Optimal Harmonic Meter Placement Using Particle Swarm Optimization Technique

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Abstract—This paper presents a method of solving optimal harmonic meter placement problem. This problem involves determining the locations and number of meters to be installed in any system to achieve a full observability depth. The result of the optimal harmonic meter placement problem is utilized to perform harmonic state estimation on power system network. The technique used to solve this problem is the Particle Swarm Optimization (PSO). The proposed technique is applied to two test systems to investigate its effectiveness in solving the problem of harmonic meter placement.

Keywords—PSO, optimal harmonic meter placement, observability depth.

I. INTRODUCTION

Modern technology promotes power electronics which has become more and more popular in recent years, which cause proliferation of non-linear loads. These non-linear loads could create a lot of harmonic currents that downgrade the power quality. These injected harmonic currents propagate throughout the entire electrical network and distort the voltages at all buses as well as the currents in all the lines of the system. Because of the distorted voltages and currents, different operational problems may occur such as equipment overheating, motor failures, mis-operation of protective equipment, inaccurate metering, and sometimes interference with communication circuits. As the amount of bus voltage distortion depends on harmonic content of the load currents, electric utilities are becoming more interested in determining the locations and magnitudes of these harmonic sources. This problem of determination of locations and magnitudes of the harmonic sources is generally termed as "reverse harmonic power flow problem" and to solve it, appropriate locations of the harmonic meters are very important. The number of harmonic measuring instruments available is always limited due to cost and the quality of the estimates is a function of the number and location of the measurement points. Therefore, systematic procedure is needed to design the optimal measurement placement.

The problem of harmonic measurement units' placement (HMU) gained a continuous increasing interest in the past few years. In [1], a method is proposed for adding measurements to an unobservable system using the gain matrix in order to make it observable. Other methods try to redesign the measurement set completely using the topology of the network, like in [2]. In which, a measurement configuration with a minimum number of measurements that satisfies the observability constraints is determined, then the measurement configuration is further optimized to reduce the number of measurement devices to be placed based on the network topology. In the decomposition technique [3], the entire network of the power system is first decomposed into smaller subsystems, in which the optimal positions for measurement placement are determined by using the minimum condition number criteria. In [4], a sequential technique for optimal sensor placement for under-determined case (number of measurements is less than the number of unknown quantities) of harmonic static state estimation has been developed. In [5], a minimum condition number criterion of measurement matrix, based on sequential elimination is proposed. In [6] and [7], genetic algorithm (GA) based method was proposed. The major advantage of using the GA is that the solution obtained is globally optimal. Based on the results obtained from the case studies and by comparing the GA with previous methods of meter placement, it was found that the GA algorithm is as accurate as the complete enumeration (CE) but sequential procedure (SP) is not so accurate as them. In the mean time, GA is faster than CE but SP is the fastest. In [8], a dynamic programming approach is used. This approach is developed to solve multi-stage decision problems.

In this paper, the problem of determining the optimal number and location of HMU is tackled using the particle swarm optimization as a powerful optimization tool.

II. Particle Swarm Optimization (PSO)

PSO was originally designed and developed as an alternative to the standard Genetic Algorithm (GA). This technique relies on the exchange of the information between the particles of the swarm. In effect, each particle adjusts its trajectory towards its own previous best position, and towards the best previous position attained by any member of its neighborhood. In the global variant of PSO, the whole swarm is considered as the neighborhood. Thus, global sharing of information takes place and particles profit from the discoveries experience of all other companions during the search for promising regions of the landscape.

A. Basic PSO

In original PSO, M particles cooperate to search for the global optimum in the n-dimensional search space [9]. The i-th (i = 1, 2, . . . , M) particle maintains a position X_i (x_{i1}, x_{i2}, . . . , x_{in}) and velocity V_i (v_{i1}, v_{i2}, . . . , v_{in}). In every iteration, each particle uses its own search experience (self cognitive) and the whole swarm’s search experience (social influence) to...
update the velocity and flies to a new position. The updating rules are as follows:

\[ V_j^{(k+1)} = w \cdot V_j^{(k)} + c_1 \cdot \text{rand}_1 \cdot [P_{\text{bestj}} - X_j^{(k)}] + c_2 \cdot \text{rand}_2 \cdot [G_{\text{bestj}}(k) - X_j^{(k)}] \quad (1) \]

\[ X_j^{(k+1)} = X_j^{(k)} + V_j^{(k+1)} \quad (2) \]

