GLOBAL KINEMATICS CHARACTERISTICS ANALYSIS OF PLANAR MECHANISMS

Guoying ZENG[1],Dengfeng ZHAO[2], Yubin LU[3]

Based on the singularity of constraint equations, the analysis method of global kinematics characteristics of planar single-loop mechanisms was studied by using singularity division of the dimension space and the solution space of mechanisms. Results show that this analysis method has two advantages, viz. all global kinematics characteristics are included in the division results and various global analysis problems can be translated into multifarious retrievals of condition-value of vertexes. Thus, this method is appropriate for analysis of complicated mechanisms by computers.

Keywords: planar mechanisms; singularity; solution space; global characteristics

1. Introduction

Traditional mechanisms analysis is to solve variation with time about kinematic parameters, under the condition that links connection relationship, links size and drive condition have been determined. Whereas, global analysis of mechanism motility performance is to discuss mechanisms performance distribution and its evolution process with links size. Traditional mechanisms analysis is based on the kinematics and dynamics, whose analysis method is already quite mature. Many software applications can solve very complicated problems. Global analysis is based on modern mathematics theories such as topology, singular bifurcation theory, group theory and so on.

The global characteristics of mechanisms were closely related with the singularity of mechanisms. However, the current researches only focus on singularity of mechanisms. In the early time, Ting et al. [1] studied the rotatability law for N-bar kinematic chains by a traditional method. Afterwards the researches on the singularity of mechanisms are widely conducted. Alici [2] focused on the determination of singularity contours for a manipulator. Jiang et al. [3] gave the singularity orientation of the Gough–Stewart platform. Hang et al. [4] presented the decoupling conditions of spherical parallel mechanisms. Wolf et al. [5] analyzed the singularities of a three degree of freedom spatial Cassino Parallel manipulators. Li et al.[6] performed a recursive matrix approach in kinematics and dynamics modelling of parallel mechanisms. Researches on the relations between the global kinematics characteristics and the singularity of mechanisms

[1] Prof, Southwest University of Science and Technology, PR China, e-mail: zgywf@163.com
[2] Prof, Southwest University of Science and Technology, PR China, e-mail: 1264879506 @qq.com
[3] Prof, Southwest University of Science and Technology, PR China, e-mail: 289232944 @qq.com

are few. Gao et al. [7] introduce the concept of solution space to the mechanisms evolution analysis, the workspace atlas of parallel planar manipulators were obtained. Zhao et al. [8, 9] using the convex hull division method explored the classification method of single-loop planar and spherical mechanisms.

If viewed from mathematical, mechanisms can be looked as nonlinear geometric constraint equations defined on variables of motion space and dimension space. Variables of motion space here changes with time, such as motion parameter of motion pair. Variables of dimension space here remain constant during movement, such as the size of links. Solution set of constraint equations in motion space is termed as the solution space. Dimension of the solution space is the difference between the amount of motion parameter and the number of constraint equation, which also is the degree of freedom of the mechanism. All characteristics of mechanism depend on the characteristics of the constraint equations, and completely reflected in the solution space. The global analysis of mechanism should include the following two main parts.

(1) The classification of complicated mechanisms

For the mechanisms with fixed connection relationship, the topology of the solution space may mutate with the change of links size. The global structure of all mechanism performance will mutate with the topology mutations of solution space. Thus most natural classification of dimension is formed. Division of high-dimensional dimension space used by all the conditions of classification would generate multitudinous geometric objects (simple or complex). The relationship of geometric objects helps us to grasp overall evolution of the mechanisms types.

(2) The analysis of solution space of the complicated mechanisms

The solution space of mechanism with fixed connection relationship and sizes is determined. To find out the overall structure of the solution space, a straightforward method is that the solution space is divided into geometric objects with simple topologic structure. The boundary of geometric objects is corresponding to the singular motion position. The characteristic of mechanisms is no singular monotonic variation in each geometric object. The adjacency relationship of geometric objects may reveal the whole distribution of mechanisms motion performance.

The global performance analysis problem of general mechanism is complex. In this paper, the global performance analysis about the simplest mechanisms, that is planar single loop mechanism, is only discussed. The analysis conclusions are given for 4,5,6-link mechanisms.

2. Constraint equations and singularity of mechanisms

A single-loop N-link planar mechanism is shown in Fig.1, where N links are connected with N revolute joints. The lengths of links are $\boldsymbol{l}=(l_1,\ldots,l_N)$, which are variables of dimension space. The angles of links $\boldsymbol{\theta}=(\theta_1,\ldots,\theta_N)$ are variables of motion space. These variables satisfy the vector constraint equations,

$$\boldsymbol{f}(\boldsymbol{\theta},\boldsymbol{l})=\begin{pmatrix} l_1\cos\theta_1+l_2\cos\theta_2\cdots+l_n\cos\theta_n \\ l_1\sin\theta_1+l_2\sin\theta_2\cdots+l_n\sin\theta_n \end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix} \tag{1}$$

The degree of freedom of the mechanism F (including the holistic freedom) equals to N-2. The solution space is an F-dimensional manifold.

