Multiple Hypothesis Tracking For Multiple Target Tracking

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Multiple hypothesis tracking (MHT) is generally accepted as the preferred method for solving the data association problem in modern multiple target tracking (MTT) systems. This paper summarizes the motivations for MHT, the basic principles behind MHT and the alternative implementations in common use. It discusses the manner in which the multiple data association hypotheses formed by MHT can be combined with multiple filter models, such as used by the interacting multiple model (IMM) method. An overview of the studies that show the advantages of MHT over the conventional single hypothesis approach is given. Important current applications and areas of future research and development for MHT are discussed.

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I. INTRODUCTION

Target tracking is an essential requirement for surveillance systems employing one or more sensors, together with computer subsystems, to interpret the environment. Typical sensor systems, such as radar, infrared (IR), and sonar, report measurements from diverse sources: targets of interest, physical background objects such as clutter, or internal error sources such as thermal noise. The target tracking objective is to collect sensor data from a field of view (FOV) containing one or more potential targets of interest and to then partition the sensor data into sets of observations, or tracks that are produced by the same object (or target). Note that the term target is used in a general sense. Once tracks are formed and confirmed (so that background and other false targets are reduced), the number of targets of interest can be estimated and quantities, such as target velocity, future predicted position, and target classification characteristics, can be computed for each track.

Since most surveillance systems must track multiple targets, multiple target tracking (MTT) is the most important tracking application. Fig. 1, taken from [1], shows the basic elements of a typical MTT system. Assume that tracks have been formed from previous data and a new set of input observations becomes available. In general observations can be received at regular intervals of time (scans or data frames) or they can occur irregularly in time. Here, we will use the general term scan to refer to any set of input measurements that were all produced at the same time. Then, the input observations are considered for inclusion in existing tracks and for initiation of new tracks. First, a gate, based upon the maximum acceptable measurement plus tracking prediction error magnitudes, is placed around the predicted track. Only those observations that are within the track gate are considered for update of the track. When closely spaced targets produce closely spaced observations there will be conflicts such that there may be multiple observations within a track's gate and an observation may be within the gates of multiple tracks. This is handled by the Observation-to-Track Association and Track Maintenance functions.

Fig. 2, also taken from [1], shows a typical conflict situation in which track gates are placed around the predicted positions (P1, P2) of two tracks, and three observations (O1, O2, O3) satisfy the gates of either (or both) of the tracks. The conventional data association method is denoted the global nearest neighbor (GNN) approach. It finds the best (most likely) assignment of input observations to existing tracks, which for example, would probably be O1 to track 1 and O2 to track 2. The term global is used to refer to the fact that the assignment is made considering all possible (within gates) associations

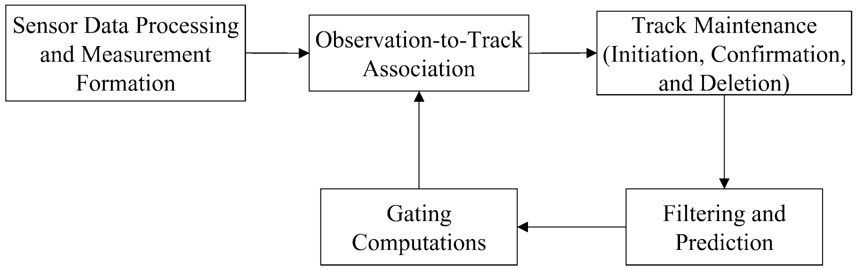

Fig. 1. Basic elements of a conventional MTT system.

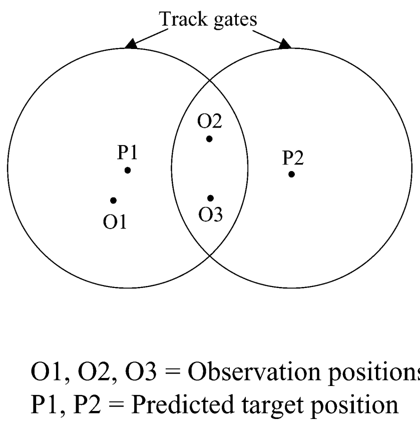

Fig. 2. Example of typical data association conflict situation.

under the constraint that an observation can be associated with at most one track. This distinguishes GNN from the archaic (but apparently still used in some systems) nearest neighbor (NN) approach in which a track is updated with the closest observation even if that observation may also be used by another track.

Only those tracks that are included in the best assignment are kept. Unassigned observations, in this case O3, initiate new tracks. Track confirmation and deletion are typically determined by rules, such as 3 detections in 4 frames of data for confirmation and N consecutive misses (typically $N = 4$ to 7) for track deletion.

Inherent in the standard GNN assignment is the assumption that an observation was produced by a single target. Tracks that do not share any common observations will be defined to be compatible. Thus, only compatible tracks can appear in the same assignment solution. Relaxation of this constraint to allow for the provision of unresolved targets that produce a single measurement will be discussed later.

Once observations are assigned to tracks, these tracks are updated during the filtering process. Conventional systems typically use a single Kalman filter. However, as discussed below, modern systems should use the interacting multiple model (IMM) approach in which several Kalman filters, tuned to different types of target maneuver, are run in parallel [1, 2]. Finally, all tracks are predicted to the time of the next set of measurements. The Kalman filter prediction covariances provide the uncertainty, in the predicted state estimate, that is required for the gating and association processes.

The GNN approach, which only considers the single most likely hypothesis for track update and new track initiation, only works well in the case of widely spaced targets, accurate measurements, and few false alarms in the track gates. For example, from results given in [1], even if the true target return is present, a single uniformly distributed false alarm in a three dimensional radar measurement space (typically range and 2 angles) reduces the probability of correct association to about 0.85. Thus, in about one out of 6 track update attempts a false alarm will be chosen rather than the correct target return. For the more usual case of multiple closely spaced targets and where missed true target detections occur, the probability of false track update is much worse. Experience indicates that often a single false update will lead to track loss and two consecutive false updates will usually lead to track loss.