Where:

- \( w \): inertia weight.
- \( V_j^{(k)} \): the \( j \)th velocity component at iteration \( k \) for particle \( j \).
- \( \text{rand}_1 \): a random number between 0 and 1.
- \( X_j^{(k)} \): the current position in the \( j \)th dimension at iteration \( k \) for particle \( j \).
- \( c_1, c_2 \): the cognition and social factors respectively.
- \( P_{\text{bestj}} \): the personal best position in the \( j \)th dimension.
- \( G_{\text{bestj}} \): the global best position in the \( j \)th dimension.

One of the most important issues that must be taken into consideration when applying PSO is parameters' selection. Parameters' selection guarantees the convergence of the solutions. It includes the following:

(a) Acceleration coefficients \( (c_1, c_2) \): The two constants \( c_1 \) and \( c_2 \) which are usually called cognition and social coefficients are used to balance the effect of self memory and group memory on the motion of the particle. Their standard values are set to 2.0.

(b) Inertia weight [10]: In PSO, the balance between global and local exploration abilities is mainly controlled by the inertia weights. Though, it was obtained that the best performance is by using an inertia weight that decreased from 0.9 to 0.4 during the iterations.

(c) The maximum velocity \( (V_{\text{max}}) \): The maximum velocity allowed actually serves as a constraint that controls the maximum global exploration ability PSO can have. Since the maximum velocity allowed affects global exploration ability indirectly and the inertia weight affects it directly, it will generally be better to control global exploration ability through inertia weight only.

(d) The constriction factor [11]: For ensuring the convergence, an analysis of the algorithm from mathematical aspects was proposed to use a constriction factor \( X \); the algorithm was named the constriction factor method (CFM).

Let \( \phi = c_1 + c_2 \)

\[ X = \frac{2}{\phi - 2 + \sqrt{(\phi^2 - 4\phi)}} \quad \text{for} \ \phi > 4 \quad (3) \]

The velocity equation becomes:

\[ V_{ji}^{(k+1)} = X[w V_{ji}^{(k)} + c_1 \cdot \text{rand}_1 \cdot [P_{\text{bestj}} - X_j^{(k)}] + c_2 \cdot \text{rand}_2 \cdot [G_{\text{bestj}}(k) - X_j^{(k)}]] \quad (4) \]

B. Binary PSO (BPSO)

The first variant proposed for discrete domains was the binary particle swarm optimization algorithm. Velocities are updated as in the standard PSO algorithm, but positions are updated using the following rule:

\[ X_j^{(k+1)} = \begin{cases} \text{if} & \text{rand} < \text{sig}(V_j^{(k)}), \\ 0 & \text{otherwise} \end{cases} \quad (5) \]

Where \( r \) is a uniformly distributed random number in the interval \([0, 1]\) and

\[ \text{sig}(x) = \frac{1}{1 + e^{-x}} \quad (6) \]

C. Discrete PSO (DPSO)

Due to its global and local exploration abilities, simplicity in coding and consistency in performance, PSO algorithm has been widely applied in many fields although PSO algorithm was originally proposed for continuous optimization problems [12]. In our research, we mainly use discrete data to process problems. Therefore, developing a mechanism to realize discrete optimization problem is attractive. Accordingly equation (1) will be modified as follows in equation (7):

\[ V_j^{(k+1)} = \text{round}[w V_{ji}^{(k)} + c_1 \cdot \text{rand}_1 \cdot [P_{\text{bestj}} - X_j^{(k)}] + c_2 \cdot \text{rand}_2 \cdot [G_{\text{bestj}}(k) - X_j^{(k)}]] \quad (7) \]

III. Problem formulation

Two different approaches are proposed to determine the optimal number and location of harmonic meters. These approaches are based on optimal allocation cost function and minimum variance criterion.