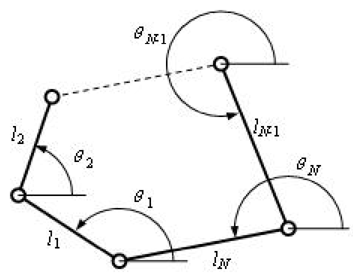


Fig.1 The planar single-loop mechanisms

According to the singularity geometry theories, the singularity of the mechanisms depend on the first-order differential of Eq.(1)

$$df=J_\theta d\theta+J_l dl+O(d\theta)^2=0 \tag{2}$$

where $O(d\theta)^2$ is the 2-order infinitesimal, J_θ and J_l is the Jacobi matrix of Eq.(1) to $\boldsymbol{l}$ and $\boldsymbol{\theta}$, respectively, i.e.

$$J_\theta=\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{\theta}}=\begin{pmatrix} -l_1\sin\theta_1 & -l_2\sin\theta_2 & \cdots & -l_n\sin\theta_n \\ l_1\cos\theta_1 & l_2\cos\theta_2 & \cdots & l_n\cos\theta_n \end{pmatrix} \qquad J_l=\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{l}}=\begin{pmatrix} \cos\theta_1 & \cos\theta_2 & \cdots & \cos\theta_n \\ \sin\theta_1 & \sin\theta_2 & \cdots & \sin\theta_n \end{pmatrix}$$

The singularity only depend on the J_θ because the lengths of links are fixed, $d\boldsymbol{l}$=0. The J_l will be used to the mutation analysis of the solution space. J_θ is a 2×N non-square matrix. The relations between singular of mechanisms and singular of matrix J_θ include three aspects as follows.

First, two columns of i, j randomly taking from J_θ can form a 2×2 square matrix J_{ij}. If J_{ij} is a singular, the determinant D_{ij} equals to 0, which is expressed as

$$D_{ij}=l_i l_j\sin(\theta_i-\theta_j)=0 \quad or \quad \varphi_{ij}=\theta_i-\theta_j=0 \text{ or } \pi \tag{3}$$

where φ_{ij} is the relative angles between link i and j. This means that the link i parallel or collinear to link j in the same direction or reverse direction. At this moment, if relative angle φ_{ij} is a driving parameter, other relative angles which are unrelated to the link i and j will not be controlled by $d\varphi_{ij}$. On the other hand, if the relative angle φ_{ij} is a driven parameter, the dead center will present. This situation is called quasi-singularity configuration.

Second, with the increase of links which are collinear or parallel, the singularity of mechanisms becomes severer. Especially, when only one motion parameter is unrelated to the collinear or parallel links, this parameter will be worst, and will be an extremum. If this parameter is a driving parameter, the position of mechanism certainly is a dead center. If this parameter is a driven parameter, all driving parameters can not control this parameter. This situation is called singularity configuration.

Third, when the rank of J_θ is equal to 1, there is a singular point in the solution space. The topology of the solution space will mutate even if the lengths of links change slightly. The global structure of all mechanism performances will mutate too. Under this situation, the determinant of the each J_{ij} equals to zero. Namely, all links in the mechanism are collinear. The lengths of links satisfy

$$\pm l_1 \pm l_2 \pm \cdots \pm l_n = 0 \tag{4}$$

Eq.(4) is the topology mutational condition of the solution space. As an essential basis of the classification of mechanisms, Eq.(4) can be proved to do not relate with the specific forms of the constraint equations.

The analyses of global kinematics characteristics include two main parts. First, the dimension space will be divided into numerous basic complexes by Eq.(4). The mechanisms can be classed by the basic complexes. Second, the solution space will be divided into numerous basic complexes by Eq.(3). The adjacency relations among those basic complexes include all information about global kinematics characteristics of mechanisms.

3. Division conditions of the dimension space and solution space

In Eq.(4), there are N "±" symbols, but the number of independent equations should be 2^{N-1}-1, not 2^N. In order to simplify the analysis and intuitively show the results, three additional conditions are necessary, which are as follows,

First, when changing lengths of all links by the same rate, the constraint equations, Eq.(1), will not change. In order to keep symmetry, a new constraint condition that the sum of lengths of all links equals to 1 is introduced. Second, each link should satisfy $l_i>0$. Hence, the division of the dimension space is limited within the first quadrant. Third, when the order of links is arbitrarily changed, Eq.(1) and Eq.(4), remain unchanged. The solution space will remain unchanged too. Hence, the division range can be further limited within the area that satisfies $l_1>l_2>\ldots>l_n$ in the first quadrant.