The fact that misassociation represents an additional error source for a Kalman filter tracker was recognized in the very early stages of tracker development [3–5]. One approach that was proposed to improve GNN performance was to increase the Kalman filter covariance matrix to reflect this additional source of uncertainty [3, 4]. A similar approach, based upon work by R. Fitzgerald, also reduces the gain for uncertain association conditions, Sec. 6.12.1 of [1].

A second approach, which has become the Joint Probabilistic Data Association (JPDA) method, “hedges” for uncertain association conditions by allowing a track to be updated by a weighted (by probability) sum of all observations in its gate [2, 5]. This also means that an observation may contribute to the update of more than one track. Thus, for the example of Fig. 2, observations O1, O2, and O3 would all contribute to the update of track 1 and observations O2 and O3 would contribute to the update of both tracks.

Both the augmented GNN approach and the JPDA method increase the Kalman filter track covariance matrix to account for the association uncertainty. However, as illustrated in [6], increasing the Kalman filter covariance matrix to account for uncertain association can exacerbate the problem whereby an increased covariance matrix leads to even more false observations in the track gate, etc. Also, the JPDA method suffers from a coalescence problem whereby tracks on closely spaced targets will tend to come together [7]. For example, from Fig. 2, since observations O2 and O3 will contribute to the updates of tracks 1 and 2, these tracks will be drawn together.

The problems that result from relatively simple upgrades to the GNN method and the recent dramatic increases in computational capabilities have led to a near universal acceptance of the multiple hypothesis tracking (MHT) approach as the preferred data association method for modern systems. MHT is a deferred decision logic in which alternative data association hypotheses are formed whenever observation-to-track conflict situations, such as shown in Fig. 2, occur. Then, rather than choosing the best hypothesis or, in effect, combining the hypotheses as in the JPDA method, the hypotheses are propagated into the future in anticipation that subsequent data will resolve the uncertainty.

Sections II and III will discuss the basic principles and commonly used implementations of MHT. Section IV discusses how modern filtering techniques (in particular IMM) can be combined with MHT. Section V outlines some important current applications of MHT and Section VI gives areas of development and extension.

II. MHT BASICS

The manner in which MHT forms multiple hypotheses and manages these hypotheses is illustrated by again referring to the example given in Fig. 2 and by referring to the overall structure shown in Fig. 3. As an example, assume that tracks T1 and T2 with predicted positions P1 and P2, represent a hypothesis (H_1) prior to the receipt of the three observations (O1, O2, O3) on the current scan. Then, there are 10 feasible hypotheses that can be generated from the initial single hypothesis. For example, the two most likely hypotheses would both update T1 with O1 but would update T2 with either O2 or O3. Another, unlikely but feasible, hypothesis would be that all observations represent new sources (false alarms or other previously undetected targets) so that neither T1 nor T2 would be updated and all observations would start new tracks.

Reid's Algorithm

Although Singer, Sea, and Housewright [8] introduced the basic idea of propagating multiple hypotheses for a single target in a false alarm background, Reid [9] first developed a

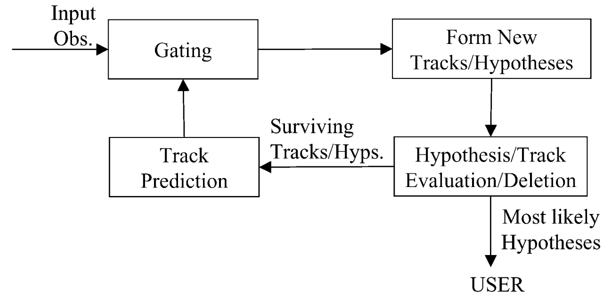

Fig. 3. MHT logic overview.

complete algorithmic approach. Reid's algorithm defines a systematic way in which multiple data (observation-to-track) association hypotheses can be formed and evaluated for the problem of multiple targets in a false alarm (and/or clutter) background. Again using the example of Fig. 2, Reid's algorithm is illustrated by defining H_1 to be the hypothesis containing T1 and T2 before the receipt of the three observations. Next, define a newly formed track

T3 (T1, O1) = track 3 formed from the association of T1 with O1

with similar definitions for T4 (T2, O2) and T5 (T2, O3). Also define NT1, NT2, and NT3 to be the new tracks initiated from O1, O2, and O3. Then, 3 of the feasible 10 hypotheses that can be formed are

$$\begin{aligned} &H_1: \quad \text{T1, T2, NT1, NT2, NT3} \\ &H_2: \quad \text{T3, T4, NT3} \\ &H_3: \quad \text{T3, T5, NT2} \\ &\vdots \end{aligned} \tag{1}$$

Tracks are defined to be compatible if they have no observations in common. As illustrated by the example above, assuming T1 and T2 share no observations, MHT hypotheses are composed of sets of compatible tracks. Again note, as discussed in more detail later, the formulation can ideally be expanded in order to address the problem of closely-spaced unresolved targets that may produce a single measurement that should be assigned to the multiple tracks that may have been formed on these unresolved targets. Using Reid's algorithm approach, hypotheses are carried over from the previous scan. Then, on the receipt of new data, each hypothesis is expanded into a set of new hypotheses by considering all observation-to-track assignments for the tracks within the hypothesis. Again, as new hypotheses are formed, the compatibility constraint for tracks within a hypothesis is maintained.

Track and Hypothesis Evaluation

The evaluation of alternative track formation hypotheses requires a probabilistic expression that includes all aspects of the data association problem. These aspects include the prior probability of target presence, the false alarm density, the detection sequences and the dynamic (kinematic) consistency of the observations contained in the tracks. Reid [9] presents such a probabilistic expression. A mathematically equivalent, but computationally preferable, approach is the log-likelihood ratio, LLR (or track score) first proposed in the pioneering paper by Sittler [10], later detailed in [11] and summarized below.

