Core Capacity Region of Energy-limited, 
Delay-tolerant Wireless Networks

Volkan Rodoplu, Member, IEEE, and Teresa H. Meng, Fellow, IEEE

Abstract

We model energy-limited, delay-tolerant wireless networks as non-transferable-utility (NTU) co-operative network flow games. We model the utility of each node as a positive linear function of the number of bits that the node sends as a source and the number of bits that it receives as a destination. Based on the core of a cooperative game, we define the “core capacity region” of these networks as the set of utility vectors that cannot be collapsed by any coalition. We show that the core capacity region is non-empty. This implies that even if the nodes have perfect side information on all of the Joules-per-bit link costs in the entire network, there still exists a stable network solution such that no subset of the nodes can improve the utilities of all of its members over those in this network solution. We present an algorithm to compute the core capacity region. We show that the only core solution under the many-to-one traffic model with a single sink and stationary nodes is the direct transmission strategy; however, when virtual prices are allowed as signals, cooperative solutions emerge within the core capacity region. We show that the core capacity region is non-empty under pricing. The core capacity region serves as a useful concept in the analysis of cooperative, energy-limited, delay-tolerant networks.

Index Terms

capacity, wireless, sensor network, energy, game theory, cooperative, delay tolerant

I. INTRODUCTION

Delay-tolerant networking [1] has been emerging as a new application area for wireless communications. Delay-tolerant networks are characterized by (1) very large end-to-end latencies (on the order of minutes to hours or even days) that can be tolerated by the applications, (2) relatively low data rates, (3) limited energy and memory of end devices, and (4) intermittent
connectivity and network partitions. The first three characteristics are typical of stationary sensor networks that deliver delay-tolerant data, such as for environmental monitoring applications [2]. The fourth characteristic typically arises from node mobility which changes the connectivity patterns over the large durations tolerable by the applications.

This paper focuses on stationary, delay-tolerant networks characterized by the first three properties and addresses the design of cooperative solutions for these networks. In the past, cooperative networking solutions [3] have focused on cooperation in the bandwidth-limited regime where the maximization of the data rate has been the main objective. In contrast, delay-tolerant networks typically generate small amounts of data which must be relayed over energy-limited devices. Hence, the physical layer design for such networks calls for cooperation strategies that are very different from those for data-rate-intensive applications.

In order to illustrate a stationary, energy-limited, delay-tolerant network, we consider the following example of a randomly deployed, terrestrial habitat monitoring network, as shown in Fig. 1. Each sensor node generates 1 kbits every 5 minutes; that is, 3.33 bits per second on average. There is a single collection site for all the data, and the transmissions are sent over multiple hops that consist of short links; thus, the heaviest relay traffic is carried by the nodes that are 1 hop away from the collection site. If there are 100 nodes in the sensor network, a node near the collection site may typically carry the traffic from 20 to 40 nodes. In the worst case, upon bad placement, such a node may have to carry the traffic from the entire network, resulting in a maximum demanded data rate of $3.33 \times 100 = 333$ bits per second on the last hop to the collection site. The popular Crossbow MPR2400CA modem delivers a data rate of 250 kbps (which is typical for many stationary RF sensor modems), and hence, needs to be turned on only a fraction $0.00133$ of the time to carry this traffic in the worst case to the collection site. (The nodes further away from the collection site need to be turned on much less.) By using localized topology control [4], a node can limit its neighbors to roughly 3 to 4, and its interference neighborhood to roughly 20 nodes. If each node selects its transmission start time randomly [5], the probability that a transmission collides with the transmission of another user in the interference neighborhood is $1 - (1 - 2 \times 0.00133)^{10} = 0.05$. (The factor of 2 is due to asynchronous transmissions.) Since the nodes operate on schedules that are randomly
and independently selected (and then fixed and revealed to the neighbor nodes), the fraction of successful transmissions is bounded below by \(100 \times (1 - 0.05) = 95\%\) in this case. This can be further improved to create essentially interference-free schedules by the techniques in [5]; that is, by choosing the transmission start times to avoid any locally detected overlaps. Note that in these systems, the available network degrees of freedom is much larger than the number of time slots required for transmission. The end-to-end latencies can be as large as 1 hour to 1 day in the collection of such data; hence, the delays caused by the sparse, random selection of transmission start times are easily tolerated.

Finding the minimum energy paths based on physical layer models [4], and maximizing the network lifetime [6] have been key objectives in the design of sensor networks. This paper uses a central concept called the “core” from cooperative game theory to arrive at stable solutions for sensor networks. Even though sensor networks appear to be made up of nodes that collectively serve a single objective (such as the detection or measurement of an underlying phenomenon), treating the nodes as different entities with their own objectives may lead to more survivable networks. This paper proposes that the nodes in sensor networks be modelled as cooperative agents, each with its own utility function, even when the sensor network as a whole might be serving a single common objective. We propose the “core capacity region”, based on the core of a cooperative game, as the set of utility vectors that are preferable from the perspective of stability.

The core solutions are the solutions that result when the nodes are given the opportunity to compete as groups in order to improve their positions. Such competitive algorithms for the maximization of group benefits can be programmed into the nodes. Fig. 2 shows an example of four nodes where node 3 wishes to transfer information to node 1, and node 2 wishes to transfer information to node 4. The figure shows the Joules-per-bit costs \(C_{ij}\) of each link \(ij\), as well as the energy supply \(E_2 = E_3 = 8\) MJ. (We model only the transmit energy consumption in this paper.) The link costs can be obtained, for example, by each node’s increasing its transmit power until it hears back from its neighbors and detects the shown set of costs by decoding transmit power stamps. (We address large-scale networks later.) Each node uses the randomly generated transmit start time selection as before to operate on essentially orthogonal channels given the
large number of non-interfering dimensions available. In Fig. 2, if nodes 1 and 3 form a group (called a “coalition” in cooperative game theory), node 3 can send 2 Mbits to node 1 on the direct link before node 3 runs out of energy. If nodes 2 and 4 form a coalition, then node 2 can send 1 Mbits to node 4 on the direct link before node 2 runs out of energy. However, if the nodes have been programmed so that groups of nodes can detect what they can achieve by cooperation, they are likely to form larger groups. In this example, node 2 can help nodes 1 and 3 by relaying their traffic, and node 3 can help nodes 2 and 4 by relaying their traffic. If all of the four nodes do this, we say that the four nodes have formed the “grand coalition”.

