Final Report of EE359 Class Project

Throughput and Delay in Wireless Ad Hoc Networks

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Abstract:

Network throughput and packet delay are the two most important parameters to evaluate the performance of wireless ad hoc networks. Generally it is difficult to achieve both high throughput and low packet delay. In this project, the objective is to achieve high throughput while keeping the packet delay under certain threshold.

We will first look at the throughput capacity theoretically in mobile ad hoc networks. Gupta and Kumar [1] show the average available throughput per node decreases as $1/\sqrt{n}$ *or* $1/\sqrt{n \lg n}$ *in a static ad hoc network, where n is the number of nodes. That means, the total network capacity increases as at most* $\sqrt{n}$*. Furthermore, Grossglauser and Tse [2] show mobility can improve the capacity. However, delay is not guaranteed in their schemes. Actually delay will increase due to possibly more hops or queueing in order to increase the throughput.*

Bansal and Liu [3] show it is possible to achieve close-to optimal capacity while keeping the delay small. In their model, each sender can achieve an average throughput of $c\frac{W \min(m,n)}{n \log^3 n}$*, where W is the maximum available bandwidth, with the packet delay at most* $\frac{2d}{v}$*, where d is the diameter of the network and v is the velocity of the mobile nodes. Based on this fact, the authors propose a routing algorithm that achieves the objective through exploiting the patterns in the mobility of nodes. The throughput achieved by this algorithm is only a poly-logarithmic factor off from the optimal.*

1. Introduction

Wireless network is becoming more and more popular in nowadays. Comparing to the traditional wired network, wireless network set up the connections through wireless channel. Generally there are two kinds of wireless networks. One has a wired backbone network in which the base stations are the boundary nodes, and the extended connections between mobile users and the base station are wireless channels. This one-hop wireless network is very popular currently, i.e., the cellular networks and WLANs. The other is wireless ad hoc network, which has more than one hop wireless channels in the connection. This kind of topology is not widely implemented yet, but it is useful sometimes, especially in military applications and sensor networks.

In our project, we will focus on the latter topology, the wireless ad hoc network, without considering any wired links. As an extension to the backbone network, wireless ad hoc network consists of nodes that communicate with each other through wireless channels only. We can describe the system as follows. Our system consists of only wireless nodes, in which all nodes can communicate with other nodes in the range of radio transmission through wireless channel. Each wireless node can act as a sender, a receiver or a router. As a sender, the node can send message to the specified destination node through some route. As a receiver, it can receive the message from other nodes. As a router, it can relay the packet to the destination or next router in the route if necessary. Each node can buffer packets when the packets need to wait for transmission.

We are interested in the capacity and delay of such kind of network. In general these two parameters are the most important performance measurement for any wireless network systems. The capacity represents the throughput (bits per second) of the whole system including all nodes, and the delay represents the average time duration of a packet transmitting in the network from a source to the destination. As in any other queueing system, there are tradeoffs between the capacity and the delay. Intuitively in order to increase the capacity, we need to keep all nodes busy with transmitting or receiving packets during all the time, which means the queue of each node is always nonempty, obviously this will lead to a longer delay. On the other hand, in order to reduce the delay, the optimal situation is, all nodes along the route can transmit the packet immediately to the next node until it reaches the destination, which means there is no packet competing for transmissions in the queues, surely this causes very low throughput. We will see this tradeoff in wireless ad hoc networks in the report. Furthermore, our objective is to find a way that the network can achieve a high throughput while keeping the delay under certain threshold.

This report will address the problem. In the following, section 2 will describe the methodology to model the problem step by step, section 3 will give out the main results in the papers and explain their meanings to the capacity and the packet delay, based on these a routing algorithm is proposed to reach our objective. Finally in section 4 we conclude the capacity and delay in wireless ad hoc networks.

2. Methodology

In this part, we will show the methodology to solve the problem step by step. Recall that our objective is to achieve high capacity in wireless ad hoc networks with keeping the packet delay under a small threshold. We will model the networks from simple to complex, from general to specific step by step. In each model, we will describe the scenario, the transmission model and the measurement metrics in details.

First of all, we develop the models for wireless ad hoc networks with static nodes. Gupta and Kumar [1] propose two models for such kind of network. For simplicity, the models scale the space so that n nodes are located in a region of area 1 m^2. Each node can transmit at W bits per second over a common wireless channel. The channel is divided to several sub-channels, each with capacity $W_1, W_2, \cdots, W_M$ bits per second, where $\sum_{m=1}^{M} W_m = W$.

2.1 Model 1 (model of arbitrary networks in [1])

First we define the scenarios. In the network the nodes and traffic patterns are arbitrarily located. Say, n nodes are arbitrarily located in a disk of unit area in the plane. Each node arbitrarily chooses a destination to send message at an arbitrary rate, and also arbitrarily choose a transmission range or power level.

Then we use two models to indicate successful reception of a transmission over one hop: the protocol model and the physical model. In the protocol model, let X_i denote the location of a node, and suppose node X_i transmits over the m^{th} sub-channel to a node X_j. This transmission causes a successful reception by node X_j if $|X_k - X_j| \geq (1+\Delta)|X_i - X_j|$ for any other node X_k simultaneously transmitting over the same sub-channel. On the other hand, in the physical model, let $\{X_k; k \in \mathrm{T}\}$ be the subset of nodes simultaneously transmitting over a certain sub-channel. Assume node X_k transmits with power P_k, for $k \in \mathrm{T}$. The transmission from a node X_i, $i \in \mathrm{T}$ causes a successful reception by node X_j if the inequality $\dfrac{\dfrac{P_i}{|X_i - X_j|^{\alpha}}}{N + \sum_{\substack{k \in \mathrm{T} \\ k \neq i}} \dfrac{P_k}{|X_k - X_j|^{\alpha}}} \geq \beta$ is satisfied, where β is a threshold of signal-to-interference ratio (SIR) for successful receptions, N is the ambient noise power level, and $\alpha > 2$ indicates the signal power decay with distance $\dfrac{1}{r^{\alpha}}$.