A. Optimal allocation cost function

The cost function is the objective function in this case [13]. The cost function (CF) consists of two main parts as follows:

\[ \text{CF} = C_{\text{F1}} + C_{\text{F2}} \quad (8) \]

The description of both CF1 and CF2 is as follows:

Observability depth term \( (C_{\text{F1}}) \):

The first part of the problem cost function, which calculates the penalty of breaking the desired observability depth \( (v) \). It is illustrated in the formula in (9) and (10). \( K_s \) in eq. (9) is a constant, which is set to be 1. MD is a matrix in which its rows represent the distances of certain meter connected to bus to all system buses, and its columns show the distance of certain system bus to each meter's connected bus.

\[ \text{CF1} = K_1 \times \sum \left[ 10^{v-1} \right] \quad (9) \]

\[ \alpha = (1^{st} \text{min} (\text{MD}) - \text{round} ((v+3)/2)) + (2^{nd} \text{min} (\text{MD}) - \text{ceil}((v+3)/2)) \quad (10) \]

Where:

- \( \text{ceil} \) : is rounding the number towards plus infinity.
Number of used meters term (CF₂):

The second part CF₂ of the cost function is for the number of meters which are used. The formula of CF₂ is illustrated in equation (11), in which \(K_2\) is a constant, set to be \(1\). \(N_{\text{meters}}\) is the number of meters used.

\[
\text{CF}_2 = K_2 \times N_{\text{meters}} \quad (11)
\]

Due to the nature of the problem studied, we have applied the binary PSO (BPSO) using the function of optimal allocation cost function on the previously stated problem.

B. Minimum variance criterion [6]

In an n-bus system, at any particular operating frequency, the bus voltages and the bus injection currents are related by:

\[
V_{\text{bus}} = Z_{\text{bus}} \cdot I_{\text{bus}} \quad (12)
\]

Where \(Z_{\text{bus}}\) is the bus impedance matrix of the system.

Suppose that bus voltages and the bus injection currents at certain buses are observed. Let these be denoted by the vectors \(V_u\) and \(I_o\), respectively. Also, let \(V_{\text{bus}}\), \(I_{\text{bus}}\) denote the vectors of the voltages and currents respectively at the remaining unmeasured buses. Partition of eq.(12) in terms of the observed and unobserved vectors yields to:

\[
\begin{bmatrix} V_u \\ V_o \end{bmatrix} = \begin{bmatrix} Z_{\text{bus}} & Z_{\text{bus}} \\ Z_{\text{bus}} & Z_{\text{bus}} \end{bmatrix} \begin{bmatrix} I_u \\ I_o \end{bmatrix} \quad (13)
\]

The basic objective of this method is to select the measurements (from the set of all possible measurement locations) that will minimize the expected value of the sum of squares of differences between estimated and actual parameter variables. Application of this minimum variance criterion to the problem of estimation of harmonic sources results in the following optimization problem:

Minimize: \(\text{Min}\{\text{min}\{\text{Et}(\hat{I}_u - I_u)^2\}\}\) \(\quad (14)\)

With respect to the locations of \(I_u\) and \(V_o\). The above problem is solved in two steps.

- In the first step, a predicted current \(\hat{I}_u\) is determined to minimize the value of the square of the difference between the unknown current vector and the true current vector \(I_u\).
- In the second step, the best measurement locations, represented by \(I_u\) and \(V_o\), that minimize the error due to the best linear predictor \(\hat{I}_u\), are found.

The theory needed to solve eq.(14) following these two steps is given below:

We assume that random vector \(I = (I_u, I_o)\) is Gaussian. This provides an adequate model for the occurrence of harmonic sources in a power system and yields a tractable mathematical model on which to base the estimation. If \(I\) and \(V\) are random variables with finite second moments, then the predictor of \(X_u\) which minimizes the variance of the error is the conditional expectation. Therefore, this conditional mean is a valid choice as an estimator of \(X_u\). Assuming \(X\) is Gaussian, we can see from eq. (13) that \(V\) is jointly Gaussian and we can write:

\[
\begin{bmatrix} V_u \\ I_u \\ V_o \\ I_o \end{bmatrix} = \begin{bmatrix} Z_{uu} & Z_{uo} \\ 1 & 0 \\ Z_{ou} & Z_{oo} \end{bmatrix} \begin{bmatrix} I_u \\ 1 \end{bmatrix} \quad (15)
\]