If these two groups cooperate, a key problem is how each node selects what part of its energy it will devote to relaying, and what part to its own traffic. In the past, many methods have been proposed for how to divide the total energy in such situations: (1) Each node may charge a price [7][8][9][10] to the other nodes, and a price equilibrium [11] may be established. In sensor networks, these prices will be “virtual prices” that do not correspond to real transfer of money, but rather serve as a signaling mechanism to lead to efficient allocations. (2) The Shapley value [12] of the resulting cooperative game may be used as a fair allocation to decide how a node partitions its energy resource between the relay traffic and its own traffic. (3) The Nash bargaining solution [12] may be used. In fact, the few papers [13][14][15] that applied cooperative game theory to communication networks in the past used the Nash bargaining solution. In our example, each of the groups \{1, 3\} and \{2, 4\} may be modelled as a single player in a 2-player bargaining game. However, the Nash bargaining solution has no notion of coalitions, and thus cannot take into account the complex cooperation structures that can form in larger scale wireless networks.

In contrast, in the general framework that we develop in this paper, we allow groups of nodes to form cooperation structures via methods that may or may not invoke pricing. We would also like the framework to generalize later to mobile networks where cooperation structures can be formed on the fly under a changing network topology. The rest of this paper\(^1\) is organized as follows: In Section II, we formulate an energy-limited, delay-tolerant wireless network as a cooperative game. In Section III, we describe the concept of the core capacity region of such

\(^1\)A part of the results of this paper has been presented by the authors in [16][17][18][19].
networks, show that the core capacity region is non-empty, and analyze the core capacity region under the many-to-one traffic model. In Section IV, we set up a virtual pricing system and show that the core capacity region is non-empty under this pricing system. In Section V, we discuss the extension of this work to interference-limited networks.

II. ENERGY-LIMITED, DELAY-TOLERANT NETWORKS AS COOPERATIVE GAMES

In this section, we formulate an energy-limited, delay-tolerant wireless network as a cooperative game. A cooperative game is a game in which each user may cooperate with others through contracts. For a network, the contract may be represented by agreements such as (1) relaying the bits that a node promises to relay, (2) charging the declared price for transmission if there is one, and (3) carrying out the network solution that was agreed to. From an implementation perspective, such contracts can be programmed into the devices. Each user $i$ is assumed to have a utility function $u_i$. A coalition is a subset of the users that have agreed to cooperate with each other at the exclusion of the other nodes that do not belong to the coalition. One of the main assumptions in this paper is that coalition structures are fluid; namely, a user may choose to break off from a coalition and join another coalition, in order to improve its utility. In cooperative game theory, such contracts are said to be non-binding [12]. One of the major foci of this paper is the determination of network solutions that are stable in the face of such fluidity.

The mathematical description of a cooperative game is a specification of what utilities are achievable by each coalition. We assume that the utility of every node is modelled separately, and that utility from one node cannot be transferred to another. In game theory, such games are called Non-Transferable-Utility (NTU) cooperative games. (Each node is taken to be a player of this game.) The following definition (adapted to this setting from [12]) formalizes this concept. Throughout the paper, we denote the set of real numbers by $\mathbb{R}$, the set of non-negative real numbers by $\mathbb{R}^+$, and the set of positive real numbers by $\mathbb{R}^+$. We use $\subset$ to denote the subset of a set, which does not imply that it is a strict subset.

**Definition 1 (NTU cooperative game):** An NTU cooperative game $(\mathbb{N}, \nu)$ is a mapping $\nu : \mathcal{P}(\mathbb{N}) \to \Phi(\mathbb{R}_N^\mathbb{N})$, where $\mathbb{N}$ is the set of players, $N$ is the cardinality of $\mathbb{N}$, $\mathcal{P}(\mathbb{N})$ is the power set of $\mathbb{N}$, and $\Phi(\mathbb{R}_N^\mathbb{N})$ is the set of subsets of $\mathbb{R}_N^\mathbb{N}$ such that for every coalition $S \subset \mathbb{N}$, (1)
\( u \in \nu(S) \Rightarrow u_i = 0 \forall i \notin S \), (2) \( \nu(S) \) is a non-empty, closed, convex subset of \( \mathcal{R}^N_+ \), and (3) \( \{ u \in \mathcal{R}^N_+ \mid u \in \nu(S) \text{ and } u_i \geq \nu_i \forall i \in S \} \) is a bounded subset of \( \mathcal{R}^N_+ \), where \( \nu_i \overset{\text{def}}{=} \max\{u_i \mid u \in \nu(\{i\})\} < +\infty \forall i \in \mathbb{N} \).

That is, an NTU cooperative game assigns to each coalition \( S \) a set of achievable utilities \( \nu(S) \). In our case, each coalition is conceived of as a network by itself, with a certain set of utilities for that coalition, which the coalition can achieve when it operates on its own. (The extension to interference-limited networks is discussed in Section V.) In the simple example of Fig. 2, assume for now that the utility of a node is equal to the sum of the amount of traffic it sends as a source and that it receives as a destination. The \( \nu(S) \) for each \( S \) is shown in Fig. 3. Note that each \( \nu(S) \) is non-empty, closed, and convex, as required by Definition 1, and that the set of achievable utilities of each coalition is bounded since the energy supply of each node is bounded.

We now begin the description of an energy-limited network. Each node \( i \) is assumed to have a bounded supply of energy \( E_i \). Each wireless link \( ij \) has an associated energy-per-bit cost \( C_{ij} \) required to transmit along that link. The “demand matrix” \( \Gamma \) specifies the set of source-destination node pairs that wish to send traffic to each other: \( \Gamma_{mn} = 1 \) if node \( m \) wishes to send traffic to \( n \), and \( \Gamma_{mn} = 0 \) otherwise. This takes as special cases, the scenarios with a single sink, several sinks, as well as peer-to-peer communications. We formalize this description as follows:

**Definition 2 (Energy-limited network):** An energy-limited network \( G \) is an ordered quadruple \( (\mathbb{N}, \mathbb{C}, \mathbb{E}, \Gamma) \) where \( \mathbb{N} \) is the node set, \( \mathbb{C} \in \mathcal{R}^{N\times N}_+ \) is the “energy-per-bit cost matrix”, \( \mathbb{E} \in \mathcal{R}^N_+ \) is the vector of node energies, and \( \Gamma \in B^{N\times N} \) is the demand matrix.

In our network model below, we assume that the transmission times of the nodes have been scheduled in a non-interfering fashion, through the techniques discussed in Section I. We use a flow-based model for the traffic on the network, and the amount of traffic actually transferred end-to-end from \( m \) to \( n \) is denoted by \( d^{(m,n)} \). In this paper, we assume that the utility of a node is a positive, linear function of the number of bits that the node sends a source to desired destinations, and the number of bits that it receives as a destination from desired sources. (We discuss the effects of a concave utility model later.)

**Definition 3 (Energy-limited network model):** An energy-limited network model \( M_G \) on an
energy-limited network $G$ is the set of the following variables and constraints: $d^{(m,n)}$ denotes the total number of bits delivered from source $m$ to source $n$, and $x^{(m,n)}_{ij}$ the total number of bits from source $m$ to destination $n$ that are transmitted on the arc $ij$. (We refer to $x^{(m,n)}_{ij}$ as “flow of commodity $(m, n)$ on arc $ij$” even though it has the units of number of bits.) We let $u_i$ denote the utility of node $i$.