A likelihood ratio (LR) for the formation of a given combination of data (including a priori probability data) into a track can be defined using a recursive relationship that follows directly from Bayes' rule

$$\mathrm{LR} = \frac{p(D \mid H_1)P_0(H_1)}{p(D \mid H_0)P_0(H_0)} \triangleq \frac{P_T}{P_F} \tag{2}$$

Hypotheses H_1 and H_0 are the true target and false alarm hypotheses with probabilities P_T and P_F, respectively, and D is the data, so that

$p(D \mid H_i)$ = probability density function evaluated with the received data under the assumption that H_i is correct

$P_0(H_i)$ = a priori probability of H_i (such as expected density of true targets in a given area for H_1)

Note that the inclusion of a priori probabilities in (2) means that LR might formally be defined to be a probability ratio. However, following the original formulation of [10], we will refer to it as a likelihood ratio.

A true target is most generally defined to be an object that will persist in the tracking volume for at least several scans. Thus, this definition includes objects, such as persistent clutter, that may not be of interest to the tracking system but that should be tracked in order to minimize their interference with tracks on targets of interest. False alarms (or false targets) refer to erroneous detection events (such as those caused by random noise or clutter) that do not persist over several scans.

It is convenient to use the log likelihood ratio (LLR) or track score [10, 11] such that

$$\mathrm{LLR} = \ln[P_T \mid P_F] \tag{3}$$

Then, LLR can be directly converted to the probability of a true target through

$$\begin{aligned} P_T/P_F &= \frac{P_T}{1-P_T} = e^{\mathrm{LLR}} \\ P_T &= e^{\mathrm{LLR}}/[1+e^{\mathrm{LLR}}] \end{aligned} \tag{4}$$

Thus, the LLR (track score) is all that needs to be computed (and maintained) in order to assess the validity of a track. Finally, as discussed further below, the track score can be used directly for track confirmation as an application of the classical sequential probability ratio test (SPRT).

The track score, $L(k)$, at scan k, can be placed in a convenient recursive form [1, 11]

$$\begin{aligned} L(k) &= L(k-1) + \Delta L(k) \\ \Delta L(k) &= \begin{cases} \ln(1-\hat{P}_D); & \text{no update on scan } k \\ \Delta L_u(k); & \text{track update on scan } k \end{cases} \end{aligned} \tag{5}$$

The loss in track score when a detection opportunity is missed is a function of the expected probability of detection ($\hat{P}_D$). As discussed in more detail in [1, 11], the gain, ΔL_u, in track score upon update is a function of the residual error (the difference between the measurement and the prediction) and its covariance matrix, the expected density of false returns, as well as $\hat{P}_D$. In addition, if signal intensity (such as signal-to-noise ratio, SNR) is measured, it may also be used in the track score.

Given the individual track scores, the hypothesis score is the sum of scores of all tracks contained in that hypothesis. Then, given hypothesis scores, the hypothesis probabilities can be computed [1, 11]. Finally, a track may be contained in multiple hypotheses so that its probability is the sum of probabilities of all hypotheses which contain it. For the example of (1), the probability of T3 would be the sum of probabilities for hypotheses H2, H3 and all other hypotheses that contain it.

To summarize, relatively simple computations can be performed to determine hypothesis and track probabilities. A theoretical objection that may be raised is that in order to compute these probabilities, such as through the track score as defined above, it is typical to assume very approximate Gaussian models for target dynamics and measurement error statistics, uniform distributions for false alarms (clutter and noise) and new targets and a nominal $\hat{P}_D$. However, all developers of practical MHT systems make essentially the same assumptions and, as discussed further below, results show that MHT with these assumptions performs substantially better than any other developed approach.

Practical Issues

As illustrated by the simple example given above, there is clearly a potential combination explosion in the number of hypotheses (and tracks within those hypotheses) that an MHT system can generate. Thus, a number of techniques have been developed to keep this potential growth in check. These techniques,

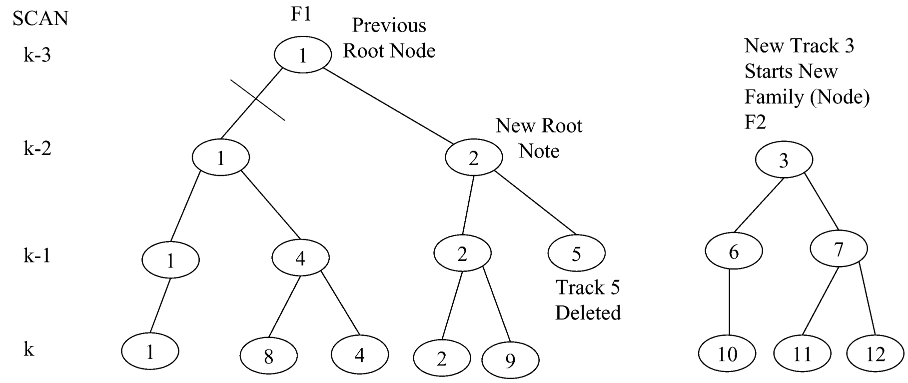

Fig. 4. Family (node) structure with N-scan pruning.

outlined next, include clustering, hypothesis and track pruning (deletion), and track merging.

The operation of clustering is performed to reduce the number of hypotheses that must be generated and evaluated. Clusters are collections of tracks that are linked by common observations. A cluster can include tracks that do not share observations directly. Thus, if track 1 shares an observation with track 2 and track 2 shares an observation with track 3, all three tracks are in the same cluster.

Clustering, in effect, decomposes a large problem into a set of smaller problems. Once clustering has been performed, the processing within each cluster can be done independently from other clusters. Thus, processing efficiencies can be achieved using a parallel processing structure whereby the processing for each cluster can be assigned to a separate processor. Then, within each cluster, hypotheses are evaluated and low probability hypotheses and tracks are deleted.

The key principle of the MHT method is that difficult data association decisions are deferred until more data are received. Thus, an important implementation feature used by all MHT developers is the family (or node) structure illustrated in Fig. 4. This structure provides a convenient mechanism for implementing a deferred decision logic and for presenting a coherent output from the MHT tracker to the user.

Fig. 4 shows how MHT track branches are formed and illustrates how a convenient structure for track pruning can be defined. Using this structure, a family is defined as a set of tracks with a common root node. Alternatively, what we define to be a family (of tracks all emanating from a single ancestor, or root node) can also be considered to be a target tree. Each branch represents a different data association hypothesis for the target and nodes are defined to be points where one track forms two or more branches. Because each branch track within the family (target tree) has at least one common node (the root node), these tracks are all incompatible with each other and can represent at most one target.

