Consistency of injective tree patterns

Claire David   Nadime Francis   Filip Murlak

U. Paris-Est Marne   ENS Cachan   U. Warsaw

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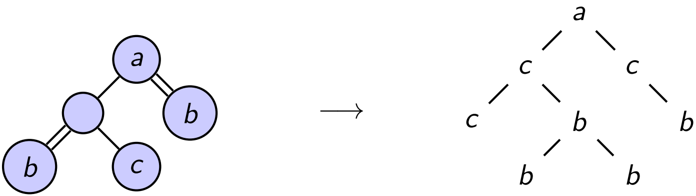
Satisfiability of tree patterns: P or NP?



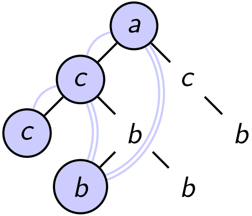
Problem

Can given pattern π be matched in some tree from regular language L?

Semantics 1: homomorphisms

Pattern π is matched in tree T if there is a homomorphism $h\colon \pi \to T$.

(Preserves labels, child and descendant relations.)



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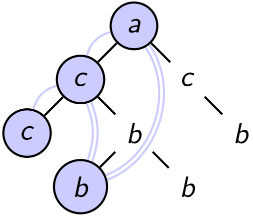

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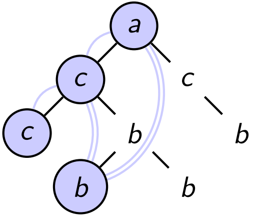

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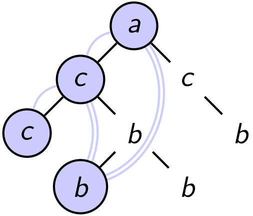

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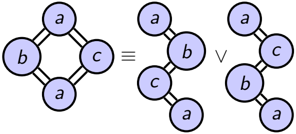

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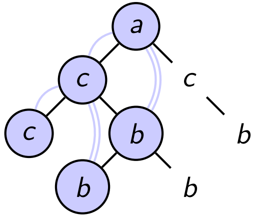

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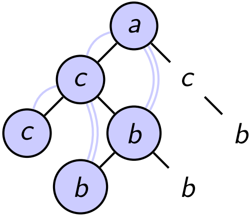

- incomplete information about XML docs in DOM model
  [Barceló, Libkin, Poggi, Sirangelo '09]
  - nodes have unique IDs
  - labels and relations may be lost
  - when ID is lost, node is lost

  pattern π: incomplete XML doc;
  regular language L: correct docs (schema);
  π is satisfiable iff incomplete doc extends to a correct doc

Regular languages

Sets of finite labelled unranked trees

- definable in monadic second order logic;
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Example: *a*-only path from root to leaf, *b*'s elsewhere

- type τ: root label *a*, immediate subtree types $\sigma^*\tau\sigma^* + \epsilon$;
- type σ: root label *b*, immediate subtree types σ^*;
- choose: τ

Satisfiability of tree patterns: complexity landscape

Semantics	Consistency (fixed L)	Satisfiability (given L)
Homomorphism		
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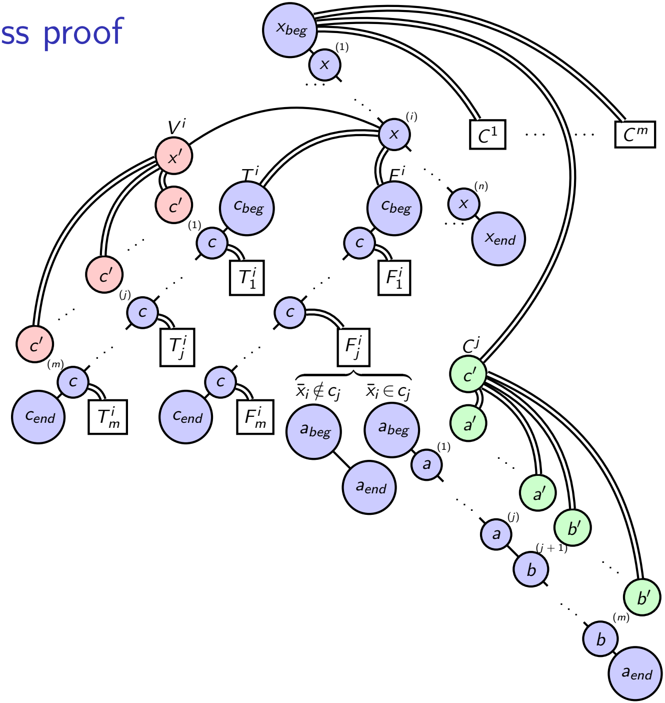
- ≤ 2 descendants on each branch: NP-complete;
- PTIME (FPT) with descendant only (without child)
  - works for injective and homomorphism semantics,
  - extends to patterns with sibling order (following-sibling).

On the hardness proof



Challenges

1. Our FPT algorithm has complexity $2^{2^{\|L\|}} \cdot \mathrm{poly}(\|\pi\|)$; the NP upper bound gives complexity $2^{\mathrm{poly}(\|L\|,\|\pi\|)}$. Inherent trade-off, or can this be reconciled?

2. Assuming we stick to descendant and following sibling, is consistency still tractable for patterns that are DAGs, not trees?