Based on the foregoing analysis, the number of division condition of the dimension space is $2^{n-1}+2n-2$,

$$\begin{cases} l_1 + l_2 + \cdots + l_n = 1 \\ l_i = 0 \qquad i = 1,2,\cdots,n \\ l_j - l_{j+1} = 0 \qquad j = 1,2,\cdots,n-1 \\ \pm l_1 \pm l_2 \pm \cdots \pm l_n = 0 \end{cases} \tag{5}$$

All division conditions are all linear equations and the matrix format is

$$\boldsymbol{V}_D = \boldsymbol{T}_D \boldsymbol{l} - \boldsymbol{a} = 0 \tag{6}$$

where $\boldsymbol{V}_D$ and $\boldsymbol{T}_D$ is termed as condition value vector and condition matrix respectively in dimension space, $\boldsymbol{a}$ is the right parts of Eq.(5).

The division conditions of the solution space is Eq.(3), viz. the condition value V_S equals to 0

$$\boldsymbol{V}_S = l_i l_j \sin(\theta_i - \theta_j) = 0 \tag{7}$$

For planar N-link single-loop mechanisms, the number of division conditions is $n(n+1)/2$. Each division condition divides the solution space to two parts, in one $V_S > 0$, in the other $V_S < 0$. The matrix format is

$$\boldsymbol{T}_S\boldsymbol{\theta} = 0 \qquad \text{and} \qquad \boldsymbol{T}_S\boldsymbol{\theta} = \pi \tag{8}$$

Where, $\boldsymbol{T}_S$ is termed as the condition matrix of the solution space.

4. Basic complexes and their singularity

The dimension space and the solution space will be divided into numerous basic complexes or simplexes with different space-dimensions by their division conditions. Because all basic complexes can be fully expressed by the 0-simplexes (vertexes), the 0-simplexes and their condition values will be basis of analysis.

The position of each 0-simplex can be specified by its coordinate. To solve the coordinates of all the 0-simplexes, the enumeration method or recursion method will be used to traverse all the possible cases.

1. The 0-simplexes in the dimension space

In the division conditions of the dimension space, Eq.(5), the first formula and N-1 other formulae compose the linear equations to solve the coordinate of a 0-simplex (vertex). If the equations have a solution and this solution is in the region limited by the second and third formula of Eq.(5), the solution is the coordinate of an effective 0-simplex. The traversal problem is a simple combination traversal. After removing repeated 0-simplexes, all the 0-simplexes in the dimension space were obtained. It is convenient that all the coordinate of 0-simplexes were listed in a matrix $\boldsymbol{C}_D$, and termed as the coordinate matrix.

The singularity characteristics of each 0-simplex can be fully expressed by the condition value sequence. All the condition value sequences of the 0-simplexes can be written in a matrix $\boldsymbol{V}_D^0$ and termed as condition value matrix, which can be calculated by Eq.(6). But the sign of the condition value is more important, and is expressed as matrix $\boldsymbol{S}_D^0$. It is termed as sign matrix and the matrix elements are

$$S_D^0(i,j) = sign\left(V_D^0(i,j)\right) = \begin{cases} 1 & V_D^0(i,j) > 0 \\ -1 & V_D^0(i,j) < 0 \\ 0 & V_D^0(i,j) = 0 \end{cases} \tag{9}$$

2. The 0-simplexes in the solution space

The constraint equations, Eq.(1), and N-3 independent formulae in singular conditions, Eq.(8), compose the equations to solve the coordinate of a 0-simplex.

This method can be treated as solving triangular angles. The links which are parallel correspond to a triangular side. The detailed steps are as follows.

(1) The integers of $0\sim3^N$-1 are translated as ternary number of N digits, and each digit have three possible values of 0, 1 and 2. Each digit corresponds to a link of the mechanism in sequence. The three values divide links into three groups

$$g_k = \{l_{k_1}, l_{k_2}, \cdots l_{k_n}\}, \quad k \in \{0,1,2\} \quad k_1, \cdots k_n \in \{1,2,\cdots,N\} \tag{10}$$

(2) Removing the case that g_k is empty.

(3) Removing the repeated cases that three groups are in different order, as $\{g_0,g_1,g_2\}$ with six various orders represent the same division for links.

(4) Because the links in each group can be collinear or parallel in same direction or reverse direction, the total length of links in each group is

$$l_{g_k} = l_{k_1} \pm l_{k_2} \pm \cdots \pm l_{k_n} \tag{11}$$

(5) After specifying a link angle θ_i arbitrarily, the direction of triangle sides can be calculated by the total length l_{g_k},

It is more convenient that the coordinate of 0-simplexes is expressed by the relative angles φ_{ij}. All coordinates of 0-simplexes can be written as matrix format, expressed as $\boldsymbol{C}_S$, and named as coordinate matrix in the solution space.