Based upon current data (including scan k), irrevocable decisions are made in the past (for the example this is scan $k-2$). Specifically, one approach finds the tracks from families F1 and F2 that are in the best current (scan k) hypothesis and goes back N scans (in this case $N = 2$) to establish a new root node. For example, if track 2 of F1 is in the best hypothesis, the new root node is track 2 at scan $k-2$. Subject to other tests, beyond the scope of this paper, if F2 does not have a track in the best hypothesis, the entire family would be deleted.

Note that the entire branch of F1 leading to tracks 1, 4, and 8 has been deleted. However, track 9 has been maintained even though track 2 was in the best hypothesis. This method is denoted N-scan pruning (or can be defined as an N-scan sliding window) and we have, for convenience of presentation, chosen $N = 2$ for the example. In practice, our experience is that N should generally be chosen to be at least 5. Also, rather than scans in the past, the decision is probably best made using N observations in the past but the basic principle is the same. Firm decisions are made in the past based upon later data.

Fig. 5, adapted from [12], shows the relationship between the families (track hypotheses for a given target) and the global (multiple track) hypotheses that are formed as collections of compatible tracks. A global hypothesis is formed by choosing at most a single track from each family.

The family representation of Fig. 4 also provides a convenient way to present MHT data to a user who typically wants one track per target, not a set of alternative tracks with probabilities. The tracks in the output trackfile are linked to the families and, at any given time, the most likely track in the family is presented to the user. This can lead to some apparent inconsistencies in the output as MHT branch probabilities change with the receipt of more data. For example, it may be that track 1 of F1 was the most likely track at scan $k-1$ but track 2 is the most likely track at scan k. Thus, a possible alternative is to provide an average state estimate, computed using the branch track probabilities, along with a

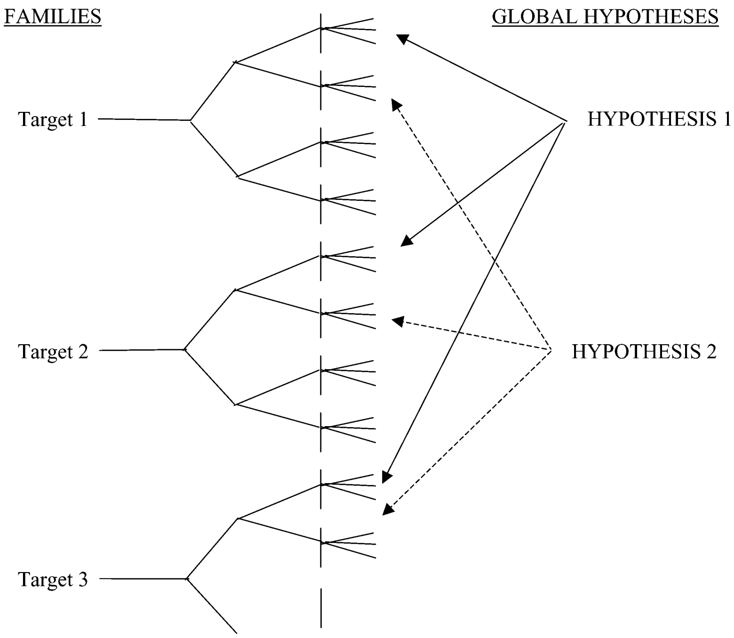

Fig. 5. Formation of hypotheses from tracks in families.

covariance that reflects the spread in the branch track state estimates. This approach is particularly useful for an agile beam radar system, discussed below, for which data association uncertainty should be used in the resource allocation logic.

III. ALTERNATIVE MHT IMPLEMENTATIONS

Although the same basic principles and mathematical models apply to all, there are several different approaches to MHT implementation. The first (hypothesis-oriented) approach follows the original work of Reid, outlined above. The computational feasibility of this approach has been greatly enhanced by the use of Murty's algorithm [13] to more efficiently generate hypotheses [14]. An alternative, track-oriented approach [1, 12] does not maintain hypotheses from scan to scan. As tracks are updated on each scan they are reformed into hypotheses. An innovative implementation of the track-oriented approach is the multidimensional (or multiple frame) assignment method [15, 16]. Finally, a Bayesian MHT approach has been proposed by van Keuk and Koch and associates [6, 17, 18]. The methods are briefly summarized below.

m-Best Implementation of Reid's Algorithm

As illustrated above, Reid's algorithm forms a large number of hypotheses that are collections of compatible tracks. These hypotheses are carried from one scan to the next where newly received observations are used to update the tracks in different ways. Thus, each hypothesis carried from the previous scan may give rise to many new hypotheses (most of which will later be discarded based upon low probability) as the tracks contained within the hypothesis are updated in different ways. This potential explosion of new hypotheses that may result from an indiscriminate expansion of the old hypotheses has been a barrier to the practical implementation of Reid's algorithm. Thus, a method to only generate "good" hypotheses is required and has been provided by the work of Cox et al. [14].

As discussed in [14], an efficient implementation of Reid's algorithm can be achieved using Murty's method for finding the m-best solutions to the assignment problem. Using this approach, given $m_p(k-1)$ hypotheses from the previous scan, the number of hypotheses formed on the current scan can be limited to $m(k)$ when m is an input parameter that could be set a priori or, presumably, could be chosen adaptively. The important principle is that the generation of many unconsequential, low probability hypotheses, that resulted from earlier implementations of Reid's algorithm, is avoided.

Track-Oriented MHT

The track-oriented approach recomputes the hypotheses using the newly updated tracks after each scan of data are received. Rather than maintaining, and expanding, hypotheses from scan to scan, the track-oriented approach discards the hypotheses formed on scan $k-1$. The tracks that survive pruning are predicted to the next scan k where new tracks are formed, using the new observations, and reformed into hypotheses. Except for the necessity to delete some tracks based upon low probability or N-scan pruning described above, no information is lost because the track scores, that are maintained, contain all the relevant statistical data. The basic, currently unresolved, issue is whether it is more efficient to expand the old hypotheses using Murty's method or to reform the hypotheses using the updated tracks and their compatibilities with other tracks.

