Joint Liability Lending with Correlated Risks

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Abstract

Group based lending with joint liability, has been a major tool microfinance institutions have employed to improve repayment rates. While this appears to be a very potent tool, there are concerns as to how it could continue to help in risk pricing especially when lenders cannot distinguish between safe agents and risky ones. A notable threat to the reliability on joint liability lending, is correlated risks.

When the returns to projects are correlated, and matching is homogeneous, lenders may not be able to use joint liability to price for risk to improve the efficiency of credit markets. In this paper, we derive an optimal lending contract for joint liability lending when risks are correlated under the context of adverse selection. We find that, correlation is bad for group based joint liability lending.

Keywords: Microfinance, Group lending, adverse selection, joint liability, correlated risks.
1 Introduction

The advent of micro-lending undoubtedly, has brought relief to many small businesses especially in developing countries, where collateral security cannot be provided by prospective borrowers because of poverty rates. It is estimated that as at 31st December 2010, more than 205 million people have been reached with loans by microfinance institutions (MFIs) (Maes and Reed 2012). Despite backlashes received by MFIs for benefiting off the poor and despite conclusions in Banerjee et al (2015) that microfinance is probably not as miraculous as we would be made to believe, the facility remains prevalent even in recent times in developing countries.

Aside being a source of funding for existing and new businesses, microfinance availability, theoretically, has been shown to aid some economies escape poverty trap in an occupational choice framework (See Banerjee and Newman, 1993). Reports from the mix market indicates that about 3652 MFI are registered, and are involved in the financial intermediation and financial services subsector of the economies of developing countries. With free entry and free exit guaranteed in the micro-lending market, successes chalked by existing MFIs has attracted potential firms both for profit and the original\(^1\) not-for-profit firms that focus mainly on outreach and maximization of borrower welfare. Particularly impressive about the performance of MFIs is the repayment rates that have gone up considerably given that, the loans are advanced to poor people perceived to have good projects but have neither capital nor collateral to secure loans from traditional banks.

Beginning as a contract between the borrower and the lender where the individual borrower was solely responsible for his debt (individual liability), other strategies such as dynamic incentive, where borrowers are promised lower interest rate (such as done by a large MFI in South Africa discussed in Karlan and Zinman 2009) or higher loan amounts in the future are used. Group lending (Joint Liability) methods have also been used and remain common in the micro-lending markets (de Quidt et al 2012). However, for-profit institutions are found to use more of individual lending while not-for-profit institutions use group-based lending methods (Cull et al 2009; de Quidt, 2012). Joint liability lending involves borrowers forming groups to access loans. The requirements are that, the liability is jointly held so that, successful group members are liable for part or the entire debt of an unsuccessful member of the group. The success of joint liability lending rests at least theoretically, (Ghatak, 1999, 2000) on the facts that borrowers are allowed to choose their own partners or (group members) and secondly, the loan is given on joint liability terms. The idea is that, in a village setting where almost everyone is known by his

\(^1\) Originally, MFIs were mainly not-for profit organizations.
or her neighbor, borrowers have local information that is not available to the lender and thus are able to partner with other borrowers with safe projects.

Despite the exploits of joint liability lending, there are some drawbacks to it being an efficient tool. Some of these issues involve side-contracting, correlated risks and group formation. The issue of how large the liability must be has already been addressed by Gangopadhyay et al (2005) who argue that, to avoid a situation where group members may lie about an unsuccessful group member, the liability should optimally be less or equal to the individual liability of the borrower, essentially, they argue that, the liability should increase with successes so that if a group member failed, the group would have no incentive to claim the failed member succeeded so as to pay less. Holmstrom and Milgram (1990), argue that side contracts can as it were, give room for arbitrage which may affect the lending contracts in group-based lending with joint liability.