1) Flow Constraints:

   - **Inflow:** $\forall m \in \mathbb{N}, \forall i \in \mathbb{N}: \sum_{j \in \mathbb{N}\setminus\{i\}} x^{(m,i)}_{ji} = d^{(m,i)}$
   - **Outflow:** $\forall n \in \mathbb{N}, \forall i \in \mathbb{N}: \sum_{k \in \mathbb{N}\setminus\{i\}} x^{(i,n)}_{ik} = d^{(i,n)}$
   - **Balance:** $\forall i \in \mathbb{N}, \forall l \in \mathbb{N}, l \neq i, \forall p \in \mathbb{N}, p \neq i: \sum_{j \in \mathbb{N}\setminus\{i\}} x^{(l,p)}_{ji} = \sum_{k \in \mathbb{N}\setminus\{i\}} x^{(l,p)}_{ik}$
   - **No Self-Shipment:** $\forall j \in \mathbb{N}, \forall m \in \mathbb{N}, \forall i \in \mathbb{N}: x^{(m,i)}_{ij} = 0$

2) Node Energy Constraints: $\forall i \in \mathbb{N}: \sum_{m \in \mathbb{N}} \sum_{n \in \mathbb{N}} \sum_{k \in \mathbb{N}} C_{ik} x^{(m,n)}_{ik} \leq E_i$

3) Traffic Model Constraints: $\forall (m, n)$ for which $\Gamma_{mn} = 0: d^{(m,n)} = 0$

4) Non-negativity Constraints: $\forall (i, j, m, n) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} \times \mathbb{N}: x^{(m,n)}_{ij} \geq 0$

5) Linear Node Utility Model: $\forall i \in \mathbb{N}$:

   $u_i = \sum_{m \in \mathbb{N}\setminus\{i\}} \alpha^{(m)}_{i} d^{(m,i)} + \sum_{n \in \mathbb{N}\setminus\{i\}} \beta^{(n)}_{i} d^{(i,n)}$  

   for non-negative parameters $\alpha^{(m)}_{i}, \beta^{(n)}_{i}$ defined $\forall (i, m, n) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}$.

Let $x \in \mathbb{R}_{+}^{N \times N \times N \times N}$ be the network flow variables, $d \in \mathbb{R}_{+}^{N \times N}$ be the traffic variables, and $u \in \mathbb{R}_{+}^{N}$ be the utility variables, all expressed in matrix form. (We call $d$ the “traffic matrix”.) An ordered triple $(x, d, u)$ is said to be “feasible” in $M_G$ if it satisfies the above constraints. A flow $x$ is said to be feasible in $M_G$ if it satisfies the above constraints. (Note that $d$ and $u$ are free variables.) A utility vector $u$ is said to be feasible in $M_G$ if there exists a $(x, d, u)$ that is feasible in $M_G$. The set of all utility vectors feasible in $M_G$ is called the “feasible set” or “achievable set” of $M_G$.

**Definition 4 (Coalitional network model):** A coalitional network model $M^S_G$ for coalition $S$, defined on the network model $M_G$, is the network model $M_G$ with the additional constraint $x^{(m,n)}_{ij} = 0 \ \forall (i, j, m, n) \notin S \times S \times S \times S$. (Note that $M^N_G = M_G$.)

That is, a coalition cannot ship any bits on any link between the nodes that do not belong to that coalition.
Definition 5 (Energy-limited network game): An energy-limited network game is an NTU cooperative game in which each node is a player, and the mapping \( \nu \) is defined on an energy-limited network model \( M_G \) and its set of coalitional network models as \( \nu(S) = \{ u \in \mathcal{R}^N_+ \mid u \text{ is feasible in } M^S_G \} \) for every \( S \subset \mathbb{R} \).

III. Core Capacity Region

In this paper, we focus on the core as our central solution concept. The core capacity region is the set of Pareto-optimal utility vectors of the entire network such that no subset of the node set can achieve a better utility for all of its own members than in this utility vector. In Fig. 3, the core capacity region is the set of utilities that lie on the closed segment between \( (2, 6) \) and \( (7, 1) \). Note, for example, that \((0, 8)\) is not in the core capacity region since \( \{1, 3\} \) can move away from that solution by using direct transmission to improve the utility of both of its members. However, the existence of a direct link is not a prerequisite for creating alternative solutions. Fig. 4 displays a 6-node network, and Fig. 5 displays the feasible set \( \nu(S) \) of each coalition, and the Pareto-optimal set of the grand coalition. (We have placed the utilities of a source and its destination on the same axis since they are identical in this example.) Note that the feasible set of the grand coalition indeed subsumes the feasible sets of all of the smaller coalitions. (The grand coalition can achieve this by choosing not to operate the nodes outside a given smaller coalition.) However, the main problem is whether the grand coalition can provide solutions to all of the smaller coalitions such that none of the smaller coalitions can move to a better solution for all of its own members. In Fig. 5, geometrically, we are looking for the Pareto-optimal utility vectors of the grand coalition such that their projections onto the coalitional utility planes do not fall in the interiors of the feasible sets of these coalitions. If the projection falls in the interior of the feasible set of some coalition, then that coalition, if it detects this problem, can improve the utility of each of its members by collapsing the current solution and picking instead a utility vector (in its own Pareto-optimal set) that is better for all of its own members. In Fig. 5, the only utility vector that cannot be collapsed in this way is \((1, 1, 1)\); hence, the core capacity region consists of a single utility vector in this case. Based on the mathematical framework of the previous section, we define the core capacity region formally as follows:
Definition 6 (Core capacity region): The core capacity region of an energy-limited network game with network model $M_G$ is the set $\{ u \in \mathbb{R}^N_+ | u$ is feasible in $M_G$, and for every $S \subset \mathbb{N}$ and every utility vector $u' \in \mathbb{R}^N_+$, the following holds: $u'_i > u_i \ \forall i \in S \Rightarrow u'$ is not feasible in $M^S_G \}$.

From a network survivability perspective, core solutions make sense because they prevent nodes’ energy supplies from being depleted unnecessarily by the other nodes. However, this is not done out of fairness, but rather by comparison to the next best alternatives available to the nodes. In Fig. 2, if we remove the direct link between each source and its destination, then the entire set of Pareto-optimal utilities becomes the core, since the smaller groups have now lost their next best alternatives. The interpretation of this from a sensor network design perspective is that when side information on the Joules-per-bit costs of alternative links are available (or can be estimated), this information may help design network solutions that are robust and rational from the perspectives of groups of users. In this context of side information, the core solutions can be interpreted as the set of solutions that cannot be collapsed by any group of users even if every group had perfect side information on all of the Joules-per-bit link costs.