A strong argument for the track-oriented approach to MHT can be made by noting that the combinatories of hypothesis formation are such that there are typically many more hypotheses formed than tracks. Typically, for difficult scenarios, there may be several thousand comparable hypotheses formed from several hundred tracks in a cluster. Then, the process of maintaining a thousand (or more) hypotheses and expanding these hypotheses using Murty's method to find the best thousand new hypotheses may be prohibitive. On the other hand, our experience with track-oriented MHT has shown that several hundred tracks can easily be maintained and expanded into new hypotheses for difficult scenarios. Typical computational results for a difficult scenario with 100 closely spaced targets and a high radar update rate indicate the feasibility of real-time operation for a track-oriented MHT [19]. This study was performed using a single 866 Mhz Pentium III computer. Newer computers and/or parallel processing with several computers would allow real-time tracking for even more difficult scenarios.

Our implementation uses a relatively simple set of heuristic search methods, based upon a breadth-first method described in [1] and the A* search method described in [20]. The multiframe assignment (MFA) method, outlined next, represents a potentially more accurate and efficient implementation of track-oriented MHT.

Multi Dimensional (Multiframe) Assignment

Deb [15] and Poore [16] and their associates independently recognized that the MTT data association problem can be placed in a form where a multi dimensional assignment approach that uses the Lagrangian relaxation method is directly applicable. Like track-oriented MHT, this approach forms and maintains tracks from scan (frame) to scan and reforms tracks into hypotheses after each new scan of data are received. It also uses a sliding window approach which is similar to the N-scan pruning method used in conventional MHT and illustrated in Fig. 4. The unique feature of this method is the manner in which a Lagrangian relaxation method is used to find the most likely hypothesis or a set of the m-best hypotheses [21].

The input is a set of tracks with their scores and their compatibilities with other tracks. Again, two tracks are defined to be incompatible, and thus cannot be in the same hypothesis, if they share one or more observations. The process of arranging these tracks into hypotheses can be formulated as an optimization problem with the goal of maximizing the hypothesis score (sum of all track scores in hypothesis) with the constraints that no tracks in the hypothesis share observations.

The basic principle of the Langrangian relaxation approach is to replace constraints (in this case that an observation can be used by at most a single track) by Lagrange multipliers in the objective function (in this case the sum of track scores) used in the maximization. The "art" of this method involves the proper choice of Lagrange multipliers so that the solution formed from maximizing the objective function approaches the best feasible solution, in which each observation is used by at most a single track.

This optimization is very complex and requires sophisticated mathematics but we will (at least attempt to) summarize the basic principles. Two solutions to the hypothesis formation problem are obtained with cost defined to be the negative of score. The first solution, defined to be the relaxed or dual solution, may not satisfy the constraints (that an observation should be used once and only once). However, Lagrange multipliers are introduced into this solution and are chosen so that constraint violations are, effectively, given high costs. Thus, the number of constraint violations should be reduced over time with successive iterations of the method.

A second solution, denoted the recovered or primal solution, is obtained from the dual solution by enforcing the constraints. For example, one method for obtaining this solution starts with the assignment of the first two scans of data that was obtained by the dual solution. Then, it adds observations from the later scans by solving an assignment matrix, that enforces the constraints, on each later scan. Thus, a feasible, but likely suboptimal, solution is obtained.

The costs of the dual solution, $q(\underline{u})$, where $\underline{u}$ represents the Lagrangian multipliers, and the primal solution, $v(\bar{z})$, represent bounds on the cost, $v(z)$, of the true, but unknown, solution

$$q(\underline{u}) \leq v(z) \leq v(\bar{z})$$

where z and $\bar{z}$ are the set of binary variables that define which tracks are included in the true and the primal solutions, respectively [1, 15, 16].

Successive iterations are performed by using updated Lagrange multipliers in an attempt to increase $q(\underline{u})$ and decrease $v(\bar{z})$ and a stopping rule is defined so that the feasible primary solution is accepted when $q(\underline{u})$ and $v(\bar{z})$ are "close enough," or when time runs out and a solution is required.

The multiscan assignment method outlined above can be used to implement the N-scan pruning method used for track-oriented MHT, as illustrated in Fig. 4. Performing N-scan pruning requires a solution to the $N + 2$ scan assignment problem.

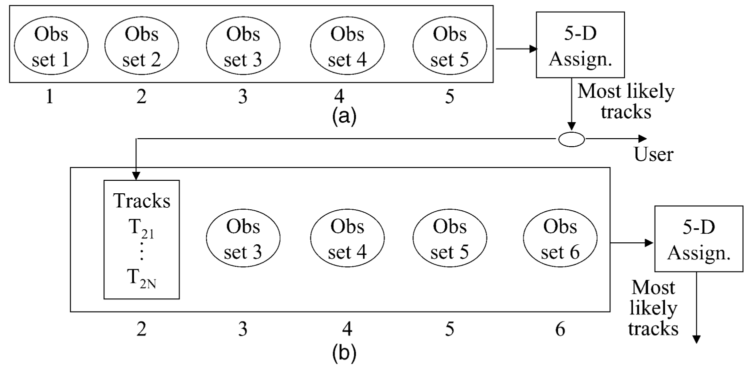

Fig. 6. Implementation of $N = 3$ scan pruning using 5D assignment. (a) First five scans of observations. (b) Tracks formed from first two scans and four scans of observations. (Adapted from notes by A. B. Poore.)

Fig. 6 illustrates the process for $N = 3$ (five-scan assignment). Referring to Fig. 6(a), initially five scans of data are collected with the observations on scan 1 effectively being the initial root nodes. The output of the 5D assignment problem will be a set of tracks in the most likely (solution) hypothesis. These tracks are traced back $N = 3$ scans to their root nodes (tracks) on scan 2. Then, all tracks that were in existence on scan 2 and that do not have one of these root node tracks as their ancestor on scan 2 are deleted.