The singular characteristics of each 0-simplex can be fully expressed by the condition value sequence of division conditions. The all condition value sequence can be written in a matrix $\boldsymbol{V}_S^0$, named as condition value matrix in the solution space. The elements can be calculated by Eq.(7), are as follows

$$V_S^0(i,j) = l_m l_n \sin(\varphi_{mn}(i)) \tag{12}$$

The row of $\boldsymbol{V}_S^0$ corresponds to the serial number of each 0-simplex, and the j-th column corresponds to j-th division condition which corresponds to φ_{mn}. The sign of condition values is more important. All sign of condition values can be written in a matrix $\boldsymbol{S}_S^0$, and termed as sign matrix in the solution space, the elements are expressed as follows,

$$S_S^0(i,j) = \begin{cases} 1 & V_S^0(i,j) > 0 \\ 0_0 & \varphi_{mn} = 0 \\ 0_\pi & \varphi_{mn} = \pi \\ -1 & V_S^0(i,j) < 0 \end{cases} \tag{13}$$

After dividing the dimension space and the solution space, numerous nonzero-dimensional basic complexes are obtained. At the same time, there are complicated adjacency relations among those basic complexes. According to the topology theory, a basic complex v can be expressed by its vertexes (0-simplexes),

$$v = < k_1, k_2, \cdots k_m > \tag{14}$$

where $k_1 \sim k_m$ are the serial number of the vertexes, m is the amount of vertexes in v. The sign matrix of vertexes of the basic complex v is a submatrix of S_S^0 (or S_D^0)

and be expressed as S_v^0. The singularly and adjacency relations of all the basic complexes can be obtained from sign matrix S_S^0 (or S_D^0).

1. The analysis principle of nonzero-complexes

(1) For the sign matrix S_v^0 of a basic complex *v*, there are not the elements with inverse sign. Hence, the all interior points have same sign and which can be calculated from the S_v^0 as follows

$$s(j) = sign\left[\sum_{i=1}^{m} S_v^0(i,j)\right] \tag{15}$$

All *s*(*j*) compose the sign sequence, S_v, of the basic complex *v*, which can fully express the singularity characteristics of the basic complex *v*. In the dimension space (or solution space), sign sequences of all the basic *m*-complexes can be written in a matrix $\boldsymbol{S}_D^m$ (or $\boldsymbol{S}_S^m$) and be termed as the sign matrix of basic *m*-complexes.

(2) Each basic complex can be identified by the sign sequence.

(3) For two basic *m*-complexes with the sign sequences S_v and S_v', if only the *k*-th sign *s*(*k*) is reverse to *s*'(*k*) and other signs are same, the two basic complexes are adjacent each other. The adjacency boundary is a basic (*m*-1)-complex whose *k*-th sign is 0 and other signs are equaled to S_v or S_v'.

(4) For a basic complex *v*, the dimension of *v* is D_v, and the dimension of embedment space of *v* is D_S. The 0 elements set of sign sequence S_v is $\{k_1^0,\ldots,k_n^0\}$ and those rows of the condition matrix $\boldsymbol{T}_S$ (or $\boldsymbol{T}_D$) compose a matrix $\boldsymbol{T}_v$. Obviously, the matrix $\boldsymbol{T}_v$ is the constraint matrix of *v*. Those parameters should satisfy

$$D_v = D_S - rank(T_v) \tag{16}$$

2. Method of obtaining all the basic nonzero-complexes

Any basic complex can be uniquely specified by its sign sequence S_v, so the enumeration method or recursion method can always traverse all possible cases, and all basic complexes can be obtained.

The traversal problem of a sequence S_v with finite length and finite value is easy. For each case, the analysis steps are as follows.

(1) According to the first and second item of the above section, "the analysis principle of nonzero-simplexes", for a specified sign sequence S_v, the vertexes of the basic complex *v*, can be obtained by searching the sign matrix of 0-simplexes S_D^0 (or S_S^0), and the sign matrix of the basic complex, S_v^0,can be obtained at the same time.

(2) According to the fourth item of the above section, "the analysis principle of nonzero-simplexes", the dimension of the basic complex, D_v, can be calculated by Eq.(16).

(3) The sign matrix, S_D^m or S_S^m can be formed by uniting all sign sequences of basic *m*-complexes.

For N-link mechanisms, the division results are many basic complexes in the dimension space or the solution space. The adjacency relations among those basic complexes can be fully expressed by the relational matrixes. If there are J basic k-complexes and I basic (k-1)-complexes, the relational matrix $\boldsymbol{L}^k$ is an I by J matrix and the elements are

$$L^k(i,j)=\begin{cases}1 & v_i\in+\partial v_j\\ -1 & v_i\in-\partial v_j\\ 0 & v_i\notin\partial v_j\end{cases} \tag{17}$$

where v_i is a basic (k-1)-complex, v_j is a basic k-complex, and ∂v_j expresses the boundary of v_j.

According to the first or fourth item in the above section, “the analysis principle of nonzero-simplexes”, all relational matrixes can be obtained.

5. Analysis of mechanisms global characteristics for typical problems

The analysis of mechanisms global characteristics is very comprehensive. In this paper, only three general problems are discussed and the 4, 5, 6-link mechanisms were given.

1. Classification of planar single-loop mechanisms

The region in the dimension space which satisfies the 1, 2, 3-th formula of Eq.(5) is divided by the mutation conditions of the solution space. To save the space, only the 0-simplexes and the (N-1)-simplexes are listed in Tab.1.