Perfect side information on the link costs is not available for large-scale networks that exercise topology control to limit the transmit energy consumption. However, because of the distance-dependent path loss, the very long links that incur a high transmit energy consumption would not provide low-cost alternatives to relaying solutions, under the one-to-one traffic model in which each node wishes to send bits to a far destination: As shown in [20], the bits-per-Joule capacities of sparse topologies that result from topology control approximate well the bits-per-Joule capacity of the complete topology in this case. Hence, practical algorithms that attempt to find core solutions under the one-to-one traffic model in randomly deployed networks may approximate these solutions by working directly on these sparse, locally-determined topologies.

In the example of Fig. 2, the core turned out to be non-empty. The main question is whether for all energy-limited networks, a core solution can be found. We show that at least one such core solution exists for any energy-limited network game. (The proofs of all of the theorems appear in Appendix B.)

Theorem 1 (Non-emptiness): The core capacity region of an energy-limited network game is
non-empty.

Even though this result was obtained under the linear utility model, it holds true under the concave utility model as well, by following steps similar to the ones in the proof. This has been omitted to present the proof in the most comprehensible setting. This theorem is a general result that states that no matter how many nodes the network has, there always exists a core solution that will be rational from the perspective of every coalition. Hence, if the grand coalition either provides this solution, or the network algorithms converge to this solution, then the resulting allocation is stable even under perfect side information on the Joules-per-bit costs of all of the links. When this side information is imperfect, a core capacity region can be defined under imperfect side information and will be larger than this one since some of the constraints that come from the bounds of what is achievable by every coalition will be removed in that case. Hence, the non-emptiness of the core is assured under imperfect side information as well. (We discuss the extension to interference-limited networks in Section V.)

In Appendix A, we describe an algorithm by which core solutions can be computed in general. In this section, we examine a specific scenario in order to gain more insight about the core. We consider the many-to-one traffic model in which every sensor node wishes to transfer bits to a single collection site. The collection site is taken to be node 1, and each node $i \neq 1$ derives utility only from its bits delivered to the collection site; that is, its utility is given by $u_i = \beta_i^{(1)} d^{(i,1)}$. We wish to determine the “core topologies” that would result when we constrain the transmission strategies to lie in the core. Surprisingly,

**Theorem 2 (Core under many-to-one traffic model):** The only utility vector in the core capacity region under the many-to-one traffic model is the one achieved by every node’s transmitting directly to the collection site.

The intuition behind this result is as follows: since the nodes are assumed to have perfect side information on the link costs, the node that has the lowest link cost to the collection site (e.g. the node closest to the collection site) reasons that no other node can help it, and thus it will devote its entire energy to transfer its own bits to the collection site. Hence, this node is out of the game, and the same reasoning is applied to the next node with the lowest cost to the collection site. Repeating this for all the nodes shows that the only core solution is the direct
transmission topology.

There are some caveats to this result: first, when every node is transmitting directly to the collection site, the essentially interference-free, delay-tolerant network assumption may no longer hold, since many network dimensions are being consumed by direct transmissions over long links. In this case, the effects of interference in the utility set $\nu(S)$ of each coalition may need to be modeled in deriving the core solutions. Second, our network model does not utilize the broadcast nature [3] of the wireless channel, and core solutions that employ relaying may be possible if it is utilized. Third, mobility is not taken into account. In fact, we have shown [21] that mobility may induce relaying within the core under the many-to-one traffic model.

From an implementation perspective, we see in the stationary case that the core solutions that transmit on such long links would quickly deplete the energies of the nodes that are far from the collection site. In order to design better solutions, we need to consider core solutions under virtual prices in order to achieve more desirable allocations. For example, if the node closest to the collection site could signal a price for the usage of its energy resources, then solutions that use relaying would indeed be obtained. In this case, the main question is whether the core capacity region would still be non-empty under such virtual pricing in sensor networks. We address this in the next section.

IV. Core Capacity Region under Pricing Systems

In this section, we describe a pricing system in which every node $i$ has the capability to “charge” any other node $\$p_i$ per Joule of energy that node $i$ expends on the other node’s traffic. These prices are not real money transfers but rather a distributed signaling system to lead to efficient, stable allocations. Let $\gamma^{(i,n)}$ denote the fraction of the price of transmission that node $i$ will be assumed to pay in order to send traffic destined to node $n$, and let $\delta^{(i,n)}$ denote the fraction of the price of transmission that node $n$ will be assumed to pay in order to receive traffic that originates from node $i$ and is destined for node $n$. We assume that the end parties share the total virtual price of any bit transmitted on the network; that is, $\gamma^{(i,n)} + \delta^{(i,n)} = 1$. Then, under the requirement that $u_i \geq 0 \ \forall i \in \mathbb{N}$, we extend the utility of a node to:

$$u_i = p_i \sum_{m \in \mathbb{N} \setminus \{i\}} \sum_{n \in \mathbb{N} \setminus \{i\}} \sum_{k \in \mathbb{N}} C_{ik} x_{ik}^{(m,n)} - \sum_{j \in \mathbb{N} \setminus \{i\}} \sum_{n \in \mathbb{N} \setminus \{i\}} \sum_{k \in \mathbb{N}} \gamma^{(i,n)} p_j C_{jk} x_{jk}^{(i,n)}$$

(2)
\[
- \sum_{j \in \mathbb{N}\{i\}} \sum_{m \in \mathbb{N}\{i\}} \sum_{k \in \mathbb{N}} \delta^{(m,i)} p_j C_{jk} x^{(m,i)}_{j,k} + \sum_{m \in \mathbb{N}\{i\}} \alpha^{(m)}_i d^{(m,i)} + \sum_{n \in \mathbb{N}\{i\}} \beta^{(n)}_i d^{(i,n)}
\]

for non-negative parameters \(\alpha^{(m)}_i, \beta^{(n)}_i\) defined \(\forall (i, m, n) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}\), and the transmit energy cost matrix \(C\). The variables are the prices \(p_i \geq 0 \forall i \in \mathbb{N}\) and the flow variables \(x^{(m,n)}_{ij} \geq 0 \forall (i, j, m, n) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} \times \mathbb{N}\). The interpretation of the terms in (2) is as follows: The first term in (2) is the “revenue” raised by node \(i\), the second term is the payment that \(i\) makes as a source node, the third term is the payment that \(i\) makes as a destination node, the fourth term is the worth to node \(i\) of the bits that \(i\) receives as a destination, and the last term is the worth to node \(i\) of the bits that it sends as a source. Here, \(\alpha^{(m)}_i\) is the worth to node \(i\) of every bit that \(i\) receives from \(m\), and \(\beta^{(n)}_i\) is the worth to node \(i\) of every bit that \(i\) sends to node \(n\). Then,

**Theorem 3 (Core capacity region under pricing):** The core capacity region is non-empty under pricing.

That is, even when each coalition can set up its own pricing system (following the utility function in (2) and using its entire set of price vectors) to create competing solutions, the grand coalition can offer network solutions, which use pricing and which cannot be collapsed by smaller coalitions. The proof of this result uses the fact that the sum of the utilities is invariant under pricing (since any virtual payment made by a node constitutes revenue for some other set of nodes).