As illustrated in Fig. 6(b), the root nodes are taken to be the tracks on scan 2 that were the ancestors of the tracks in the most likely hypothesis. The next scan of data is used to update the tracks that survived pruning on the previous scan. The process continues with, in general, new observations received on scan $k+1$, a sliding window of observations received on scans $k, k-1, \ldots, k-N+1$ and the root node tracks on scan $k-N$. This process is illustrated in Fig. 6(b) for $k = 5$ and $N = 3$. See [22, 23] for more details on efficient implementation.

Bayesian MHT

The technique denoted Bayesian MHT [6, 17, 18] is designed to more closely represent the probability density functions (PDF) of alternative data association hypotheses. The PDF is represented as a Gaussian mixture that represents the joint distribution of the targets under track. Thus, the method effectively requires knowledge, or assumption, of the number of targets in track. Reference [18] addresses the problem of estimating this number.

IV. MHT AND MULTIPLE MODEL FILTERING

It is widely accepted that accurate tracking of dynamic targets requires the use of multiple Kalman filter models. The basic idea of all multiple model approaches, as applied to tracking maneuvering targets, is that maneuvers are typically abrupt deviations from basically straight-line target motion. Because this process is very difficult to represent with a single maneuver model, multiple models, representing different potential target maneuver states, are run in parallel and continuously evaluated using filter residual histories. Bayes' rule and the residuals are used to determine the probabilities of validity of the models. The output is then typically a probability-weighted composite of the individual filters.

There are two basic approaches that can be used to combine MHT with multiple model filtering. The first, outlined in [12], is to add a set of maneuver hypotheses to the MHT data association hypotheses. Thus, an additional set of hypotheses which differ in target dynamics history will be formed. Use of interacting multiple model (IMM) filtering appears to be difficult for this approach.

IMM filtering has become generally accepted as the best method for using multiple filter models [2]. The unique feature of the IMM approach is the manner in which the state estimates and the covariance matrices are combined via the process defined to be mixing. The basic principle is that the currently more accurate (as determined by the computed model probabilities) models transfer their state estimates to the less accurate models. For example, in the case of a maneuvering target, the state estimates from the maneuver models, that should follow the target motion fairly well, are transferred to the nonmaneuver filter that otherwise would develop a large lag.

In order to conveniently do IMM filtering within an MHT framework, we believe that it is most convenient to define tracks according to their data association history. A similar approach is presented in [24]. Then, the track score (or probability) is computed using all component IMM filter models. Thus, hypothesis formation and pruning are done on the composite tracks, containing contributions from all IMM filter models, rather than on the IMM model tracks independently. Using this approach the mixing process is conveniently done for each track, rather than requiring mixing across tracks.

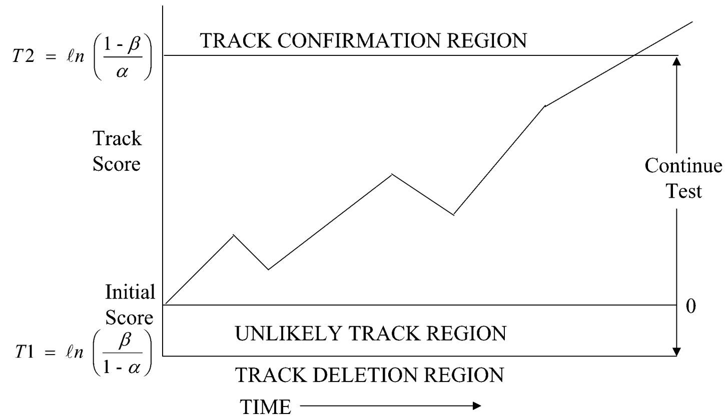

Fig. 7. Score function approach as an application of SPRT.

Given that hypothesis formation and pruning are performed using all the IMM filter models for each track, the next issue is how to perform gating and how to update the combined track score. One approach is to form a composite track state estimate and covariance matrix before gating and to perform gating using the composite quantities.

The composite state estimate and covariance matrix are formed from weighted (by the filter model probabilities) sums of the state estimates and covariance matrices of the individual filters. Alternatively, using a second approach, each filter model can be used separately for gating. In this case, there will be separate state estimates (and corresponding covariance matrices) that will be individually compared with the candidate observations. The observation-to-track gating test will then be satisfied if the gating test is satisfied for any filter model. Similarly, the track score can be computed from the composite track residual (and residual statistics) or a combined track score can be computed using the individual residual data from the different IMM filter models.

It has been our experience that the second approach is preferred. During times of nonmaneuver, the composite state and covariance matrix (and resulting gate) may become so heavily weighted towards the nonmaneuver models that an abrupt target maneuver can lead to track loss. Finally, the extension to the track score required when multiple filter models are used is straight forward [1].

V. MHT APPLICATIONS

The actual practical implementation of MHT has been impeded by the, currently incorrect [19], perception that its complexity precludes real-time application. Also, the security restrictions that surround technologies, such as tracking, being developed for current military applications and company proprietary policies have greatly restricted the ability of MHT tracker developers to publish and compare their results, and to share ideas. Another problem is that very little comparative study of MHT performance, versus that of alternative tracking methods, has been reported in the tracking literature. However, the brief summary of reported comparative studies, such as [25], given in [1] and the growing acceptance of MHT among those in the tracking community clearly indicate that MHT is the currently preferred method for difficult tracking problems. We next summarize some important applications with which the author is familiar.

Track Confirmation and Maintenance for Dim Targets in Clutter

As illustrated by Fig. 7, and discussed further in [1, 18, 26], a confirmation test that uses the track score (LLR) is essentially an application of the classical sequential probability ratio test (SPRT). Then, as detailed in [1], the choice of confirmation and deletion thresholds (T1 and T2, respectively, shown in Fig. 7) can be related to tracking requirements (such as the number of false tracks allowed per hour) through the parameters α = false track confirmation, and β = true track deletion probability. This approach also provides a convenient analysis tool for preliminary system design [1, 26].

