Paper

Optimal Pump Scheduling for Large Scale Water Transmission System by Linear Programming

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Abstract—Large scale potable water transmission system considered in this paper is the Toronto Water System, one of the largest potable water supply networks in North America. The main objective of the ongoing Transmission Operations Optimizer project consists in developing an advanced tool for providing such pumping schedules for 153 pumps, that all quantitative requirements with respect to the system operation are met, while the energy costs are minimized. We describe here a linear, so-called Simplified Model (SM), based on mass-balance equations, which is solved on week horizon and delivers boundary conditions for so-called Full Model (FM), which is nonlinear and takes into account hydraulic phenomena and water quality.

***Keywords*—linear programming, minimum cost operative planning, pump scheduling, water supply.**

1. Introduction

Toronto Water System (TWS) delivering water to 4 million people is the largest potable water supply network in Canada and the fifth largest in North America. It includes the whole City of Toronto (COT) and southern portion of the Region of York (ROY). TWS is supplied by 4 water filtration plants located at the north shore of Lake Ontario, and additionally by a number of wells at southern part of ROY. The average daily water demand from TWS is 2500 ML, while the total storage of reservoirs 2200 ML. It has 1300 km of pipelines, 153 pumps in 29 pumping stations, 19 pressure districts, 28 reservoirs and elevated tanks (many with two or more cells). The annual cost of water pumping is about 36 millions CAD (data from 2007). Since the electrical tariffs and costs structure are very volatile and unstable (changes are from hour to hour, and even at 15-minutes intervals), there is a need for an automatic control system of the network, reacting in a proper way to both the changes in customers demands and the market energy prices.

The main objective of the ongoing Transmission Operations Optimizer (TOO) project consists in developing an advanced tool for providing such pumping schedules for 153 TWS pumps that all quantitative requirements with respect to the system operation are met, while the energy costs are minimized [5]. It is assumed that TOO should produce detailed optimal schedules for all pumps which will be further passed to water transmission system by a Supervisory Control And Data Acquisition (SCADA) module.

The following modules of TOO has been developed: demand forecasting module, energy rates forecasting module, pumping schedule optimizer and, finally, an assessment module consisting mainly of hydraulic, EPANET based, TWS simulator (Fig. 1).

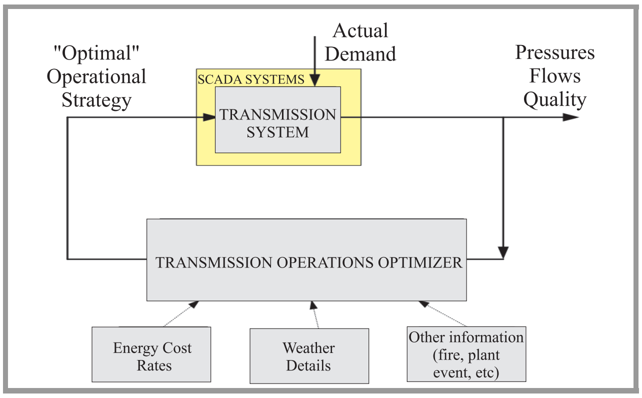


Fig. 1. TOO functionality.

This paper presents one of the key components of TOO, namely, the Simplified Model (SM), based on the solution of a linear programming (LP) problem with complex objective function expressing the cost of electrical energy consumption at pumping stations.

This component of TOO uses water distribution model based on mass-balance equations for pressure district volumes consisting of aggregated volumes of reservoirs and elevated tanks. For the 7-day control horizon with one-hour discretization and aggregation of pump's flows at pumping stations, the resulting LP problem is solved. After that, the optimal aggregated flows at pumping stations are disaggregated by a scheduler into individual pump flows and their start/stop times.

The solution obtained from SM is supposed to deliver terminal conditions for the precise, based on hydraulic de-

pendencies, Full Model (FM) at 24th or 48th hour of the control horizon, and the reference control trajectories for those obtained from the FM.
In the paper, a part of the overall system was studied. This included all the facilities within the City of Toronto which account about 97% of the energy used. The Region of York facilities (3% energy use) were not included. One-hour and 15-minute intervals were used as the discretization step. The computations were started with reservoir levels at 95%, and at the end of the 7-day period they were taken back to 95% of the total capacity (or higher).