If we now assume that harmonic sources at distinct buses are uncorrelated, the solution of eq. (14) is simplified. In this case, the variance matrix for the existence of harmonic sources is diagonal. Thus, a priori information about the likelihood of occurrence of the harmonics at each bus may be easily incorporated in such a model by matrix:

\[
\sigma^2 = \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_o^2 \end{bmatrix} \quad (16)
\]

Where: \(\sigma_u\) and \(\sigma_o\) are diagonal sub-matrices modeling a priori probability of existence of harmonic sources for the unknown (not measured) buses and observed (measured) buses, respectively. The values in eq. (16) are to be determined from bus load levels and from prior information on the likelihood of particular buses being harmonic sources. Any a priori knowledge should be incorporated since it has some useful value. If a bus has no load, it cannot be a source of harmonics. Also, if a bus consists mostly of industrial customers, then it should be assigned a higher probability of being a harmonics source than a bus that has mostly residential customers.

Using (14) - (16) to determine the covariance matrices and cross-covariance matrices, we obtain:

\[
\text{Cov} \begin{bmatrix} V_u \\ I_u \\ V_o \\ I_o \end{bmatrix} = \begin{bmatrix} \sigma_u^2 & \sigma_u \sigma_o^T & Z_{uo}^T & 1 \\ \sigma_o \sigma_u^T & \sigma_o^2 & Z_{uo}^T & 0 \\ Z_{uo} & Z_{uo}^T & \sigma_o^2 & \sigma_o^2 \sigma_o^T \\ 1 & 0 & Z_{uo}^T & Z_{oo}^T \end{bmatrix} \quad (17)
\]

Rewriting eq. (17) in terms of the variables needed for the minimization problem yields to:

\[
\text{Cov} \begin{bmatrix} V_u \\ I_u \\ V_o \\ I_o \end{bmatrix} = \begin{bmatrix} \sigma_u^2 & \sigma_u \sigma_o^T & 0 \\ \sigma_o \sigma_u^T & \sigma_o^2 & \sigma_o \sigma_o^T + Z_{uo} \sigma_o \sigma_o^T & Z_{uo} \sigma_o^T \\ 0 & \sigma_o \sigma_o^T & \sigma_o^2 \end{bmatrix} \quad (18)
\]

The covariance of the estimation error is given by:

\[
\text{Cov}(\hat{I}_u - I_u) = \text{Var}(\hat{I}_u) - \text{Cov}(I_u, V_o I_o) \text{Var}(V_o I_o)^{-1} \text{Cov}(V_o I_o, I_o) \quad (19)
\]

By using (14) - (19) to solve for the conditional error covariance matrix, we obtain:
Cov(I_u-I_u) = \sigma_u^2 - \sigma_u^2 Z_{ou}(Z_{ou} \sigma_u^2 Z_{ou}^T)^{-1} Z_{ou} \sigma_u^2 \quad (20)

In the above expression, \( \sigma_u \) is a constant matrix representing a priori probability of existence of harmonic sources for the unknown (not measured) buses. This matrix is obtained by appropriately partitioning the matrix \( \sigma \), which models a priori probability of existence of harmonic sources at all the buses in the system. Hence, the objective of the above optimization problem is to determine the appropriate \( Z_{ou} \) matrix (i.e. to choose the appropriate measurement locations) so as to minimize the trace of the covariance Cov \( (I_u-I_u) \).

Due to the nature of the problem we have applied the discrete PSO (DPSO) using function of minimum variance criterion on the previously stated problem.

IV. Proposed algorithm
1. Start with random initial values for position and velocity. The number of particles is chosen to be \( N \), where \( N \) is the number of buses.
2. Calculate the objective function (OF) to determine the global best value (G_best).
3. Calculate the velocity and position for each particle using PSO.
4. Calculate the objective function to determine the local best (P_best) in addition to (G_best).
5. Repeat steps (3) to (5) for certain number of iterations (maximum number of iterations) or reach the target value of objective function (if known).