In the example of Fig. 2, when pricing is introduced, the prices for core solutions can be shown to satisfy \(1/8 \leq p_2 \leq 3/4\) and \(1/4 \leq p_3 \leq 7/8\); however, the core capacity region under pricing turns out to be identical to that in Fig. 3 because in this example, the entire core capacity region in the absence of pricing already achieves the maximum unconstrained \(\sum_{i \in \mathbb{N}} u_i\). In contrast, in Fig. 5, even though the \(\{\nu(S)\}_{S \neq \emptyset}\) do not change under pricing, the Pareto-optimal boundary of \(\nu(\mathbb{N})\) is enlarged (towards the axial corners) to \(\{u \in \mathbb{R}^3_+ | u_{1,2} + u_{3,4} + u_{5,6} = 3\}\); however, the core capacity region still remains equal to that in the absence of pricing in this case. In contrast, Fig. 6 shows a network with two sensor nodes under the many-to-one traffic model, and Fig. 7 shows the core capacity region of this example with and without pricing. Without pricing, \((2, 2)\) is the only core utility vector and is achieved by direct transmissions to the collection site. Under pricing, core solutions require that \(1/2 \leq p_2 \leq 3/4\). Here, \(p_2 = 1/2\) corresponds to the
case where node 2 charges the minimum price to recoup its opportunity cost of relaying. This produces the utility vector $(2, 3)$, which is an extreme point of the core capacity region. The price $p_2 = 3/4$ corresponds to the case where node 2 charges the maximum price that can be borne by node 3 (after which node 3 would switch to direct transmission). This produces the utility vector $(3, 2)$, which is the other extreme point of the core capacity region. Hence, this example shows that pricing may induce relaying within the core capacity region, in particular for the many-to-one traffic model where no such core solutions that use relaying exist without pricing.

V. EXTENSION TO INTERFERENCE-LIMITED NETWORKS

In this section, we discuss the extension of this work to interference-limited networks. A key feature of our model has been that when each coalition operates on its own, its utility set is not affected by the nodes outside the coalition. If the network were interference-limited, the effects of interference on achievable utilities would need to be modelled. Such games would exhibit a complex hybrid structure of competition between coalitions for shared resources (such as the available bandwidth) and cooperation within coalitions. The core capacity region could still be defined as an equilibrium concept for such networks; however, the concept of the achievable utility set $\nu(S)$ of a coalition $S$ must be generalized. One possible generalization is as follows. Note that for each transmission strategy $\sigma(S)$ of $S$ co-existing with a transmission strategy $\sigma(\mathcal{N} \setminus S)$, there exists a set of utilities that are jointly achievable by the members of $S$ and $\mathcal{N} \setminus S$. Then, a core transmission strategy $\sigma^{(\text{core})}(\mathcal{N})$ can be taken to be one such that for every $S \subset \mathcal{N}$, neither $S$ nor $\mathcal{N} \setminus S$ can achieve a better utility for all of its own members over the entire set of joint transmission strategies than the one achieved under this transmission strategy $\sigma^{(\text{core})}(\mathcal{N})$. Note that the solutions that would drive up the transmit power levels of both $S$ and $\mathcal{N} \setminus S$ and thus radically shrink the utility set of each are quickly ruled out as being outside the core; hence, such core equilibria are indeed dramatically different from the Nash equilibria of non-cooperative games. However, it must still be examined whether the core will be non-empty in this case, and distributed algorithms must be designed to drive toward such core solutions.
APPENDIX A: CORE ALGORITHM

In this appendix, we present a centralized algorithm that computes a utility vector in the core capacity region of an energy-limited network. First, we review the following definition of a Transferable-Utility (TU) game [12] from cooperative game theory:

**Definition 7 (TU cooperative game):** A TU cooperative game \((\mathbb{N}, \tilde{\nu})\) is a mapping \(\tilde{\nu} : \mathcal{P}(\mathbb{N}) \to \mathbb{R}_+\), where \(\mathbb{N}\) is the set of players, and \(\mathcal{P}(\mathbb{N})\) is the power set of \(\mathbb{N}\).

That is, a TU cooperative game is a game where the payoff to each coalition is given, not by the individual utilities of the players in that coalition, but rather by a single real number, which may, for example, be the sum of the utilities of the players. The core of a TU game \((\mathbb{N}, \tilde{\nu})\) is defined as \(\{u \in \mathbb{R}^N_+ | \sum_{i \in \mathbb{N}} u_i = \tilde{\nu}(\mathbb{N}), \text{ and } \sum_{i \in S} u_i \geq \tilde{\nu}(S) \forall S \subset \mathbb{N}, S \neq \mathbb{N}\}\).

The geometric idea behind finding a utility vector in the core capacity region is as follows: We want to find a point on the Pareto-optimal boundary of \(\nu(\mathbb{N})\) such that when we project that point onto any of the coalitional subspaces, the projection does not fall in the interior of \(\nu(S)\) for any \(S \neq \mathbb{N}\). We solve this problem as follows: Let \(\lambda \in \mathbb{R}^N_+\) and \(\lambda > 0\) be given. We seek to find a relationship between our NTU game and a derived TU game. Consider the TU game that assigns to each \(S\) the value \(\sum_{i \in S} \lambda_i u_i\). Let \(\tilde{\nu}_\lambda(S)\) be the maximum of this value over \(\nu(S)\).

The key observation is that, if there is a Pareto-optimal utility vector \(u \in \nu(\mathbb{N})\) that achieves at least \(\tilde{\nu}_\lambda(S)\) for every \(S \subset \mathbb{N}, S \neq \mathbb{N}\), then \(u\) must be in the core of the NTU game. That such a vector \(u\) always exists follows from the fact that the cores of all of the coalitional games of the derived TU game are non-empty. We now present the precise statement of the algorithm, which we call the “core algorithm”.

**Algorithm 1 (Core Algorithm):** Fix \(\lambda \in \mathbb{R}_+^N\). For every \(S \subset \mathbb{N}\), define \(\tilde{\nu}_\lambda(S) \defeq \max_{u \in \nu(S)} \sum_{i \in S} \lambda_i u_i\). Compute \(\tilde{\nu}_\lambda(S)\) for every \(S \neq \mathbb{N}\). Then, solve the linear program:

\[
\tilde{\nu}_\lambda^{\text{core}}(\mathbb{N}) \defeq \max_{u \in \nu(\mathbb{N})} \sum_{i \in \mathbb{N}} \lambda_i u_i \quad \text{subject to} \quad \sum_{i \in S} \lambda_i u_i \geq \tilde{\nu}_\lambda(S), \forall S \subset \mathbb{N}, S \neq \mathbb{N} \tag{3}
\]

The simplicity of this algorithm, which exploits the network structure of the game, contrasts sharply with the general procedures designed to compute the core of NTU cooperative games, such as Scarf’s algorithm [22], and the generalization [23] to a path-following method for finitely polyhedral NTU games.