2. Optimization Problem Formulations for the SM

2.1. Optimization Problem 1

A simplified discrete-time system model (mass-balance) for the entire system can be written as:

$$V_i(k+1) = V_i(k) + \sum_{j=1}^{N_{\mathrm{PS}}} b_{ij} \frac{T}{24} u_j(k) - d_i(k),$$
$$i = 1,2,\ldots,N_D, \quad k = 0,1,\ldots,N-1, \tag{1}$$

where:

- N_D – number of pressure districts,
- N_{PS} – number of pumping stations,
- $V_i(k)$ – volume stored in district i at time k (in ML),
- $b_{ij} = c_{ij} Q_j$ – matrix (units are ML/D, Firm Capacity matrix),
- c_{ij} – 1 if pumping station j is pumping into district i,
  -1 if pumping station j is pumping out of district i,
  0 otherwise,
- $Q_j = \sum_{m=1}^{N_{jp}} Q_{jm}$ – total station capacity (in ML/D),
- Q_{jm} – pump capacity for the m-th pump at the j-th pumping station,
- $u_j(k)$ – accumulated "control vector" at time k, $u_j(k) \in [0,1]$ (a continuous variable),
- $d_i(k)$ – demand/consumption from the i-th pressure district in period between k and $k+1$ time instant (in ML),
- T – time interval (typically one-hour),
- N – number of time intervals (= 168 for 7-days and $T = 1$ hour).

We must take into account time varying minimum and maximum reservoir levels:

$$V_{i,\min}(k) \le V_i(k) \le V_{i,\max}(k), \tag{2}$$

where $V_{i,\min}(k)$ and $V_{i,\max}(k)$ are the minimum and maximum storage volumes specified (typically these will be constants with respect to time k).

The total cost is the sum of pumping stations energy cost and water production cost:

$$J_{\mathrm{TOTAL}} = J_{\mathrm{STATIONS}} + J_{\mathrm{PLANTS}}, \tag{3}$$

where J_{STATIONS} is total cost for all stations:

$$J_{\mathrm{STATIONS}} = \sum_{j=1}^{N_{\mathrm{PS}}} J_j, \tag{4}$$

and total cost for a week (7 days) for station j is:

$$\begin{aligned} J_j = \sum_{k=0}^{N-1} & \mathrm{CC}_j(k) + (\mathrm{DCR}_j - \mathrm{TAR}_j)\,\mathrm{MaxKVA}_j \\ & + \mathrm{TCNR}_j\,\mathrm{PeakKW}_j + \mathrm{TCCR}_j\,\mathrm{MaxKW}_j \\ & + \mathrm{DRCR}_j\,\mathrm{PKWHtotal}_j \\ & + \mathrm{WOCR}_j\,\mathrm{LFactor}\,\mathrm{PKWHtotal}_j, \end{aligned} \tag{5}$$

where:

- CC_j – Commodity Charge, per kWh; flat or increasing block tariffs charge,
- DCR_j – Distribution Charge, per maximum KVA through the week,
- TAR_j – Transmission Allowance, per maximum KVA through the week,
- TCNR_j – Transmission Charge – Network, per maximum kW from 7:00 a.m. to 7:00 p.m. weekdays (referred to as "peak kW"), through the week,
- TCCR_j – Transmission Charge – Connection, per maximum kW from 7:00 p.m. to 7:00 a.m., through the week,
- DRCR_j – Debt Retirement Charge, per kWh in the week,
- WOCR_j – Wholesale Operation Charge, per kWh in the week; cost is multiplied by a loss factor (eg., 1.0376),

and

$$\mathrm{PKWHtotal}_j = \sum_{k=0}^{N-1} \mathrm{PKWH}_j(k) \tag{6}$$

$$\mathrm{PKWH}_j(k) = \mathrm{PKW}_j(k)\,T \tag{7}$$

$$\mathrm{PKW}_j(k) = P_j u_j(k) \tag{8}$$

$$P_j = \sum_{m=1}^{N_{jp}} \mathrm{PRATING}_{jm}, \tag{9}$$

where $\mathrm{PKW}_j(k)$ is used power (kW) at station j at time k, N_{jp} is the number of pumps at the j-th pumping station and $\mathrm{PRATING}_{jm}$ is the pump power rating for the m-th pump at the j-th pumping station.
Maximum KVA through the week is:

$$\mathrm{MaxKVA}_j = \max\left\{\mathrm{PKVA}_j(k)\right\}_{k=0}^{N-1} \tag{10}$$

$$\mathrm{PKVA}_j(k) = \frac{\mathrm{PKW}_j(k)}{\mathrm{PF}_j}, \tag{11}$$

where PF_j is the power factor for the j-th pumping station (eg., 0.92).