Although operations of MFIs are seen to be with small non-agricultural businesses, loans advanced to agricultural workers are also granted on joint liability basis, by rural banks and agricultural development bank This raises concerns about the ability of joint liability lending to help in pricing for risk or helping in repayment rates if project returns of borrowers are correlated or there exist correlation in output such that the probabilities of success are correlated. That is the bit this paper explores. As Besley (1994) rightly puts it “A special feature of agriculture which provides the income of most rural residents is the risk of income shocks. These include weather fluctuations that affect all the producers of a particular commodity. Such shocks affect the operation of credit markets if they create the potential for group of farmers to default at the same time”. To be able to price for risk and improve on repayments, lenders must learn from the repayment behavior of borrowers. But as conveyed in Ahlin and Waters (2014), and Ghatak (2000), consider a case of perfect serial correlation, then dynamic incentive strategy is not useful as a tool since only one draw for example, is obtained from a borrower’s distribution even in a two-period lending cycle. This serial correlation appears to be more relevant in small businesses settings. On the other hand, consider also the case of perfect spatial correlation, then group lending could be disastrous if group members fail-- since groups are shown to be formed on homogeneous matching of risk (Ghatak 1999, 2000) and also based on the type of risk exposed to (Ahlin, 2009 Working paper).

The focus of this paper is on spatial correlation. This paper aims to derive an optimal lending contract in an adverse selection model with correlated risks. This problem is very important empirically and theoretically. Empirically, we can easily see how correlated risk is a great threat to the functioning or financial intermediation efforts of MFIs. It could demotivate lenders from handing out loans leading to inefficiencies in the credit market. Theoretically, it would be interesting to see how the results differ from
the independent risks case. We would then be able to see to what extent is correlation bad for group lending methods. Ahlin (2009-BREAD working paper), is the closest to this paper in terms of delving into correlated risks. It puts a simple structure on the correlation to understand how groups are formed, and adds to the already showed homogeneous group formation. Findings in Ahlin (2009 working paper), that borrowers would anti-diversify risk in group formation in order to lower the occurrence of having to carry the debt burden of a group member, lays credence to how important it is to understand further, how the theoretical optimal lending contract and parameter space for efficiency are altered with the introduction of correlated risks. We compare results from this to the individual lending and group lending under independent risks.

The rest of the paper is organized as follows; the next section discusses some related literature, the third section outlines the basic models. Section four discusses how we introduce correlation in the project returns using the probabilities of success and optimal group lending contracts with correlated risk is derived in this section. Section five concludes and discusses the next steps.

2 Literature Review

In the absence of collateral security from borrowers, micro-credit lenders have resorted to strategies such as joint liability lending. They seek to price for risk and induce quick repayment of loans on the part of borrowers. Theoretically, group lending has been shown to improve efficiency when compared to the traditional individual liabilities loans (Ghatak 1999, 2000; Van Tassel1999) both papers were under the context of adverse selection.

A plethora of the literature on group lending under adverse selection has focused on how group lending can perform better than the individual loan contracts. The works of Ghatak (1999,2000), Van Tassel(1999), Gangopadhyay et al (2005) have all been in that direction except that, Gandopadhay et al (2005) added to Ghatak (1999) by raising the concerns that separating equilibrium could be unattainable if borrowers found it more profitable to lie that a failed group member succeeded. Bhole and Ogden
as well as de Quidt et al (2013) compares dynamic individual lending to its counterpart (dynamic group lending) but in a strategic default setting. They find welfare of borrowers to be higher with group lending than individual lending under certain conditions. One such condition is when penalty is allowed to be different among members of a group. Although under a moral hazard context, Chowdhury (2007) shows that if loans are not advanced to group members sequentially, dynamic group lending is not any better than dynamic individual lending.

The very early works done by Varian (1990) and Stiglitz (1990) have investigated the potency of joint liability in harnessing local information to induce high repayment rates. In Varian (1990) however, a model is proposed which takes the form that, the bank does its own screening and does not rely on local information among borrowers. In that model, banks interviewed group members. The eligibility or otherwise of a group member, sufficed to know whether the entire group would have a loan advanced to them or not. Besley and Coate (1995) have also looked at how joint liability affects the willingness to pay on the part of borrowers. Even in the situation where borrowers have imperfect knowledge about the project types of their group members, Armendarize de Aghion and Gollier (1998) show that, joint liability can lead to lower interest rate and help overcome some credit market inefficiencies. This is also true with the work of Laffont and N’Guessan (1999).

Much earlier, even before the seminal contributions of Ghatak (1999, 2000), Besley (1994) identifies three major things that make rural credit markets in developing countries different from other developed countries. These he highlights as collateral security unavailability, covariant risk and under developed states of related institutions. Not many papers to the best of our knowledge have looked at the optimal lending contract in Joint-liability contracts when there are correlated risks under contexts such as adverse selection, limited liability and the constraint on the liability imposed on Gangopadhyay et al (2005). Ghatak (2000), and Ahlin and Waters (2014) talk about the repercussions of correlated risks on group lending and dynamic lending but do not delve into it.