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Theorem 4 (Correctness): Let $\lambda > 0$. Algorithm 1 computes a $(\tilde{x}, \tilde{u})$ such that $\tilde{u}$ is in the core capacity region of the energy-limited network game.

The proof of this theorem will be presented in Appendix B, after sufficient tools have been developed. Although the number of constraints in the core algorithm appears to be exponential in the number of nodes, the relevant number of constraints can be empirically much smaller. For example, for the Gupta-Kumar one-to-one traffic model [24] with traffic scheduled in a multi-hop fashion to far-away destinations, it suffices to take into account only the large coalitions (of which there are only a polynomial number in the number of nodes) in the application of the constraints in (3).

APPENDIX B

In this appendix, we present the proofs of the theorems in this paper. We first review two definitions from cooperative game theory [25]:

Definition 8 (Balanced family): A balanced family $\mathcal{B}$ of coalitions in a cooperative game is a collection of non-empty coalitions of $\mathbb{N}$ such that for each coalition $S' \in \mathcal{B}$ there exists a scalar $\delta(S')$ with the property that $0 \leq \delta(S') \leq 1$ and $\forall i \in \mathbb{N}$, $\sum_{S' \in \mathcal{B}_i} \delta(S') = 1$ where $\mathcal{B}_i \overset{\text{def}}{=} \{ S \in \mathcal{B} | i \in S \}$.

In the development below, we will use the following notation: For any given vector $u \in \mathbb{R}^N_+$ and any coalition $S \subset \mathbb{N}$, we define $u_S$ component-wise as follows: $(u_S)_i \overset{\text{def}}{=} u_i$ if $i \in S$ and $(u_S)_i \overset{\text{def}}{=} 0$ otherwise.

Definition 9 (Balanced NTU game): Let $(\mathbb{N}, \nu)$ be an NTU cooperative game such that $\nu(S)$ is closed in $\mathbb{R}^N_+$ for every $S \subset \mathbb{N}$. The NTU cooperative game $(\mathbb{N}, \nu)$ is said to be balanced, if for every balanced family $\mathcal{B}$ of coalitions of $\mathbb{N}$, $\{ u \in \mathbb{R}^N_+ \text{ and } u_S \in \nu(S) \text{ for every } S \in \mathcal{B} \} \Rightarrow u \in \nu(\mathbb{N})$.

Proof of Theorem 1: The Bondereva-Scarf Theorem [25] states that a balanced NTU game has a non-empty core. We will show that this network game is balanced. Then, by the Bondereva-Scarf Theorem, it will follow that the core of this game is non-empty. In order to show that the game is balanced, we first note that $\nu(S)$ is closed in $\mathbb{R}^N_+$ $\forall S \subset \mathbb{N}$. Next, we let $\mathcal{B}$ be a balanced family of coalitions of $\mathbb{N}$, and let $u \in \mathbb{R}^N_+$ be such that $u_S \in \nu(S) \forall S \in \mathcal{B}$. We must show that
u ∈ ν(ℵ). The main idea of the proof is as follows: For any \( S ∈ B \), \( u_S ∈ ν(S) \) implies that there exists a \( (x^{(S)}, d^{(S)}, u_S) \) feasible in \( M_G^S \). If we can construct a \( (x, d, u) \) feasible in \( M_G \) for some \( (x, d) \), then we will have shown that \( u ∈ ν(ℵ) \). We will construct such an \( x \) by superposing the \( \{x^{(S)}\}_{S ∈ B} \) using the balancing scalars of \( B \) for the weights in the superposition. We will show that the \( x \) constructed this way (A) is a feasible flow in \( M_G \), and (B) achieves the utility vector \( u \). This will imply that \( u \) is feasible in \( M_G \). To this end, let \( x \triangleq \sum_{S ∈ B} \delta^{(S)} x^{(S)} \). To show (A), we first show that \( x \) satisfies the node energy constraint of the flow-based network model:

\[
\sum_{m \in ℵ} \sum_{n \in ℵ} \sum_{k \in ℵ} C_{ik} x_{ik}^{(m,n)} = \sum_{m \in ℵ} \sum_{n \in ℵ} \sum_{k \in ℵ} C_{ik} \sum_{S ∈ B} \delta^{(S)} \cdot (x^{(S)})_{ik}^{(m,n)} \tag{4}
\]

\[
= \sum_{S ∈ B_i} \delta^{(S)} \sum_{m \in S} \sum_{n \in S} \sum_{k \in ℵ} C_{ik} \cdot (x^{(S)})_{ik}^{(m,n)} \tag{5}
\]

\[
\leq E_i \sum_{S ∈ B_i} \delta^{(S)} \tag{6}
\]

\[
= E_i \tag{7}
\]

This shows that \( x \) is a feasible flow in \( M_G \). Now, to show (B), let \( \tilde{d} \) be the traffic matrix and \( \tilde{u} \) be the utility vector achieved by the flow \( x \). Then,

\[
\tilde{u}_i = \sum_{m \in ℵ} \alpha_i^{(m)} \tilde{d}^{(m,i)} + \sum_{n \in ℵ} \beta_i^{(n)} \tilde{d}^{(i,n)} \tag{8}
\]

\[
= \sum_{m \in ℵ} \alpha_i^{(m)} \sum_{j ∈ ℵ} x_{ji}^{(m,i)} + \sum_{n \in ℵ} \beta_i^{(n)} \sum_{k \in ℵ} x_{ik}^{(i,n)} \tag{9}
\]

\[
= \sum_{m \in ℵ} \alpha_i^{(m)} \sum_{j ∈ ℵ} \sum_{S ∈ B} \delta^{(S)} (x^{(S)})_{ji}^{(m,i)} + \sum_{n \in ℵ} \beta_i^{(n)} \sum_{k \in ℵ} \sum_{S ∈ B} \delta^{(S)} (x^{(S)})_{ik}^{(i,n)} \tag{10}
\]

\[
= \sum_{S ∈ B_i} \delta^{(S)} \left( \sum_{m \in S} \alpha_i^{(m)} \sum_{j ∈ S} (x^{(S)})_{ji}^{(m,i)} + \sum_{n \in S} \beta_i^{(n)} \sum_{k \in ℵ} (x^{(S)})_{ik}^{(i,n)} \right) \tag{11}
\]

\[
= \sum_{S ∈ B_i} \delta^{(S)} (u_S)_i \tag{12}
\]

\[
= u_i \sum_{S ∈ B_i} \delta^{(S)} \tag{13}
\]

\[
= u_i \tag{14}
\]

This shows that the utility vector achieved by \( x \) is precisely \( u \). Hence, \( u ∈ ν(ℵ) \), which completes the proof.