Table 1

Main division results of the dimension space for the 4, 5, 6-link mechanisms

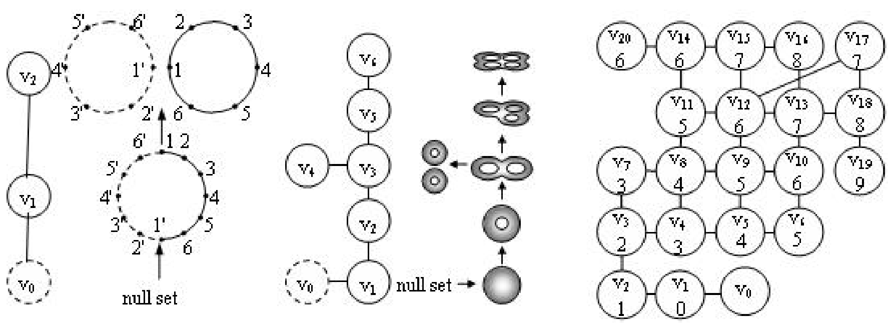
Coordinates of 0-simplexes					
1	0	0	0	0	0
1/2	1/2	0	0	0	0
1/2	1/4	1/4	0	0	0
1/3	1/3	1/3	0	0	0
1/2	1/6	1/6	1/6	0	0
1/4	1/4	1/4	1/4	0	0
1/2	1/8	1/8	1/8	1/8	0
1/3	1/6	1/6	1/6	1/6	0
1/4	1/4	1/6	1/6	1/6	0
1/4	1/4	1/4	1/8	1/8	0
1/5	1/5	1/5	1/5	1/5	0
1/2	1/10	1/10	1/10	1/10	1/10
3/8	1/8	1/8	1/8	1/8	1/8
1/3	1/6	1/6	1/6	1/12	1/12
3/10	1/5	1/5	1/10	1/10	1/10
1/4	1/4	1/4	1/12	1/12	1/12
1/4	1/4	1/8	1/8	1/8	1/8
1/6	1/6	1/6	1/6	1/6	1/6

Basic (N-1)-complexes					
4-link	v_0<1,2,3,5>	6-link	v_0<1,2,3,5,7,12>	6-link	v_{11}<2,3,6,8,10,15>
4-link	v_1<2,3,5,6>	6-link	v_1<2,3,5,7,12,13>	6-link	v_{12}<2,6,8,9,10,15>
4-link	v_2<2,3,4,6>	6-link	v_2<2,3,5,7,8,13>	6-link	v_{13}<6,8,10,15,17,18>
5-link	v_0<1,2,3,5,7>	6-link	v_3<2,3,5,8,13,14>	6-link	v_{14}<2,3,6,10,15,16>
5-link	v_1<2,3,5,7,8>	6-link	v_4<2,3,8,13,14,15>	6-link	v_{15}<2,6,10,15,16,17>
5-link	v_2<2,3,5,6,8>	6-link	v_5<2,8,13,14,15,17>	6-link	v_{16}<6,10,15,16,17, 18>
5-link	v_3<2,3,6,8,10>	6-link	v_6<8,13,14,15,17,18>	6-link	v_{17}<2,6,8, 9,10,17>
5-link	v_4<2,3,4,6,10>	6-link	v_7<2,3,5,6,8,14>	6-link	v_{18}<6,8, 9,10,17,18>
5-link	v_5<2,6,8,9,10)>	6-link	v_8<2,3,6,8,14,15>	6-link	v_{19}<6,8,9,10,11,18>
5-link	v_6<6,8,9,10,11>	6-link	v_9<2,6,8,14,15,17>	6-link	v_{20}<2,3,4,6,10,16>
5-link		6-link	v_{10}<6,8,14,15,17,18>	6-link	

There are 18 0-simplexes in the division results of the 6-link mechanisms. The 1~11-th 0-simplexes which satisfy l_6=0, are the 0-simplexes of 5-link mechanisms. The 1~6-th 0-simplexes which satisfy l_6=l_5=0, are the 0-simplexes in of 4-link mechanisms.

All the (N-1)-simplexes correspond to basic types of mechanisms. Other simplexes are on the boundary of the (N-1)-simplexes and correspond to the transition types among basic types. All interior points of the (N-1)-simplex v_0 satisfy the link l_1 greater than the sum of links, which can not form a mechanism.

For 4, 5, 6-link mechanisms, the evolvement of the basic types are shown in Fig.2. Only the (N-2)-dimensional evolvement paths were given. In addition, the lower dimension of a basic complex, the more singular of mechanisms.



(a) (b) (c)

Fig.2 The evolvement process of the basic types of mechanisms

2. The topology of the solution space of basic mechanism types

The topology of the solution space can be studied by the singular division method. The amount of basic complexes is listed in Tab.2 and the adjacency relations among basic complexes are not listed. Various problems about global characteristics can be solved by retrieving the sign sequences of 0-simplexes.