The following figure illustrates the flowchart of the proposed method.

To test the effectiveness of the proposed PSO technique for optimal harmonic meter placement, simulation studies have been carried out in two different systems.

a. IEEE 14-bus system:
Figure (2) shows the single line diagram and the location of the non-linear loads of this system. The complete data of this system is given in [14]. In this system the harmonic sources are already known to be present at buses 4, 5, 6 and 14. The priori probability matrix \( \sigma \) is assumed to be diagonal \([0.1, 0.1, 0.1, 1, 1, 1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 1]\).

The results obtained by using the optimal allocation cost function are:
- The number of meters shall be installed is six meters.
- Those meters shall be at buses 2, 8, 11, 12, 13 and 14.

The results obtained by using the minimum variance criterion are:
- The number of meters shall be installed is six meters.
- Those meters shall be at buses 2, 4, 6, 7, 9 and 10.

The two proposed PSO methods was applied to the above test system and the results are to be compared to that's obtained in [7] using complete enumeration (CE), sequential method (SM) and binary genetic algorithm (BGA).

It was observed that the results obtained by applying PSO on both objective functions is optimal compared to the other previously proposed methods as we obtained minimum number of buses used with full observability depth.

The following table illustrates the results using different solution techniques.
Table (1): Comparison between solution techniques for IEEE 14-bus system

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of meters</th>
<th>Location of meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete Enumeration (CE)</td>
<td>9</td>
<td>1, 3, 6, 7, 8, 10, 11, 13 and 14</td>
</tr>
<tr>
<td>Sequential Method (SM)</td>
<td>9</td>
<td>1, 3, 9-11, 6, 7, 8, 13 and 14</td>
</tr>
<tr>
<td>Binary Genetic Algorithm (BGA)</td>
<td>9</td>
<td>1, 3, 6, 7, 8, 10, 11, 13 and 14</td>
</tr>
<tr>
<td>Binary Particle Swarm Optimization (BPSO)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>using Optimal allocation cost function</td>
<td>6</td>
<td>2, 8, 11, 12, 13 and 14</td>
</tr>
<tr>
<td>Discrete Particle Swarm Optimization (DPSO)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>using Minimum variance criterion</td>
<td>6</td>
<td>2, 4, 6, 7, 9 and 10</td>
</tr>
</tbody>
</table>

It was observed from the results obtained from both functions, the minimum variance approach gives more optimum solution than that of optimal allocation cost function at large systems.

Table (2): Comparison between solution techniques of IEEE 30-bus system

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of meters</th>
<th>Location of meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binary Particle Swarm Optimization (BPSO)</td>
<td>17</td>
<td>2, 4, 7, 8, 9, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25 and 27</td>
</tr>
<tr>
<td>using Optimal allocation cost function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discrete Particle Swarm Optimization (DPSO)</td>
<td>16</td>
<td>1, 4, 6, 7, 9, 10, 11, 13, 15, 16, 19, 21, 22, 25, 26 and 27</td>
</tr>
</tbody>
</table>

b. IEEE 30-bus system:

For the sake of the conclusion support, we have to apply the proposed algorithm to another system. Figure (3) shows the single line diagram and the location of the non-linear loads of this system. The complete data of this system is given in [15]. In this system the harmonic sources are already known to be present at buses 5, 7, 8 and 12. The priori probability matrix σ is assumed to be diagonal [0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1].

The results obtained by using the minimum allocation cost function are:
- The number of meters shall be installed is seventeen meters.
- Those meters shall be install at buses 2, 4, 7, 8, 9, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25 and 27.

The results obtained by using the minimum variance criterion are:
- The number of meters shall be installed is sixteen meters.
- Those meters shall be at buses 1, 4, 6, 7, 9, 10, 11, 13, 15, 16, 19, 21, 22, 25, 26 and 27.

VI. Conclusions

We applied the proposed particle swarm optimization algorithm on test systems to solve the problem of harmonic meter placement. The simulation results clear that:
- PSO technique can be used to determine the optimal number and locations for harmonic meter placement in power system network, a goal which is achieved in a precise way comparable to other techniques.
- Full observability is obtained using this technique.

V. References


Reference Number: W10-0037