We will use the following lemma to prove Theorem 2:
Lemma 1 (Maximum Demand): Let \((x^*, d^*, u^*)\) be the solution achieved by direct transmissions to the collection site on \(M_G^S\) for a given coalition \(S\), under the many-to-one traffic model. Then \((x^*, d^*, u^*)\) is the optimal solution to the linear program: 
\[
\max_{(x,d,u)} \sum_{i \in S} d(i,1) \text{ subject to } u \in \nu(S).
\]
Proof: By way of contradiction, let \((x, d, u)\) be an optimal solution of the program and assume that \(x \neq x^*\). The fact that \((x, d, u)\) is optimal implies that \(d\) is a Pareto-optimal traffic vector in \(M_G^S\). Since the set of feasible traffic vectors is closed, bounded and convex, and the objective function is linear, an optimal solution that is also a local maximum must exist. We will show that \(d\) is not a local maximum; hence, \(d\) is not optimal, and we will conclude that \((x^*, d^*, u^*)\) must be the optimal solution. To show that \(d\) is not a local maximum, order the nodes as \(k_1, k_{NS}\) where \(NS\) is the cardinality of \(S\), in increasing transmit energy cost to the collection site. Since \(x \neq x^*\) and \(d\) is Pareto-optimal, there exists a pair of nodes \((l, m)\) such that \(x_{l1}^{\text{m,1}} = x_{nl}^{\text{m,1}} > 0\) for some node \(n\) (and \(n = m\) is possible). Let \(E^*\) be the energy that \(l\) devotes to the part of \(m\)’s traffic that it sends directly to the collection site. Now, if \(l\) used this energy instead, for its own traffic to the collection site, it would be able to transmit just as many bits of its own traffic with it. This would release a positive energy, say \(E_i^*\), at \(n\), while keeping \(\sum_{i \in S} d(i,1)\) constant. Then, \(n\) could use this released energy to transmit an extra \(\frac{E_i^*}{C_{nl}}\) bits to the collection site, hence increasing \(\sum_{i \in S} d(i,1)\) by \(\frac{E_i^*}{C_{nl}}\) bits. This shows that \(d\) is not an optimal traffic vector, which completes the proof.

Proof of Theorem 2: Let \(u^* \in \mathcal{R}_+^N\) be the utility vector achieved by direct transmissions to the collection site, and let \(C\) denote the core capacity region of \(M_G\). We must show that (A) \(u^* \in C\), and (B) \(\{u' \in \mathcal{R}_+^N \text{ and } u' \neq u^*\} \Rightarrow u' \notin C\). To show (A): From Lemma 1, it follows that \(u^*\) is Pareto-optimal in \(M_G\). To show that \(u^*\) is in the core of the network game, we must show that \(\forall S \subset \mathcal{R}, S \neq \mathcal{R}, \text{ there exists no } u' \in \nu(S) \text{ with } u'_i \geq u^*_i \forall i \in S \text{ and } u'_j > u^*_j \text{ for some } j \in S\). By way of contradiction, assume that such a pair \((S, u')\) exists. Then, since \(u_i = \beta_i^{(1)} d(i,1)\) \(\forall i \in \mathcal{R} \setminus \{1\}\) under the many-to-one traffic model, it follows that \(\sum_{i \in S} d'_i > \sum_{i \in S} d^*_i\), where \(d'\) is a traffic vector that corresponds to \(u'\). But this contradicts Lemma 1. Hence, \(u^* \in C\). To show (B): Let \(u' \in \mathcal{R}_+^N\) be feasible in \(M_G\). If \(u'_i < u^*_i\) for some \(i \in \mathcal{R} \setminus \{1\}\), then \(u'\) can be blocked by the coalition \(\{i, 1\}\) and hence \(u' \notin C\) in this case. Then, by (A), it follows that \(u^*\) is the only
utility vector in the core capacity region, which completes the proof.

The following definition [25] from cooperative game theory will be used to prove Theorem 4.

**Definition 10 (Totally Balanced TU game):** A TU cooperative game \((N, \tilde{\nu})\) is said to be totally balanced if for every \(S' \subset N\), the following holds: For every balanced family \(B\) of coalitions of \(S'\), \(\sum_{S \in B} \delta(S) \tilde{\nu}(S) \leq \tilde{\nu}(S')\).

**Proof of Theorem 4:** We will prove that (A) an optimal solution to the program in (3) exists, and that (B) this optimal solution is in the core capacity region. In order to show Part (A) of the proof, we will prove that this TU game is totally balanced. Then, it will follow that the program in (3) is feasible. Since the optimization maximizes a linear function over a compact, convex feasible set, it will follow that an optimum \((\tilde{x}, \tilde{u})\) exists.

In order to show that the TU game \((N, \tilde{\nu}_\lambda)\) is totally balanced, fix \(S' \subset N\), and let \(B\) be a non-empty balanced family of coalitions of \(S'\), with balancing scalars \(\delta(S)\) defined for each \(S \in B\). Let \(u^{(S')} \in \mathcal{R}^N_+\) be a utility vector in \(\nu(S')\) that achieves \(\tilde{\nu}_\lambda(S')\). Then, \(\tilde{\nu}_\lambda(S') = \sum_{i \in S'} \lambda_i u_i^{(S')}\). For each \(S \in B\), let \(u(S) \in \mathcal{R}^N_+\) be a utility vector in \(\nu(S)\) that achieves \(\tilde{\nu}_\lambda(S)\). Then \(\tilde{\nu}_\lambda(S) = \sum_{i \in S} \lambda_i u_i^{(S)}\). Define \(\hat{u} \in \mathcal{R}^N\) component-wise as \(\hat{u}_i = \sum_{S \in B} \delta(S) u_i^{(S)}\) for every \(i \in S'\). Let \(x^{(S)}\) be a flow that is feasible in \(\nu(S)\) and achieves \(u^{(S)}\). Let \(x \triangleq \sum_{S \in B} \delta(S) x^{(S)}\). Then, by following a sequence of steps similar to (4)-(7), we have \(\sum_{m \in S'} \sum_{n \in S'} \sum_{k \in S'} C_{ik} x_{ik}^{(m,n)} \leq E_i\). This shows that \(x\) is a feasible flow in \(M^{S'}_G\). Now, let \(\bar{d}\) be the traffic matrix and \(\hat{u}\) be the utility vector achieved by the flow \(x\). Then, by following a sequence of steps similar to (8)-(14), we have \(\tilde{u}_i = \hat{u}_i\). This shows that the utility vector achieved by \(x\) is precisely \(\hat{u}\). Hence, \(\hat{u} \in \nu(S')\). Then, \(\sum_{S \in B} \delta(S) \tilde{\nu}_\lambda(S) = \sum_{S \in B} \delta(S) \sum_{i \in S} \lambda_i u_i^{(S)} = \sum_{S \in S'} \lambda_i \sum_{S \in B} \delta(S) u_i^{(S')} = \sum_{i \in S'} \lambda_i \hat{u}_i \leq \sum_{i \in S'} \lambda_i u_i^{(S')} = \tilde{\nu}_\lambda(S')\). This shows that the TU game \((N, \tilde{\nu}_\lambda)\) is totally balanced and thus completes Part (A) of the proof.