Again, Ahlin and Townsend (2007) also touch on correlated risks. In testing joint liability models’ repayment implications using data from Thai borrowing groups, find that “a higher correlation of output and borrower’s ability to act cooperatively can raise or lower repayment, depending on the model.” The papers which discuss correlated risks in some detail as this work seeks to do are Ahlin (2009, BREAD working paper), and Ahlin and Townsend (2007).2 The paper finds evidence of homogenous sorting by risk and risk anti-diversifying strategy among group members albeit not on the lines of

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2 Journal of Econometrics.
occupation. This corroborates the claims that group lending helps alleviates inefficiencies in the credit market.

However, worrying is that the anti-diversification attitude of borrowers work contrary to the expectation that borrowers form groups as a way of sharing risks- again, raising the concerns and need for us to study how the optimal lending contract is affected with correlated risks. If a group member would anti-diversify risk, the strength of joint liability lending can be restrained because it does not accrue to the benefit of the lender. See Boot (2000) for a detailed empirical and theoretical review of the literature in relationship lending.

3 Baseline Model

3.0 Economic environment

We assume there is a continuum of agents that are risk neutral and has measure one. Each agent is endowed with a unit of labor which she supplies inelastically. Agents are endowed with a project that requires one unit of capital and one unit of labor. However, agents have no endowment of capital. Agents’ projects are known to differ in risk type \( p \in [p, 1) \) although there is an outside option that yields exogenously given net return of \( \bar{u} \geq 0 \). Agents would need to borrow a unit of capital to start their projects since they have no initial wealth.

The project of a type \( p \) agent pays \( R_p \) with probability \( p \) and pays 0 otherwise. We assume the riskiness of the project is private information known to the agent but not the lender. As in Stiglitz and Weiss (1981), we assume that all projects have the same expected return \( \bar{R} = R_p \cdot p \), \( \forall p \in [p, 1) \). This means that safer projects pay less when agent is successful. We also assume limited liability, which means agents who are unsuccessful have no liability. Output can be verified as either successful or failed but lender cannot verify various shades of success. The assumption of limited liability and costly verification of output makes debt contracts the only feasible contracts. Borrowers who are able to repay their debts do and so, there are no enforcement problems. There is a single lender who is risk neutral and would be willing to lend provided it earns an expected return of \( \rho \) where \( \rho \), is the opportunity cost of
capital per loan. The lender or bank knows the distribution of borrowers but not their probabilities of success.

We assume $\bar{R} > \rho + \bar{u}$, which makes all projects have higher expected return than costs of capital and labor invested. With social surplus strictly increasing in the number of projects funded, fully efficient market would lend to all agents.

Suppose we have two types of agents $p \in \{p_r, p_s\}$ where $0 < p_r < p_s < 1$ and $R_s < R_r$. Let $0 < \theta < 1$ be the population of risky borrowers. Let the population average of any function $g(p)$ be denoted by $\bar{g}(p) = \theta g(p_r) + (1 - \theta) g(p_s)$. Similarly, $\bar{p}$ denotes the mean risk-type and $\bar{p}^2$ the mean squared-type.

### 3.1 Static Individual Lending

Under full information where agents types are known lender can price for risk by charging

$$p_r r_r = \rho$$

So that $r_r = \frac{\rho}{p_r}$ for each risk type $\tau$. $r_r(r)$ is the loan amount plus interest. This is the first best outcome which can be seen to be efficient and equitable with all surpluses accruing to the borrowers.

Now, for the case where agent’s risk types are unknown to the lender,

Let $\bar{p} = \theta p_r + (1 - \theta) p_s$ be the average success probability and $r$ the repayment amount. An agent of type $\tau \in \{r, s\}$, will borrow to undertake the project if and only if,

$$\bar{R} - p_r r \geq \bar{u} \iff r \leq \hat{r}_\tau \equiv \frac{\bar{R} - \bar{u}}{p_\tau}.$$  

The first inequality says that the expected returns from the project less the expected repayment amount should exceed the outside option’s return. The second inequality follows from a rearrangement of the first and gives us a reservation interest rate $\hat{r}_\tau$, above which an agent of type $\tau$ will choose the outside option instead. If safe borrowers borrow, then so will the risky ones since safe borrowers succeed often and repay with a higher probability. Thus $r \leq \hat{r}_s = \frac{\bar{R} - \bar{u}}{p_s}$ is necessary for fully efficient lending. It is however necessary and sufficient for $r \leq \bar{R}_s$ if both types are to find it affordable to borrow.
Now the lender would be willing to give out the loan if \( \bar{p}r = \rho \) thus the break-even interest rate is \( r = \frac{\rho}{\bar{p}} \).