For the 4-link mechanisms, the solution space is 1-dimension. There are 12 0-simplexes and 12 1-simplexes. The adjacency relations were shown in Fig.2(a) and Tab.2. The two types of the solution space are homeomorphic to one and two circles, respectively. The vertexes and their sign sequence of the 4-link mechanisms are listed in Tab.3. For the 5, 6-link mechanisms, the division results are very complicated. For example, there are 588 0-simplexs and numerous other basic complexes of the type v_{13} of 6-link mechanisms.

The topological invariants, such as the Euler's characteristic and homology group, can express the topology structure of the solution space. The calculation of the topological invariants is very convenient by the division results.

If there are m_k basic k-complexes, the Euler's characteristic γ is

$$\gamma = \sum_{k=0}^{D_s} m_k (-1)^k \tag{18}$$

Table 2

The division results and topological invariants of the solution space

Mechanisms & types		topological invariants			Basic complexes							
		γ	D^0	D^1	Point	Line	Trigon	Tetragon	Pentagon	4-hedron	5-hedron	6-hedron
4-link	v_1	0	1	1	12	12						
	v_2	0	2	2	12	12						
5-link	v_1	2	1	0	50	120	48	24	0			
	v_2	0	1	2	72	168	48	48	0			
	v_3	-2	1	4	86	204	60	52	4			
	v_4	0	2	4	72	168	48	48	0			
	v_5	-4	1	6	92	228	84	36	12			
	v_6	-6	1	8	90	240	120	0	24			
6-link	v_1	0	1	0	180	780	720	360	0	240	240	0
	v_2	0	1	1	300	1260	960	720	0	288	384	48
	v_3	0	1	2	396	1668	1224	960	24	360	480	96
	v_4	0	1	3	468	2004	1552	1080	72	456	528	144
	v_5	0	1	4	516	2268	1824	1080	144	576	528	192
	v_6	0	1	5	540	2460	2160	960	240	720	480	240
	v_7	0	1	3	432	1776	1152	1152	0	288	576	96
	v_8	0	1	4	504	2112	1440	1272	48	384	624	144
	v_9	0	1	5	552	2376	1752	1272	120	504	624	192
	v_{10}	0	1	6	576	2568	2088	1152	216	648	576	240
	v_{11}	0	1	5	516	2148	1416	1336	40	360	656	144
	v_{12}	0	1	6	564	2412	1728	1336	112	480	656	192
	v_{13}	0	1	7	588	2604	2064	1216	208	624	608	240
	v_{14}	0	1	6	504	2112	1440	1272	48	384	624	144
	v_{15}	0	1	7	552	2376	1752	1272	120	504	624	192
	v_{16}	0	1	8	576	2568	2088	1152	216	648	576	240
	v_{17}	0	1	7	552	2376	1752	1272	120	504	624	192
	v_{18}	0	1	8	576	2568	2088	1152	216	648	576	240
	v_{19}	0	1	9	540	2460	2160	960	240	720	480	240
	v_{20}	0	2	6	432	1776	1152	1152	0	288	576	96

For a 2-dimension closed space, the genus g is $(\gamma\text{-}2)/2$. But for an odd-dimension closed space, γ equals to 0. Hence, for the 5-link mechanisms, the relations between genus of the solution space and mechanism types are v_1 to 0, v_2 to 1, v_3 to 2, v_5 to 3, and v_6 to 4. But the 4, 6-link mechanisms satisfy $\gamma \equiv 0$.

Table 3

3 All the 0-simplexes and their sign sequences of the 4-link mechanisms

	Type v_1												Type v_2											
	1	1’	2	2’	3	3’	4	4’	5	5’	6	6’	1	1’	2	2’	3	3’	4	4’	5	5’	6	6’
φ_{12}	0_π	0_π	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
φ_{13}	-1	1	0_π	0_π	-1	1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
φ_{14}	1	-1	1	-1	1	-1	0_π	0_π	-1	1	1	-1	0_π	0_π	0_0	0_0	1	-1	-1	1	-1	1	1	-1
φ_{23}	1	-1	-1	1	0_0	0_0	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
φ_{24}	-1	1	-1	1	-1	1	-1	1	0_0	0_0	-1	1	-1	1	1	-1	0_π	0_π	0_0	0_0	1	-1	-1	1
φ_{34}	-1	1	-1	1	-1	1	1	-1	1	-1	0_0	0_0	1	-1	-1	1	-1	1	1	-1	0_π	0_π	0_0	0_0

For a solution space G and the integer group Z, the k-dimensional homology group is expressed as $H^k(G,Z)$ and its dimension D^k is

$$D^k = m_k - rank(L^k) - rank(L^{k+1}) \tag{19}$$

where $rank(L^k)$ is the rank of the relational matrix L^k. For an m-dimensional solution space, there are m+1 homology groups. But their dimension satisfy D^k=D^{m-k}, so only the D^1 and D^0 are listed in Tab.2. The D^0 is number of connected branches and the D^1 corresponds to the genus. The types of mechanisms that the number of connected branches is equal to 2 include the v_2 of 4-link, v_4 of 5-link and v_{20} of 6-link mechanism. For 6-link mechanisms, genus of the solution space increases 1 at each evolvement.

