^c \neq 0_Y \). And, since \( f \) is somewhat fuzzy irresolute \( \gamma \)-open, there exists a \( \gamma \)-open set \( \delta \neq 0_Y \) on \( Y \) such that \( \delta < f(\mu^c) = (f(\mu))^c \). Consequently, \( f(\mu) < \delta = \nu \neq 1_Y \) and \( \nu \) is a fuzzy \( \gamma \)-closed set on \( Y \).

(2) implies (1): Let \( \mu \) be a fuzzy \( \gamma \)-open set on \( X \) such that \( f(\mu) \neq 0_Y \). Then \( \mu^c \) is a fuzzy \( \gamma \)-closed set on \( X \) and \( f(\mu^c) \neq 1_Y \). Hence there exists a fuzzy \( \gamma \)-closed set \( \nu \neq 1_Y \) on \( Y \) such that \( f(\mu^c) < \nu \). Since \( f \) is bijective, \( f(\mu^c) = (f(\mu))^c \). Suppose there exists no fuzzy \( \gamma \)-open set \( \nu^c \neq 0_X \) on \( Y \) such that \( f(\mu^c) < \nu^c \). Thus there exists a \( \gamma \)-closed set \( \delta^c \) on \( Y \) such that \( \nu < \delta^c < 1 \). This is a contradiction. Hence \( f^{-1}(\nu) \) is fuzzy \( \gamma \)-dense on \( X \).

Theorem 3.13. Let \( f : X \to Y \) be a surjection. Then the following are equivalent:

1. \( f \) is somewhat fuzzy irresolute \( \gamma \)-open.
2. If \( \nu \) is a fuzzy \( \gamma \)-dense set on \( X \), then \( f^{-1}(\nu) \) is a fuzzy \( \gamma \)-dense set on \( X \).

Proof. (1) implies (2): Let \( \nu \) be a fuzzy \( \gamma \)-dense set on \( Y \). Suppose \( f^{-1}(\nu) \) is not fuzzy \( \gamma \)-dense on \( X \). Then there exists a fuzzy \( \gamma \)-closed set \( \mu \) on \( X \) such that \( f^{-1}(\nu) < \mu \leq 1 \). Since \( f \) is somewhat fuzzy irresolute \( \gamma \)-open and \( \mu^c \) is a fuzzy \( \gamma \)-open set on \( X \), there exists a fuzzy \( \gamma \)-open set \( \delta \neq 0_Y \) on \( Y \) such that \( f(\text{Int} \mu^c) \leq f(\mu^c) \). Since \( f \) is surjective, \( \delta \leq f(\mu^c) < f(f^{-1}(\nu^c)) = \nu^c \). Thus there exists a \( \gamma \)-closed set \( \delta^c \) on \( Y \) such that \( \nu < \delta^c < 1 \). This is a contradiction. Hence \( f^{-1}(\nu) \) is fuzzy \( \gamma \)-dense on \( X \).

(2) implies (1): Let \( \mu \) be a fuzzy open set on \( X \) and \( f(\mu) \neq 0_Y \). Suppose there exists no fuzzy \( \gamma \)-open \( \nu \neq 0_Y \) on \( Y \) such that \( \nu < f(\mu) \). Then \( (f(\mu))^c \) is a fuzzy set on \( Y \) such that there exists no fuzzy \( \gamma \)-closed set \( \delta \) on \( Y \) with \( (f(\mu))^c \) \( \leq \delta \). This means that \( (f(\mu))^c \) is fuzzy \( \gamma \)-dense on \( X \). Hence \( f^{-1}((f(\mu))^c) \) is fuzzy \( \gamma \)-dense on \( X \). But \( f^{-1}((f(\mu))^c) = f^{-1}(f(\mu))^c \leq \mu^c < 1 \). This is a contradiction to the fact that \( f^{-1}(f(\nu))^c \) is fuzzy \( \gamma \)-dense on \( X \). Hence there exists a \( \gamma \)-open set \( \nu \neq 0_Y \) on \( Y \) such that \( \nu < f(\mu) \). Therefore, \( f \) is somewhat fuzzy irresolute \( \gamma \)-open.

References

2. I. M. Hanafy, \( \gamma \)-open sets and fuzzy \( \gamma \)-continuity, J. Fuzzy Math. 7 (1999), 419-430.

**Young Bin Im** received his B.S. and Ph.D. at Dongguk University under the direction of Professor K. D. Park. Since 2009 he has been a professor at Dongguk University. His research interests are fuzzy topological space and fuzzy matrix.
Faculty of General Education, Dongguk University, Seoul 100-715, Korea.
*e-mail*: philpen@dongguk.edu

**Joo Sung Lee** received his B.S. from Dongguk University and Ph.D. at University of Florida under the direction of Professor B. Brechner. Since 1995 he has been a professor at Dongguk University. His research interests are topological dynamics and fuzzy theory.
Department of Mathematics, Dongguk University, Seoul 100-715, Korea.
*e-mail*: jsl@dongguk.edu

**Yung Duk Cho** received his B.S. and Ph.D. at Dongguk University under the direction of Professor J. C. Lee. Since 2008 he has been a professor at Dongguk University. His research interests are fuzzy category and fuzzy algebra.
Faculty of General Education, Dongguk University, Seoul 100-715, Korea.
*e-mail*: joyd@dongguk.edu