To attract all borrowers, we need \( \frac{\rho}{\bar{p}} \leq \frac{\bar{R} - \bar{u}}{\bar{p}_s} \) ie \( N \geq \frac{\bar{p}_s}{\rho} \) where \( N = \frac{\bar{R} - \bar{u}}{\rho} \) and for affordability as before, \( r \leq R_s \) or \( \frac{\rho}{\bar{p}} \leq \frac{\bar{R}}{\bar{p}_s} \) ie \( \frac{\bar{p}_s}{\rho} \leq \frac{\bar{R}}{\rho} \).

We interpret \( N = \frac{\bar{R} - \bar{u}}{\rho} \) to mean, efficient lending is achieved if the net excess return to capital in this market is larger than the extent of asymmetric information, represented by \( \frac{\bar{p}_s}{\rho} \). There is however a second best option which involves lending to only risky agents. In which case there is inefficiency as the lender is unable to price for risk.

### 3.2 Group Lending with independent risks

In this section, we review very quickly the model of joint liability lending presented in Ghatak (2000) and results from Gangopadhyay et al. (2005). Relying on the environment discussed earlier, a joint liability contract requires a borrower to pay a joint liability say \( c \) in addition to the repayment amount \( r \) on her own loan if the group member fails while she succeeds. Here, agents are assumed to know each other’s types but this is unknown to the lender. Ghatak, (2000) shows that borrowers would form groups homogenously under joint liability contracts. For a borrower of type \( \tau \in \{r, s\} \), the expected payoff under homogenous matching is given as;

\[
\bar{R} - p_r r - p_s (1 - p_r)c = \bar{R} - p_r [r + (1 - p_r)c] \ 
\]

The effective interest rate \( r + (1 - p_r)c \), is seen to vary positively with risk type of the borrower and penalizes risky borrowers as full information similarly does, although the lender has no information on risk types of borrowers. Since contract may attract only risky borrowers, for group lending to achieve full efficiency, we consider maximizing safe borrower’s payoff subject to the constraints that lender breaks even and monotonicity constraint. Again, assuming homogenous matching obtains.

\[\text{we assume a group is of size two throughout this work.}\]

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ie, maximize, $\bar{R} - p_3 r - p_3 (1 - p_3) c$ \hspace{1cm} \text{subject to} \hspace{1cm} c \leq r, \text{ and } \bar{p} r + p (1 - p) c \geq \rho

Solving, we have the optimal contract as

$c = r = \frac{\rho}{p(2 - p)}$ \hspace{1cm} \text{Assuming affordability is not an issue. That is } R_s \geq r + c

on the other hand, when $r = c$ is not affordable, we have the optimal contract derived in Ahlin and Waters (2014). The optimal pooling contract is given as

$$(r, c) = \left\{ \frac{p_3 - p (1 - p) G}{p_3 p^2}, p \frac{G - p_3}{p_3 p^2} \right\}$$

Where $G = \frac{\bar{R}}{\rho}$. See paper for details and the restrictions on parameters.

4 \hspace{1cm} \textbf{Group Lending with Correlated Risk}

We introduce correlation of risk type into the group lending model in a manner that preserves the individual probabilities of success. A similar modification was done in (Ahlin and Townsend, 2007) in the spirit of Holstrom and Milgrom (1990). Suppose there are two borrowers (1 and 2) with probabilities of success $p_i$ and $p_j$ respectively. We can have the joint distribution in the table below

<table>
<thead>
<tr>
<th></th>
<th>2 Succeeds ($p_{ij}$)</th>
<th>2 Fails (1-$p_{ij}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 succeeds ($p_i$)</td>
<td>$p_i p_j + \varepsilon$</td>
<td>$p_i (1 - p_j) - \varepsilon$</td>
</tr>
<tr>
<td>1 Fails (1-$p_i$)</td>
<td>$(1 - p_i) p_j - \varepsilon$</td>
<td>$(1-p_i)(1 - p_j) + \varepsilon$</td>
</tr>
</tbody>
</table>

Clearly, the columns and rows add up to equal the individual probabilities of success. And of course $\varepsilon = 0$ corresponds to the no correlation case. We focus on positive correlation only since that is what pertains in most settings and appears to be what pertains if groups are formed homogeneously. We maintain the assumption that borrower types are unknown to the lender. In addition, we assume here that
the lender knows that project returns are correlated and knows the joint distribution of the probabilities of success. A positive $\epsilon$ adds to the probabilities that both either succeed together or fail together and subtracts from the probabilities that one succeeds and the partner fails.