The application of SPRT theory to MHT track confirmation assumes that false alarms are uncorrelated in time. In practice, such as for tracking targets against a background of ground clutter, clutter returns tend to be correlated in time. In this case, it is best to maintain tracks on the stationary sources of ground clutter that produce the returns. Thus, special logic using motion or signal characteristics is developed to inhibit the output of these tracks to the user [27].

A number of studies, discussed further in [1], have indicated that an MHT tracker will provide performance that is comparable to the conventional, single hypothesis (GNN) method at 10 to 100 times the false alarm density of the GNN method. This allows a system using MHT to operate at a lower detection threshold, in order to detect and track dim targets [28]. However, the comparative study given in [29] showed that a well-designed track-before-detect (TBD) approach that, in effect, combines the detection and tracking functions, will confirm tracks on nonmaneuvering dim targets at much lower SNR (about 4–5 dB lower for the cases considered in [29]).

Agile Beam Radar

Efficient allocation of radar resources is one of the major issues in the design of an agile beam (or electronically scanned) radar tracking system. Moreover, following [1, 30–32] use of an MHT tracker can greatly enhance the effectiveness of an allocation scheme. Specifically, the combined use of MHT data association and IMM filtering and prediction methods provides the most accurate estimates of tracking error that are required for efficient sensor allocation. The IMM filter model probabilities and covariance matrices provide estimates of the error due to target maneuver and the potential error due to data association is computed from alternative MHT hypotheses. Further discussions of the radar benchmark study that demonstrated the effectiveness of an IMM/MHT solution to the agile beam radar resource allocation problem are given in [30] and the Introduction of [33].

Missile Defense Systems

Post boost tracking scenarios for missile defense systems are characterized by a large number (potentially hundreds or even thousands) of closely spaced objects. These objects are deployed over time by the post boost vehicle (PBV or bus) and very accurate tracks are required for impact point prediction. In addition, track purity (defined to be the proportion of observations in a track that were produced by the same source) must be high so that discrimination can be successfully performed. Discrimination methods employ Bayesian or Dempster-Shafer reasoning to determine the target type using the characteristics (such as intensity profile) of the measurements in the track as examined over time. For example, it is very important to discriminate between the lethal reentry vehicle (RV) and decoys that are employed to "trick" the tracking and discrimination algorithms.

Both radar and space-based infrared (SBIR) tracking systems are being developed. Given the stringent tracking requirements, it is generally accepted that MHT should be used for both types of sensors and there are several special features, outlined next, that must be addressed for these applications.

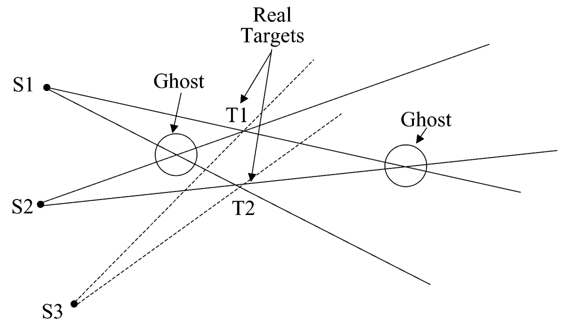

Fig. 8. Triangulation with angle tracks leads to false intersections that can be resolved with MHT and later data.

First, objects (RVs and various types of decoys) are deployed from the PBV with basically the same velocity as the PBV. Thus, a "warm start" track initiation (or spawning) procedure is used in order to quickly obtain the required tracking accuracy.

Referring to Fig. 2, observations O2 and O3 would be candidates for "warm start" new track initiation (in addition to the hypotheses that they update T2 or possibly T1 also). Thus, the new tracks would be given a position estimate based upon the measurement and a velocity estimate based upon the velocity of the parent tracks (either T1 or T2 or an average of the two) for which they satisfied a gating relationship. For the SBIR system, in which only angle measurements are available, the range from the sensor platform, as well as the platform position, will be used along with the measured angles to form the initial position estimate. The initial "warm start" track filter covariance matrices are defined using the measurement error variances and the parent track range (for SBIR) and velocity error covariance matrices. Also, terms to account for the potential differences in velocity of the newly detected (resolved) object and the parent track object are added. Finally, once spawning occurs, MHT processing will take over to determine, using later data, which observations (in our example O2 or O3) should start new tracks and which should update existing tracks (or possibly be discarded).

An additional source of data association uncertainty occurs for the angle-only measurements of an SBIR system. Tracks on targets that are separated from existing tracks (so that spawning cannot be accurately used) must be initiated by the triangulation process, illustrated in Fig. 8 and discussed further in [1]. Using this procedure, the intersections (or near intersections) of mono (angle-only) tracks from two platforms (S1, S2) are used to initiate stereo (3D position and velocity) tracks. The problem, shown in Fig. 8, is that, for closely spaced targets, there may be false intersections that form ghost tracks, as well as the correct intersection where the targets actually exist. Thus, an MHT approach is required so that all feasible tracks are maintained until either the evolving

TABLE I
Comparative Conventional and MHT Tracking Errors Referenced to an Idealized System

	Early Time		Intermediate		Late	
System	Position	Velocity	Position	Velocity	Position	Velocity
Conventional	3.3	2.9	2.8	5.0	3.0	3.3
MHT	1.7	2.1	1.4	2.0	1.5	1.2
Idealized	1.0	1.0	1.0	1.0	1.0	1.0

geometry or data from additional sensors (S3) allows the system to sort out the ghosts from the true target tracks.

In order to illustrate the advantages of MHT over conventional single hypothesis (GNN) tracking, Table I gives recent comparative results for a difficult SBIR application. Table I gives comparative 97 percent Monte Carlo simulation derived position and velocity errors for three tracking systems. The 97 percent level values were defined such that only 3 percent of the tracking error (averaged over multiple targets and multiple Monte Carlo runs) exceeded these values at the sampling times. A highly optimistic reference for Table I was an idealized system for which perfect observation-to-track association was performed. The observations were assigned target truth tags, which were used for the association, but the effects of unresolved targets and missed detections were included.