Peak KW through the week is:

$$\text{PeakKW}_j = \max\left\{\text{PKW}_j(k),\; k=7\text{ a.m. to 7 p.m. weekdays}\right\}_{k=0}^{N-1} \quad (12)$$

Maximum KW through the week is:

$$\text{MaxKW}_j = \max\left\{\text{PKW}_j(k),\; k=7\text{ p.m. to 7 a.m. weekdays}\right\}_{k=0}^{N-1} \quad (13)$$

The cost function (5) depends on the maximum values over the time period of optimization:

$$J_{\text{MAX},j} = (\text{DCR}_j - \text{TAR}_j)\,\text{MaxKVA}_j + \text{TCNR}_j\,\text{PeakKW}_j + \text{TCCR}_j\,\text{MaxKW}_j \quad (14)$$

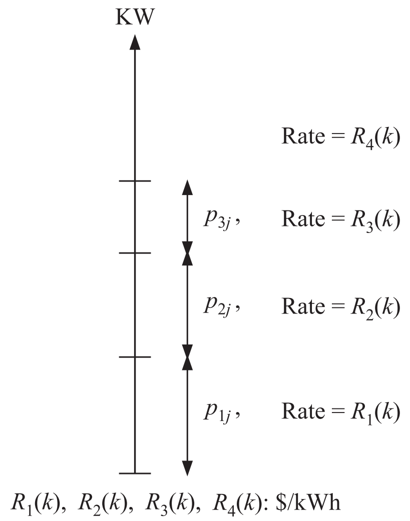
The above component can be converted into a conventional linear programming form by introducing auxiliary variables z_{1j}, z_{2j} and z_{3j} to represent peak factors. We express the transformed model as

$$\bar{J}_{\text{MAX},j} = (\text{DCR}_j - \text{TAR}_j)\,z_{1j} + \text{TCNR}_j\,z_{2j} + \text{TCCR}_j\,z_{3j} \quad (15)$$

subject to constraints:

$$\begin{aligned} \text{PKVA}_j(k) &\le z_{1j}, \quad k=0,\dots,N-1 \\ \text{PKW}_j(k) &\le z_{2j}, \quad k=7\text{ a.m. to 7 p.m. weekdays and } k=0,\dots,N-1 \\ \text{PKW}_j(k) &\le z_{3j}, \quad k=7\text{ p.m. to 7 a.m. weekdays and } k=0,\dots,N-1 \end{aligned} \quad (16)$$

The commodity charge (CC_j) is variable, dependent on the time of a day and the rate structure. It is assumed that there are a maximum of four blocks for the cost rates, as depicted below:



If $\text{PKW}_j(k) < p_{1j}$

$$\text{CC}_j(k) = R_1(k)\,\text{PKWH}_j(k) \quad (17)$$

Else if $\text{PKW}_j(k) < (p_{1j}+p_{2j})$

$$\text{CC}_j(k) = (R_1(k)-R_2(k))p_{1j}T + R_2(k)\,\text{PKWH}_j(k) \quad (18)$$

Else if $\text{PKW}_j(k) < (p_{1j}+p_{2j}+p_{3j})$

$$\text{CC}_j(k) = (R_1(k)-R_3(k))p_{1j}T + (R_2(k)-R_3(k))p_{2j}T + R_3(k)\,\text{PKWH}_j(k) \quad (19)$$

Else

$$\text{CC}_j(k) = (R_1(k)-R_4(k))p_{1j}T + (R_2(k) - R_4(k))p_{2j}T + (R_3(k)-R_4(k))p_{3j}T + R_4(k)\,\text{PKWH}_j(k) \quad (20)$$