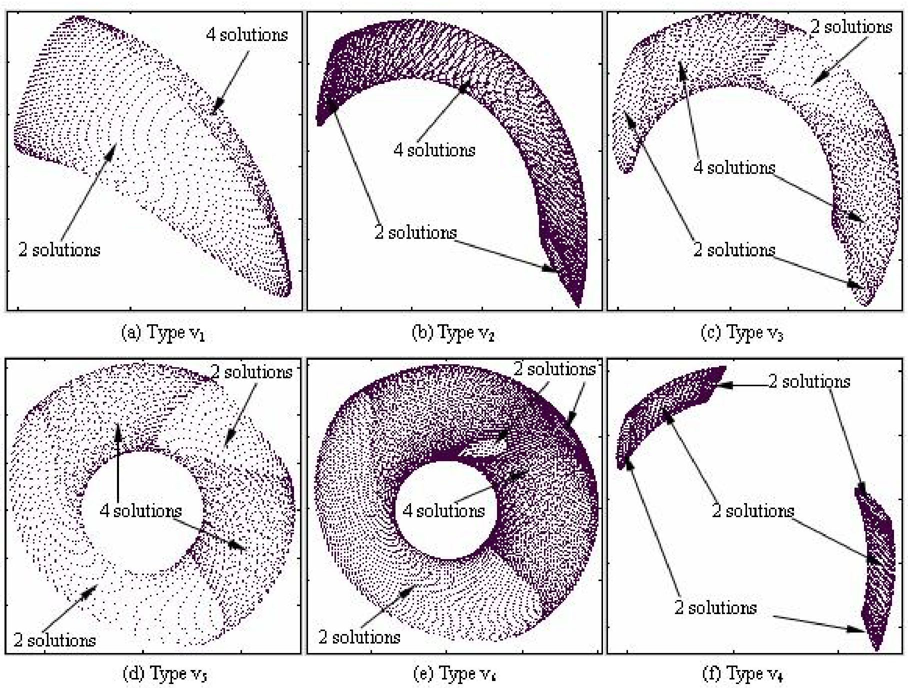
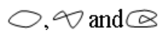


Fig.3 The work space of the 5-link mechanisms

The work space of a mechanism can be treated as the projection of the solution space on the physics space. For a mechanism with N_s-dimension solution space and N_w-dimension work space, a generic point in work space correspond to a (N_s-N_w)-dimension space. The work space can be divided into many regions by some singularly conditions and the topology is different in different regions.

For a 5-link mechanism, typical work space is shown in Fig.3. The topology of solution space of the 6 types of mechanisms can be clearly observed from the work space. The work space is divided into several regions. The number of solutions is different in different regions. There are two types of regions, two solutions and four solutions. For example, the 4-link mechanisms have three types of the work spaces, . Deep discussion can be referenced in Ref. [9].

3. Rotatability analysis of the mechanisms

The rotatability analysis of a mechanism is to ascertain whether a relative angle φ_{ij} (or combination of relative angles) can fully rotate. A relative angle which can fully rotate be termed as full-angle. After division, rotatability analysis can be completed by retrieving the sign matrix S_S^0. For a φ_{ij}, if the sign 0_0 and 0_π are included simultaneously in the signs of all 0-simplexes, the φ_{ij} is a full angle.

For the 4-link mechanisms, the full-angles can be found easily from the Tab.3. There are not any full-angles in the type v_1 and the relative angles φ_{14}, φ_{24} and φ_{34} are full-angle in the type v_2. But for 5, 6-link mechanisms, computers should be used to find the full-angles and the results are shown in Tab.4. The brackets in Tab.4 represent that the combination of relative angles is full-angle.