$\epsilon$ generally, is expected to depend on the probabilities of success. However, we consider two cases in this paper. In the first case, we assume that it is constant $\epsilon(p_i, p_j) = \epsilon > 0$ for all $p_i$ and $p_j$ and in the second case, we assume $\epsilon(p_i, p_j) > 0$ is not constant for all $p_i$ and $p_j$.

Finally, given the requirement that elements in the cells in table 1, must be less than 1 and greater than zero, it follows that, $\epsilon \leq p_r(1 - p_s)$ is sufficient to ensure this.

4.1 Constant $\epsilon$ (case 1)

Suppose $\epsilon(p_i, p_j) = \epsilon > 0$ for all $p_i$ and $p_j$.

The results on the independent risk types hinges on the homogeneous matching results derived in Ghatak (1999). With the introduction of correlation in project returns, we show below that the homogenous matching result is unaffected under case 1. To show that homogeneous matching obtains, is efficient and is the equilibrium, we compare the sum of group payoff from homogenous matching to group payoff from heterogeneous matching. Here, consider two groups each of size two.

Consider the following:

The group payoff of a safe borrower for partnering with another safe borrower,

$$\bar{R} - p_s r - [p_s(1 - p_s) - \epsilon]c + \bar{R} - p_s r - [p_s(1 - p_s) - \epsilon]c$$  

(1)

The group payoff of a risky borrower pairing with another risky borrower,

$$\bar{R} - p_r r - [p_r(1 - p_r) - \epsilon]c + \bar{R} - p_r r - [p_r(1 - p_r) - \epsilon]c$$  

(2)

a non-homogenous group’s payoff (1st agent safe, second risky) is...
\[ \tilde{R} - p_s r - [p_s(1 - p_r) - \varepsilon]c + \tilde{R} - p_r r - [p_r(1 - p_s) - \varepsilon]c \]  
(1st agent risky and second agent safe)

\[ \tilde{R} - p_r r - [p_r(1 - p_s) - \varepsilon]c + \tilde{R} - p_s r - [p_s(1 - p_r) - \varepsilon]c \]  

(1) + (2) yields

\[
2\tilde{R} - 2p_s r - 2p_s(1 - p_s)c + 2\varepsilon c + 2\tilde{R} - 2p_r r - 2p_r(1 - p_r)c + 2\varepsilon c
\]

\[= 4\tilde{R} - 2p_s r - 2p_r r - 2p_s(1 - p_s)c - 2p_r(1 - p_r)c + 4\varepsilon c \]  
(5)

And (3) + (4) yields

\[4\tilde{R} - 2p_s r - 2p_r r - 2p_s(1 - p_r)c - 2p_r(1 - p_s)c + 4\varepsilon c \]  
(6)

After some algebra, it can be shown that (5) is greater than (6) since \((p_s - p_r)^2 \geq 0\)

This demonstrates that homogeneous matching obtains.

**Observation 1:**

Firstly, it can be seen, from the safe borrower’s payoff, that, the safe borrower in the presence of correlated risk pays the joint liability less often than under independent risk. Because they either both succeed or fail with a higher probability compared to independent risk.

**Observation 2:**

Secondly, the participation constraint of the bank or the zero profit condition is tighter for the lender compared to the independent risk case. Suggesting that, the lender would need to charge a higher interest as well as higher liability cost to be able to participate.

As mentioned already, given the assumptions in the model, a joint liability contract that attracts safe borrowers would also attract risky ones simply because the safe borrowers succeed more often and repay the joint liability with a higher probability. As such, to investigate the feasibility of fully efficient lending, we zero in on what can attract the safe borrower into accepting the contract. It must be noted that,
to attract safe borrowers who succeed with higher probability, the contract should put more burden on
states of the world with more failures—on risky borrowers. It is then easy to see that, if full liability is
affordable, the best contract should have $c = r$ and extract the maximum possible if $c = r$ is not
affordable.