The $\text{CC}_j(k)$ can be modeled as a piecewise-linear function, and because costs are non-decreasing, i.e., $R_1(k) \le R_2(k) \le R_3(k) \le R_4(k)$, it is also convex, so as a result of total modeling, we obtain large-scale linear programming model (without binary variables).
J_{PLANTS} is the cost of producing water at the four water treatment plants:

$$\begin{aligned} J_{\text{PLANTS}} = {} & \lambda_{\text{RLC}} \cdot \text{VOLtotal}_{\text{R.L.CLARK}} \\ & + \lambda_{\text{RCH}} \cdot \text{VOLtotal}_{\text{R.C.HARRIS}} \\ & + \lambda_{\text{FJH}} \cdot \text{VOLtotal}_{\text{F.J.HORGAN}} \\ & + \lambda_{\text{ISLAND}} \cdot \text{VOLtotal}_{\text{ISLAND}}, \end{aligned} \quad (21)$$

where λ_s are the production costs in \$/ML for the respective plants. The total volume from the plants is formulated in the following way, e.g.:

$$\text{VOLtotal}_{\text{R.L.CLARK}} = \sum_{k=0}^{N-1} \text{VOL}_{\text{R.L.CLARK}}(k), \quad (22)$$

and

$$\text{VOL}_{\text{R.L.CLARK}}(k) = \text{Flow}_{\text{R.L.CLARK}}(k) \cdot T \quad (23)$$

Each of the pumping station flows can be expressed at time k as:

$$\text{Flow}_j = Q_j \frac{T}{24} u_j(k). \quad (24)$$

Optimization goal is to find $u_j(k), k=0,1,\dots,N-1$ to minimize J_{TOTAL} Eq. (3) subject to mass-balance equations (1), bounds (2) and $0 \le u_j(k) \le 1$.

2.2. Optimization Problem 2

This optimization problem is similar to Optimization Problem 1 with the only difference that we have individual control variables for each pump $u_{jm}(k)$, $0 \le u_{jm}(k) \le 1$, for $j=1,\dots,N_{\text{PS}}$, $m=1,\dots,N_{jp}$, and $k=0,1,\dots,N-1$.

3. Disaggregation in Optimization Problem 1

Our disaggregation problem which is solved at every stage (every hour or every quarter of an hour; for simplicity we will omit the time index k) and for every pumping station can be described as follows:

$$\min_{u^j} \sum_{m=1}^{N_{jp}} PRATING_{jm} u_{jm} \tag{25}$$

$$\sum_{m=1}^{N_{jp}} \eta_{jm} Q_{jm} u_{jm} = \hat{Q}_j, \tag{26}$$

$$u_{jm} \in [0,1], \forall m \tag{27}$$

where:

u_{jm} – individual pumping as a continuous variable (% of the interval T when the pump is ON) for the m-th pump at the j-th PS,

$u^j = (u_{j1}, u_{j2}, \ldots, u_{j\overline{\overline{PS_j}}})$ – vector of all pumpings at the j-th PS,

η_{jm} – the efficiency of the m-th pump at the j-th PS,

$\hat{Q}_j = Q_j \frac{T}{24} \hat{u}_j$ – the desired flow of the j-th PS; it results from the solution of the Optimization Problem 1.

The cost of the energy in Eq. (25) is proportional to the power used. Because price is the same for all pumps, it is proportional to the sum of the power used by all pumps.

The number of these LP problems is not bigger than NN_{PS}. Probably, there would be much less of them, because we omit these PS-es for which $\hat{Q}_j$ equal zero (then automatically all $\hat{u}_{jm}$ equal zero too).

We will disaggregate control in such a way that we will get at every stage (of the length T) the minimal power used.

Let us replace now the components $\eta Q u$ with the new variables y:

$$y_{jm} = \eta_{jm} Q_{jm} \ u_{jm} \tag{28}$$

Hence:

$$u_{jm} = y_{jm}/(Q_{jm} \eta_{jm}) \tag{29}$$

Let us denote:

$$\alpha_{jm} = PRATING_{jm}/(Q_{jm} \eta_{jm}) \tag{30}$$

In the new variables we will have the problem:

$$\min_{y} \sum_{m=1}^{N_{jp}} \alpha_{jm} y_{jm} \tag{31}$$

$$\sum_{m=1}^{N_{jp}} y_{jm} = \hat{Q}_j, \tag{32}$$

$$y_{jm} \in [0, \eta_{jm} Q_{jm}], \forall j \tag{33}$$

This is nothing, but an auction problem.