Table 4

The rotatability analysis results of the 4, 5, 6-link mechanisms

Types		Full-angles	Types		Full-angles	Types		Full-angles
4-link	v_1	none	6-lnk	v_3	$\varphi_{i6}, \varphi_{i5}$	6-ink	v_{13}	$(\varphi_{i6}, \varphi_{i5}), (\varphi_{i6}, \varphi_{i4})$, $\varphi_{i3}, \varphi_{i2}$
	v_2	φ_{i4}		v_4	$\varphi_{i6}, \varphi_{i5}, \varphi_{i4}$		v_{14}	$(\varphi_{i6}, \varphi_{i5}), (\varphi_{i6}, \varphi_{i4}), (\varphi_{i5}, \varphi_{i4})$
5-link	v_1	none		v_5	$\varphi_{i6}, \varphi_{i5}, \varphi_{i4}, \varphi_{i3}$		v_{15}	$(\varphi_{i6}, \varphi_{i5}), (\varphi_{i6}, \varphi_{i4}), (\varphi_{i5}, \varphi_{i4}), \varphi_{i3}$
	v_2	φ_{i5}		v_6	$\varphi_{i6}, \varphi_{i5}, \varphi_{i4}, \varphi_{i3}, \varphi_{i2}$		v_{16}	$(\varphi_{i6}, \varphi_{i5}), (\varphi_{i6}, \varphi_{i4}), (\varphi_{i5}, \varphi_{i4})$, $\varphi_{i3}, \varphi_{i2}$
	v_3	$\varphi_{i5}, \varphi_{i4}$		v_7	$(\varphi_{i6}, \varphi_{i5})$		v_{17}	$(\varphi_{i6}, \varphi_{i5}), (\varphi_{i6}, \varphi_{i4}), (\varphi_{i3}, \varphi_{i6})$
	v_4	$(\varphi_{i5}, \varphi_{i4})$		v_8	$(\varphi_{i6}, \varphi_{i5}), \varphi_{i4}$		v_{18}	$(\varphi_{i6}, \varphi_{i5}), (\varphi_{i6}, \varphi_{i4}), (\varphi_{i3}, \varphi_{i6}), \varphi_{i2}$
	v_5	$\varphi_{i5}, \varphi_{i4}, \varphi_{i3}$		v_9	$(\varphi_{i6}, \varphi_{i5}), \varphi_{i4}, \varphi_{i3}$		v_{19}	$(\varphi_{i6}, \varphi_{i5}), (\varphi_{i6}, \varphi_{i4}), (\varphi_{i3}, \varphi_{i6})$, $(\varphi_{i6}, \varphi_{i2})$
	v_6	$\varphi_{i5}, \varphi_{i4}, \varphi_{i3}, \varphi_{i2}$		v_{10}	$(\varphi_{i6}, \varphi_{i5}), \varphi_{i4}, \varphi_{i3}, \varphi_{i2}$		v_{20}	$(\varphi_{i6}, \varphi_{i5}, \varphi_{i4})$
6-link	v_1	none		v_{11}	$(\varphi_{i6}, \varphi_{i5}), (\varphi_{i6}, \varphi_{i4})$			
	v_2	φ_{i6}		v_{12}	$(\varphi_{i6}, \varphi_{i5}), (\varphi_{i6}, \varphi_{i4}), \varphi_{i3}$			

In addition, for multi-degree of freedom mechanisms, motion path which evade the singularity configurations can be programmed by retrieving of the condition value sign of 0-simplexes.

6. Conclusions

The analysis method discussed in this paper is mainly consisted of three parts. (1) All the singularity conditions of the planar single-loop mechanisms are obtained from the singularity of constraint equations. (2) The singularity division method is the basic method. (3) The global characteristics analysis is translated into the multifarious retrieval for the sign matrix of 0-simplexes. The analysis method is suitable for computerized analysis on more complex mechanisms.

However, the current study is still inadequate. Further research will be conducting in the following aspects, i.e., popularizing this method to more complicated mechanisms, and developing analysis software concerning the global characteristics of the mechanisms.

Acknowledgment

This paper was supported by National Natural Science Foundation of China (51375410).

R E F E R E N C E S

[1] *K.L. Ting and Y.W. Liu,* Rotatability laws for N-bar kinematic chains and their proof. ASME Journal of Mechanisms Transmissions and Automation in Design, vol.108,no.1, Jan.1991,pp 32-39
[2] *G. Alici,* Determination of singularity contours for five-bar planar parallel manipulators. Robotica, vol.18, no. 5, May.2000, pp.569- 575
[3] *Q.M. Jiang and C.M. Gosselin,* Determination of the maximal singularity-free orientation workspace for the Gough–Stewart platform. Mechanism and Machine Theory, vol.44, no.6, Jun.2009, pp.1281-1293
[4] *L.B. Hang and Y. Wang,* Study on the decoupling conditions of the spherical parallel mechanism based on the criterion for topologically decoupled parallel mechanisms, Chinese Journal of Mechanical Engineering, vol. 41, no. 9, Sep. 2005, pp.28-32
[5] *Alon Wolf, Erika Ottaviano, Moshe Shoham, Marco Ceccarelli,* Application of line geometry and linear complex approximation to singularity analysis of the 3-DOF CaPaMan parallel manipulator, Mechanism and Machine Theory, vol.39, no.1, Jan.2004, pp.75-95
[6] *Yangmin Li, Stefan Staicu,* Inverse dynamics of a 3-PRC parallel kinematic machine, Nonlinear Dynamics, vol.67, no.1, Jan.2012, pp. 1031-1041

[7] *F. Gao, X.Q Zhang and Y.S. Zhao,* A physical model of the solution space and the atlas of reachable workspace for 2-DOF parallel planar manipulators, Mechanism and Machine Theory vol.31, no.2, Feb. 1996, pp. 173-184
[8] *D.F. Zhao,* Dimension Classification Method of Planar Mechanism Based on Convex Hull Division, Chinese Journal of Mechanical Engineering , vol.45 , no.4, Apr.2009, pp.100-104
[9] *D.F. Zhao, G.Y. Zeng, Y.B. Lu,* The classification method of spherical single-loop mechanisms, Mechanism and Machine Theory,vol. 51, no. 5, May.2012, pp.46-57