The MHT and conventional (GNN) tracking system RMS position and velocity tracking errors are normalized with respect to the idealized system errors. Results are presented at three times. The initial (early) time is when targets are beginning to become resolved so that by the last (late) time nearly all targets were resolved. Of course, the tracking errors decreased for all systems (even though some ratios increased) with time but the comparative advantage of the MHT system is clearly apparent over the entire scenario. Finally, note that the MHT errors closely approach those of the idealized system towards the end of the scenario while the conventional tracker errors remain at about 3 times the values for the idealized system.

Ground Target Tracking

Probably the most important, and challenging, current tracking application uses data from airborne (or spaced-based) sensors to track ground targets. Difficult target dynamics include move-stop-move and on and off-road target motion as well as closely spaced targets moving in groups (convoys). Sensor difficulties result from potentially long revisit times (greater than 10 sec.), obscured (by mountains or building) sensor line-of-sight, unresolved targets, out of sequence measurements in multiple sensor systems, and the fact that a radar operating in the standard ground moving target detection (GMTI) mode will not detect stopped (or slow moving) targets that cannot be distinguished from the ground clutter.

The difficulty of tracking ground targets has led to the consensus that multiple filter models, for on and off-road tracking, and MHT data association are required. For example, see [34–38] and Chapt. 6 of [33].

An example of the interesting challenges of the ground target tracking problem are targets that use move-stop-move motion in order to evade detection by GMTI radar. This necessitates the development of a special stopping target filter model and the inclusion of the hypothesis that a missing detection results from a stopped target, rather than a random miss or an incorrect track prediction [36–38]. In particular, the lack of detection can actually be used to infer target position by forming the hypothesis that a missed detection results from the fact that the target has stopped [37].

VI. MHT RESEARCH AND DEVELOPMENT AREAS

As stated by Daum several years ago [39], a major, mostly ignored, tracking problem is the presence of unresolved, or partially resolved, measurements produced by closely spaced targets. In closely-spaced target scenarios, such as aircraft flying in formation, an observation will often be produced by two, or more, targets. Thus, for these conditions, the standard MHT assumption that an observation was produced by a single target, and thus can only be assigned to a single track, must be modified. This issue becomes particularly important when tracking with sensors of different resolution capability, such as radar and IR. References [1, 40–42] and [33, ch. 4] present methods that are applicable to the extension of MHT to include hypotheses that allow a potentially merged observation to update more than one track.

Another important area of research is the combination of MHT with group tracking. An example where combined group and MHT tracking will be required is the missile defense problem where large numbers of objects may be deployed from the PBV (bus) in a short time period [43, 44]. As discussed in [44], there may be time intervals, as the targets are first deployed, when the proliferation of closely spaced targets may cause the number of MHT hypotheses formed to become prohibitive. The proposed solution [44] uses group tracking until the targets separate sufficiently to allow feasible MHT tracking of individual targets. The determination of when (and how) to make the transition from a group track to MHT tracking of individual targets is the major issue. Other applications where combined MHT and group tracking will be required for optimal performance include tracking formations of aircraft [18] and convoys of ground moving targets [45].

As shown in [28], the track-before-detect (TBD) approach may significantly outperform MHT for the task of track confirmation of dim nonmaneuvering targets. However, the TBD approach, which essentially integrates signal intensity along a set of potential, nearly straight line paths, is not applicable to highly maneuvering targets and has questionable applicability in dense target environments. Thus, the goal is to combine use of the powerful TBD methods, such as the dynamic programming algorithm, DPA [1, 46] and Bayesian tracking [28, 47], for detecting and tracking widely-spaced dim targets with IMM/MHT techniques that are most applicable to maneuvering targets in dense environments. Reference [46] discusses a combined DPA/MHT tracking system.

Standard track and hypothesis evaluation methods currently only use metric (measured position, range rate, etc.) and possibly intensity (measured SNR, etc.) data. The increased capability of sensors to measure other feature data, such as high range resolution (HRR) and jet engine modulation (JEM) radar measurements, and the development of multiple sensor tracking systems dictate that features, attributes and target classification/ID should be used to improve data association. This is particularly true for the problem of maintaining tracks on high priority targets for the ground target tracking problem [48].

A basic issue is how to weight attribute/ID data versus metric measurements. For example, a radar return might contain JEM information regarding engine type that is consistent with other target type information contained in the track, but the measured range rate may differ significantly from the track's predicted range rate. How should the observation-to-track score reflect these two different, and possibly inconsistent, data sources? As outlined in [1, 49] a mapping to likelihood (or LLR) is required. However, to the author's knowledge this approach has not yet been implemented for a practical system.

As discussed further in [33, ch. 1], the multisensor distributed tracking problem is of great practical importance. One basic issue/goal for a distributed platform system is to attempt to ensure that all platforms have a Single Integrated Air Picture (SIAP) so that, for example, track 1 on platform 1 represents the same target as track 1 on platform 2, etc. Methods for maintaining SIAP for conventional (single hypothesis) tracking use an associated measurement report (AMR) that is sent from the platform that receives a measurement. The AMR contains the association decision, made by the platform that produced the measurement, which is broadcast to all other platforms in the network who update their tracks accordingly, without any further association logic being performed.

Maintaining SIAP for an MHT system is much more difficult because multiple current association hypotheses are maintained so that, as shown in Fig. 4, final irrevocable decisions are delayed. In the meanwhile, as the result of imperfect communication (missing and out-of-sequence data), the family structures on the different platforms may diverge. Also, track initiation and confirmation decisions may differ as different platforms use different sequences of measurements to initiate duplicate tracks on the same target. This is an important area of current research with approaches discussed in [50–52] and [33, ch. 6].

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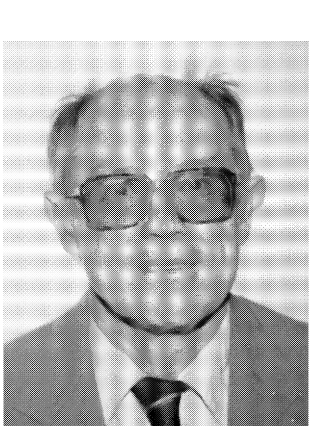

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