More formally, we maximize a safe borrower’s payoff in a homogenous matching

$$\text{Max } \bar{Y} = \bar{R} - p_s r - [p_s (1 - p_s) - \varepsilon]c \quad \text{subject to}$$

1. $0 \leq c \leq r$, Monotonicity [m]
2. $\bar{p} r + p (1 - p) c - \varepsilon c \geq \rho$ Lender break even or zero profit constraint $[\mu]$

Where $m$ and $\mu$ are the Lagrange multipliers respectively

The first order conditions are

$[r]: -p_s + \mu \bar{p} + m = 0$

$[c]: -p_s (1 - p_s) + \varepsilon + \mu p (1 - p) - \varepsilon \mu - m = 0$

solve to get

$$\mu = \frac{p_s (2 - p_s) - \varepsilon}{p (2 - p) - \varepsilon} > 0 \quad \text{and} \quad m = \frac{\theta p_s (p_s - p) (p_s - \rho - \varepsilon)}{p (2 - p) - \varepsilon} > 0 \quad \text{if we assume } p_s p_r > \varepsilon \text{ in which case we have the solutions as} ; \quad r = c = \frac{\rho}{p (2 - p) - \varepsilon}$$

The remaining affordability constraint is satisfied if

$$R_s \geq r + c$$

$$R_s \geq 2r = 2 \frac{\rho}{p (2 - p) - \varepsilon}$$

$$R_s p_s \geq 2p_s \frac{\rho}{p (2 - p) - \varepsilon}$$

$$\frac{\bar{R}}{\rho} \geq \frac{2p_s}{p (2 - p) - \varepsilon}$$

$$\Rightarrow G \geq \frac{2p_s}{p (2 - p) - \varepsilon}$$
To be worth the investment, we need the expected payoff from the project to exceed the outside option. The condition is the inequality below.

\[ R - p_s r - [p_s (1 - p_s) - \varepsilon] r \geq \bar{u} \]

\[ \Rightarrow N \geq \frac{p_s (2 - p_s) - \varepsilon}{p (2 - p) - \varepsilon} \]

Now, when affordability is an issue, (when \( \frac{p_s}{p} \leq G < \frac{2 p_s}{p (2 - p) - \varepsilon} \))

The problem is

\[
\text{Max } \bar{Y} = R - p_s r - [p_s (1 - p_s) - \varepsilon] c \quad \text{subject to the following constraints}
\]

\[ R_s \geq r + c \quad [\lambda] \]

\[ \bar{p} r + p (1 - p) c - \varepsilon c \geq \rho \quad [\mu] \]

\[ [r]: -p_s - \lambda + \mu \bar{p} = 0 \]

\[ [c]: -[p_s (1 - p_s) - \varepsilon] - \lambda + \mu p (1 - p) - \varepsilon \mu = 0 \]

solve to obtain the Lagrange multipliers as

\[ \mu = \frac{p_s^2}{p (2 + \varepsilon)} > 0 \quad \text{and} \quad \lambda = \frac{\theta (p_s - p_r) (p_s p_r - \varepsilon)}{p_s (p^2 + \varepsilon)} > 0 \quad \text{as before, if we assume } p_s p_r > \varepsilon, \]

then the contract can be derived as

\[ (r, c) = \left\{ \rho \frac{p_s - [p (1 - p) - \varepsilon] G}{p_s (p^2 + \varepsilon)}, \rho \frac{\bar{p} G - p_s}{p_s (p^2 + \varepsilon)} \right\} \]

To be worth the investment, we can show that we need the following condition,

\[ N \geq \frac{p_s^2 + \varepsilon}{p_s (p^2 + \varepsilon)} - \left[ \frac{\theta (p_s - p_r) (p_s p_r - \varepsilon)}{p_s (p^2 + \varepsilon)} \right] G \]

\[ \text{we expect individual lending to be feasible at least even if affordability of a joint liability is an issue.} \]
The results we have derived so far in this section assumed that $p_s p_r > \varepsilon$. Now assume $p_s p_r < \varepsilon$

then the constraints in the full affordability case becomes

\[ c \geq 0 \quad [q] \]
\[ \bar{p}r + p(1 - p)c - \varepsilon c \geq \rho \quad [\mu] \]

The objective function is the same as before. We solve to have the lagrange multipliers as

\[ \mu = \frac{p_s}{\bar{p}} > 0 \quad \text{and} \quad q = \mu [\theta p_r^2 - \theta p_r p_s + \varepsilon] - \varepsilon > 0 \]

Hence $c = 0$ and $\frac{\rho}{\bar{p}}$. The results here show that the initial group contract reduces to the individual lending contract as discussed in section 3. This result suggests that, correlation is bad for group lending and the correlation does not need to be too high to disable group lending with joint liability. It suffices for $p_s p_r < \varepsilon$.