Now, to show Part (B) of the proof, let \((x, u)\) denote an optimal solution to (3). Let \(C\) denote the core capacity region. Fix \(\lambda > 0\). Note that \(u\) is Pareto-optimal in \(M_G\). By way of contradiction, assume that \(u \notin C\). Then, there exists a coalition \(S \neq N\) and a \(u' \in \nu(S)\) such that \(u'_i > u_i \forall i \in S\). Then, \(\sum_{i \in S} \lambda_i u'_i > \sum_{i \in S} \lambda_i u_i \geq \tilde{\nu}_\lambda(S)\), where the last inequality follows from the fact that \(u\) is a feasible solution of (3). Then, \(u' \notin \nu(S)\), which is a contradiction. Hence, \(u \in C\). This completes the proof.
Proof of Theorem 3: Consider the program in (3) in the statement of Algorithm 1 with the restriction that $\lambda = 1$. Note that the sets $\{\nu(S)\}$ in Algorithm 1 denote the feasible sets of the coalitions under no pricing. This program is feasible as shown in the proof of Theorem 4; hence, it has an optimal solution $(x^*, u^*)$. Consider the set of solutions $(x^*, p \geq 0, u)$ under pricing with the flow matrix fixed as $x^*$. Note that $\forall S \subseteq \mathbb{N}$, $\sum_{i \in S} u_i$ is invariant with respect to $p$. The feasible set of utilities under pricing, denoted by $\nu^{(p)}(\mathbb{N})$, is bounded since $\sum_{i \in S} u_i$ is invariant and $\nu(\mathbb{N})$ is bounded, and $u \geq 0$ in our pricing system. Further, $\nu^{(p)}(\mathbb{N})$ is closed since no strict inequalities appear in the finite number of constraints on the variables. Since $u^* \in \nu(\mathbb{N})$, it follows that $u^* \in \nu^{(p)}(\mathbb{N})$ by taking $p = 0$. Because $\nu^{(p)}(\mathbb{N})$ is closed and bounded, and $u^* \in \nu^{(p)}(\mathbb{N})$, there exists a utility vector $\bar{u}$ that is Pareto-optimal in $\nu^{(p)}(\mathbb{N})$ such that $\bar{u}_i \geq u^*_i \forall i \in \mathbb{N}$. We will show that $\bar{u}$ is in the core capacity region under pricing. By way of contradiction, assume that there exists a $S \neq \mathbb{N}$ and a $(x^{(S)}, p^{(S)}, u^{(S)})$ that is feasible in $M_G^{(S)}$ under pricing such that $u^{(S)}_i > \bar{u}_i \forall i \in S$. Then, $\sum_{i \in S} u^{(S)}_i > \sum_{i \in S} \bar{u}_i \geq \sum_{i \in S} u^*_i \geq \bar{\nu}_1(S)$. However, $\sum_{i \in S} u^{(S)}_i = \sum_{i \in S} \bar{u}^{(S)}_i \leq \bar{\nu}_1(S)$, where $\bar{u}^{(S)}$ denotes the utility vector obtained with the flow matrix $x^{(S)}$ under no pricing. This contradiction establishes that $\bar{u}$ is in the core capacity region under pricing.

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Fig. 1. An example of a habitat monitoring network with a single collection site.
Traffic Model:
Node 3 wants to send to Node 1
Node 2 wants to send to Node 4

Traffic Model:
Node 3 wants to send to Node 1
Node 2 wants to send to Node 4

\[ C_{31} = 4 \]
\[ C_{21} = 1 \]
\[ C_{23} = 1 \]
\[ C_{32} = 1 \]
\[ C_{34} = 1 \]

\[ \alpha_2^{(1)} = \alpha_4^{(3)} = 1 \]
\[ \beta_1^{(2)} = \beta_3^{(4)} = 1 \]

\[ C_{24} = 8 \]
\[ E_2 = 8MJ \]
\[ E_3 = 8MJ \]

All link costs are in Joules per bit.

Fig. 2. A 4-node wireless energy-limited network topology.
Fig. 3. The feasible set and the core capacity region of the example of Fig. 2. The units of $u_i$ are Megautils.
Traffic Model:
Node 1 wants to send to Node 2
Node 3 wants to send to Node 4
Node 5 wants to send to Node 6

Each link consumes 1 Joule/bit.
Each of the nodes has 2 MJoules.

\[ S_1 = \{1, 2, 3, 4\} \]
\[ S_2 = \{3, 4, 5, 6\} \]
\[ S_3 = \{1, 2, 5, 6\} \]

\[ \alpha_2^{(1)} = \alpha_4^{(3)} = \alpha_6^{(5)} = 1 \]
\[ \beta_1^{(2)} = \beta_3^{(4)} = \beta_5^{(6)} = 1 \]

Fig. 4. A 6-node wireless energy-limited network topology. The links that are not shown are assumed to have infinite Joules-per-bit costs.
\[ \nu(S_1) = \{ u \mid u_{1,2} + u_{3,4} \leq 2 \} \]
\[ \nu(S_2) = \{ u \mid u_{3,4} + u_{5,6} \leq 2 \} \]
\[ \nu(S_3) = \{ u \mid u_{1,2} + u_{5,6} \leq 2 \} \]

(1,1,1) is the only vector in the core capacity region. Hexagonal planar surface is the Pareto-optimal set of the grand coalition.

Fig. 5. The feasible sets of the coalitions and the core capacity region for the example of Fig. 4. The units of \( u_i \) are Megautils.
Traffic Model:
Both Node 2 and Node 3 want to send to Node 1.

Collection Site (Node 1)  
Node 2  

\[ C_{21} = 2 \]

Node 2  

\[ C_{31} = 4 \]

\[ E_2 = 8MJ \]

Node 3  

\[ C_{32} = 1 \]

\[ E_3 = 8MJ \]

\[ \beta_2^{(1)} = \frac{1}{2} \]

\[ \beta_3^{(1)} = 1 \]

All link costs are in Joules per bit.

Fig. 6. Many-to-one traffic model with two energy-limited nodes.
Fig. 7. The feasible set and the core capacity region for the example of Fig. 6 with and without pricing. The units of $u_i$ are Megautils.