We can get the optimal solution by sorting (before the optimization) for every PS the elements α_{jm} from the smallest to the largest and allocate the maximum, that is $y_{jm} = \eta_{jm} Q_{jm}$ (=the pump is ON over the whole stage) until their sum reaches $\hat{Q}_j$. The last element before reaching $\hat{Q}_j$ will be usually smaller then $\eta_{jm} Q_{jm}$ (this pump will be ON over a fraction of T), the remaining pumps (with the larger coefficients α_{jm}) will be OFF during the given stage.

4. Numerical Results

The full 7-day model with discrete variables for pump switches was intractable in reasonable time period (the obtained computation times for 2-day subproblems were much longer than 5-minute time limit assumed for TOO), for two popular commercial mixed-linear optimizers: CPLEX, Xpress-MP and one mixed-nonlinear optimizer: MINLPBB, so we decided to use continuous control variables $u_{jm}(k)$ from interval $[0,1]$ for each pump individually.

We solved Optimization Problems 1 and 2 (linear) for flat and increasing energy tariffs, for full 7-day optimization horizon with a one-hour and 15-minute intervals. The cost of optimized operations, problem statistics, and solution times are summarized in Table 1. For the optimization we used commercial Xpress-MP solver (version 2008A, on evaluation license, 64-bit Linux binary) with options `barrier, barthreads=4, mipthreads=4` (multithreaded mode).

It is seen from the Table 1 that, quite surprisingly, the value of costs in both flat and increasing block energy tariffs case does not depend on the time discretization. Hence, there is no motivation to use time step equal 15 minutes instead of 1 hour. The cost of suboptimal, aggregated solution is not more than 2% higher than that of the optimal one, so the approach presented as Optimization Problem 1 is rather acceptable.

5. Conclusions and Future Work

We have implemented a simplified mass-balance based model for COT. The resulting continuous LP problem is solved very fast by both commercial and free LP solvers. The results obtained for the aggregation of pumping variables case are satisfactory. However, there are some doubts about applicability of the SM, because it neglects hydraulic phenomena in the network, such as flows and head-losses in pipes and valves, dynamics of individual reservoirs and elevated tanks, pumpage and efficiency curves of pumps, continuity laws for junctions, etc. Owing to this, the next step will be the full hydraulic model of the system. The current solvers allow for solving such problems on only shorter horizon – 24 or 48 hour long. The presented

Table 1
Optimization results with Xpress-MP 2008A solver for problems with one-hour (1h) or 15-minute (15m) intervals, and flat (F) or increasing (I) block energy tariffs (n – number of linear variables, m – number of linear constraints, $J_{\mathrm{STATIONS}} = J_{\mathrm{CC}} + J_{\mathrm{OTHER}} + J_{\mathrm{MAX}}$, T – optimization time)

Problem	n	m	J_{TOTAL} [\$]	J_{CC} [\$]	J_{OTHER} [\$]	J_{MAX} [\$]	J_{PLANTS} [\$]	T [sec]
OP1-F-1h	8651	9528	528590	219980	61070	244569	2970	1
OP2-F-1h	23603	9528	521747	214514	60151	244111	2970	1
OP1-I-1h	12179	10872	539119	226849	61960	247340	2970	1
OP2-I-1h	27131	10872	529622	222655	60866	243129	2970	2
OP1-F-15m	34307	38001	528590	219980	61070	244569	2970	9
OP2-F-15m	94115	38001	521747	214514	60151	244111	2970	19
OP1-I-15m	48356	43353	539119	226849	61960	247340	2970	8
OP2-I-15m	108164	43353	529622	222655	60866	243129	2970	10

SM model will be used to deliver for this future FM terminal conditions, as well as the reference control and state trajectories.

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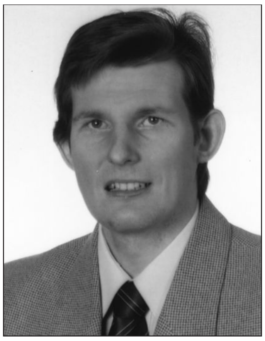

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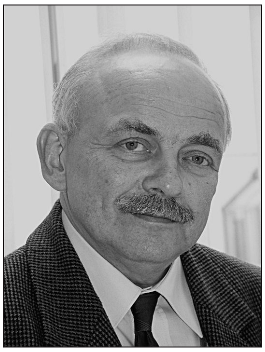

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