To summarize our results in this section, we first define the following

\[ C_1^5 = B_1 = \frac{p_s}{\bar{p}}, \quad B_2 = \frac{p_s(2 - p_s) - \varepsilon}{p(2 - p) - \varepsilon} \quad \text{and} \quad C_2 = \frac{2p_s}{p(2 - p) - \varepsilon} \]

Proposition 1

Under the assumptions in this paper, a group contract with joint liability that maximizes borrower surplus subject to the following conditions: homogenous matching, limited liability on borrower, lender breaking even and monotonicity achieves full efficiency if and only if;

\[ N \geq \begin{cases} B_1 - \frac{B_1 - B_2}{C_2 - C_1} [G - C_1] & G \in [C_1, C_2] \\ B_2 & G \geq C_2 \end{cases} \]

Otherwise, only risky agents would be seen to be going for loans.

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5 The subscript (1) means contract is for 1 borrower
Proposition 2

Under our assumptions, and in the presence of correlated risks, group lending with joint liability breaks down if and only if \( p_s p_r < \varepsilon \)

### 4.2 Non-constant \( \varepsilon \) (case 2)

To put some structure to the form of \( \varepsilon \), so that it is not constant but depends on the probabilities of success, we make the assumption that \( \varepsilon(p_i, p_j) = \tilde{\rho} \ast \min\{p_i(1-p_j), p_j(1-p_i)\} \) where \( \tilde{\rho} \) is the correlation coefficient for homogenous groups. In this second case also, we need to show that homogeneous matching is an efficient equilibrium and thus the only equilibrium. To do this, we compare the sum of group payoff from homogenous matching to non-homogenous matching.

The group payoff of a safe borrower for partnering with another safe borrower,

\[
\bar{R} - p_s r - [p_s(1-p_s) - \varepsilon_{ss} ]c + \bar{R} - p_s r - [p_s(1-p_s) - \varepsilon_{ss} ]c \quad (7)
\]

group payoff of a risky borrower pairing with another risky borrower,

\[
\bar{R} - p_r r - [p_r(1-p_r) - \varepsilon_{rr} ]c + \bar{R} - p_r r - [p_r(1-p_r) - \varepsilon_{rr} ]c \quad (8)
\]

a non-homogenous group’s payoff (1st agent safe, second risky) is

\[
\bar{R} - p_s r - [p_s(1-p_r) - \varepsilon_{sr} ]c + \bar{R} - p_r r - [p_r(1-p_s) - \varepsilon_{sr} ]c \quad (9)
\]

(1st agent risky and second agent safe)

\[
\bar{R} - p_r r - [p_r(1-p_s) - \varepsilon_{rs} ]c + \bar{R} - p_s r - [p_s(1-p_r) - \varepsilon_{rs} ]c \quad (10)
\]

(7) + (8) yields

\[6 \varepsilon_{ss} \text{ Represents the } \varepsilon \text{ when a safe agent partners with another safe agent} \]
\[2 \bar{R} - 2p_s r - 2p_s (1 - p_s) c + 2 \varepsilon_{ss} c + 2 \bar{R} - 2p_r r - 2p_r (1 - p_r) c + 2 \varepsilon_{rr} c\]

\[= 4 \bar{R} - 2p_s r - 2p_r r - 2p_s (1 - p_s) c - 2p_r (1 - p_r) c + 2 \varepsilon_{ss} c + 2 \varepsilon_{rr} c\]  \hspace{1cm} (11)

And (9) + (10) yields

\[4 \bar{R} - 2p_s r - 2p_r r - 2p_s (1 - p_s) c - 2p_r (1 - p_r) c + 2 \varepsilon_{sr} c + 2 \varepsilon_{rs} c\]  \hspace{1cm} (12)

This implies that we need

\[-2p_s (1 - p_s) c - 2p_r (1 - p_r) c + 2 \varepsilon_{sr} c + 2 \varepsilon_{rs} c \geq -2p_s (1 - p_s) c - 2p_r (1 - p_s) c + 2 \varepsilon_{sr} c + 2 \varepsilon_{rs} c\]

After some algebra using the assumption about the form of the correlation parameter \(\varepsilon\) in this second case, we can have that,

\[(p_s - p_r)^2 - \Sigma \geq 0 \hspace{0.5cm} \text{where} \hspace{0.5cm} \Sigma = \varepsilon_{sr} + \varepsilon_{rs} - \varepsilon_{ss} - \varepsilon_{rr} \hspace{0.5cm} \text{\(\Sigma \leq \bar{\rho} p_s (1 - p_r) + \bar{\rho} p_r (1 - p_s) - \bar{\rho} p_s (1 - p_s) - \bar{\rho} p_r (1 - p_r) = \bar{\rho} (p_s - p_r)^2\). Thus, homogenous matching to obtains in this case also.}

The contracts are similar to the constant \(\varepsilon\) case except that we replace \(\varepsilon\) with \(\bar{\rho} p_s (1 - p_s)\) which indicates that, the contracts depend on the correlation coefficient.

When affordability is not an issue the contract is

\[r = c = \frac{\rho}{p(2-p) - \bar{\rho} p_s (1 - p_s)}\]

and when affordability is an issue, we have the contract as

\[(r, c) = \left\{ \rho \frac{p_s - [p(1 - \bar{\rho} p_s (1 - p_s))]}{p_s (p^2 + \bar{\rho} p_s (1 - p_s))}, \rho \frac{\bar{\rho} p_s}{p_s (p^2 + \bar{\rho} p_s (1 - p_s))} \right\}\]

Consider for instance the condition \(N \geq \frac{p_s (2 - p_s) - \bar{\rho} p_s (1 - p_s)}{p(2-p) - \bar{\rho} p_s (1 - p_s)}\) which is obtained under full affordability.

Let \(M = \frac{p_s (2 - p_s) - \bar{\rho} p_s (1 - p_s)}{p(2-p) - \bar{\rho} p_s (1 - p_s)}\)

Differentiating \(M\), the right hand side of the above inequality with respect to \(\bar{\rho}\) yields,

\[\begin{align*}
\frac{dM}{d\bar{\rho}} &= \frac{\theta p_s p_s (p_s - p_r)}{[p(2-p) - \bar{\rho} p_s (1 - p_s)]^2} > 0
\end{align*}\]

Which means that, \(M\) increases in \(\bar{\rho}\) and thus, the higher correlation is the worse, group lending performs in terms of efficiency.
To summarize our results in this section, we define the following as before

\[ C'_1 = B'_1 = \frac{p_s}{\hat{\rho}}, \quad B'_2 = \frac{p_d(2-p_s) - \hat{\rho} p_s (1-p_s)}{p(2-p) - \hat{\rho} p_s (1-p_s)}, \quad \text{and} \quad C'_2 = \frac{2p_s}{p(2-p) - \hat{\rho} p_s (1-p_s)} \]

where \( \hat{\rho} \) is as defined before.

Proposition 3

Under the assumptions in the model, and under homogenous matching, a group contract with joint liability that maximizes borrower surplus subject to the following conditions; homogenous matching, limited liability on borrower, lender breaking even and monotonicity achieve full efficiency if and only if;

\[
N \geq \begin{cases} 
B'_1 - \frac{B'_1 - B'_2}{C'_2 - C'_1}[G - C_1] & G \in [C'_1, C'_2] \\
B'_2 & G \geq C'_2 
\end{cases}
\]

Otherwise, only risky agents would be seen to be going for loans.

Proposition 4:

The efficiency region for joint liability group-lending contract narrows as the correlation between homogenous agents increases.

5. Conclusion

The parameter space for efficient lending under correlated risk is smaller compared to independent risk case. Correlation is bad for group lending with joint liability. Correlation between project return needs not be very high for group lending to be less useful.
Next steps:

Next steps in this paper would be to look at a lender who is faced with two category of borrowers. A fraction of the borrowers have correlated risk and the other fraction has independent risks. We study how that impacts the matching pattern of borrowers, how the optimal contracts look like and whether it affects efficiency.
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