Finally in the model we need to define the measurement metrics. We define *bit-meter* as the product of the number of bits and the distances over which the bits are carried. According to this, the capacity is defined as the sum of all *bit-meter* in the network.

From Model 1, we can compute the upper bound and lower bound of the capacity, thus get some knowledge about the actual capacities, which is describe in details in section 3.1. While this model is quite general, a further model with more information on the location and traffic pattern of the nodes will give us more useful results.

2.2 Model 2 (model of random networks in [1])

Similarly first we describe the scenarios. There are some tiny differences from Model 1. n nodes are independently and uniformly distributed on the surface S^2 of a three-dimensional sphere of area 1 m^2. We adopt this change in order to eliminate the boundary effects. Each node randomly and independently chooses a destination to send message with the rate of $\lambda(n)$ bits per second.

Then for the transmission model we adopt both a protocol model and a physical model for indicating the successful reception, just like what we do in Model 1. The only difference is that we introduce a common range r for all transmissions and the inequality in the protocol model changes to $\left|X_i - X_j\right| \leq r$ and $\left|X_k - X_j\right| \geq (1+\Delta)r$.

Finally in order to compute the throughput of the network, we define a throughput of $\lambda(n)$ bits per second for each node is *feasible* if there is a spatial and temporal scheme for scheduling transmissions, such that every node can send $\lambda(n)$ bits per second on average to its chosen destination node through the intermediate nodes and some buffering strategy in the intermediate nodes.

Based on this model, similarly we can calculate a lower bound and an upper bound of the capacity. However, because node relaying and buffering are introduced, it is possible that some packets will have long delays. Furthermore, in both models we only consider the network consists of static nodes. As we know, one of the biggest advantages of wireless networks is the mobility. In the next we will extend the model to include mobile nodes. Grossglauser and Tse [2] study influence of mobility on the capacity of wireless networks.

2.3 Model 3 (model with mobile nodes in [2])

First we describe the scenarios. The network still consists of n nodes lying in the disk of unit area, but different from Model 1 and 2, all the nodes are mobile. Denote the location of the i^{th} user at time t as $X_i(t)$. Assume the trajectories of different users are independent and identically distributed, and each node is both a source node for one session and a destination node for another session. Let $d(i)$ represent the destination node of node i. During the transmission, we assume that each source node has infinite number of packets to send to its destination. Furthermore, this S-D association does not change with time no matter how the nodes move.

We will use the same physical model as in Model 1 and 2 for transmission. Furthermore, in order to ensure a high long-term throughput for each S-D pair, a *scheduler* is introduced to determine which nodes will transmit packets, which packets they will transmit, and at which power levels $P_i(t)$ the packets will be transmitted from node *i*.

As a measurement metric, the throughput $\lambda(n)$ is a random quantity depending on the random locations and movements of the nodes. The capacity of the network is considered as the total throughput to all S-D pairs.

Based on this model with mobility, we can compute the theoretical results for the capacity in case of either without relaying nodes or with relaying nodes, which are discussed in details in section 3.3.

Until now we have satisfactory models for the wireless ad hoc networks with both static and mobile nodes. However, as we have seen, in Model 1, 2 and 3, we only consider the capacity of the networks without any considerations on the delay. In order to achieve high capacity, we assume packets can be relayed and buffered in the intermediate nodes, this might cause very large delay when the buffer length is long and the number of intermediate nodes is large. So it is necessary and important to get some ideas on the packet delay in the networks. Bansal and Liu [3] set up a model to address this problem, with more assumptions on the mobility pattern and traffic pattern.

2.4 Model 4 (delay model in [3])

First, the ad hoc network consists of *n* static nodes and *m* mobile nodes lying in a disk of unit area. The static nodes are uniformly distributed over the unit circular disk and never move. The mobile nodes are randomly distributed in the disk initially, later they will change positions and velocities with a mobility model. There are many models to do so, here a uniform mobility model is used. Initially each mobile node moves at speed *v* inside the unit circular disk. The directions of movement are independent and uniformly distributed in $[0,2\pi)$. At subsequent time the node picks a direction uniformly distributed in $(0,2\pi]$ and moves in that direction for a distance *d* at speed *v*, where *d* is an exponentially distributed random variable with mean μ. And so on. When the node reaches the boundary of the disk, it is reflected back to the disk again.

Similarly we use the physical model for transmission with minor modifications. At time *t*, let $S_1, S_2, \cdots, S_m$ be the senders with positions $X_1, X_2, \cdots, X_m$ and let *R* be the receiver with position X_0. If S_i use power $P_i(t)$ for transmission, it causes a successful reception by node R if $\dfrac{P_i(t)\|X_i - X_0\|^{-\alpha}}{N + \sum_{k \neq i} P_k(t)\|X_k - X_0\|^{-\alpha}} \geq \beta$.

The same performance metric is used as in the first 3 models. But besides capacity, the packet delay is also considered.

From this model, Bansal and Liu proves it is possible to achieve a high throughput while keeping the delay under some threshold, furthermore, a routing protocol is proposed to implement the objective, which is described in section 3.4.

With studying the evolution of the models of wireless ad hoc networks, we almost solve our proposed problem by adding more and more assumptions to the simple model. This is a very important methodology to do research.

3. Main Results

We have described the models to solve the problem step by step. In this section we will list the main results for different models and explain the importance in designing a wireless ad hoc networks. We won't go through the derivations of those results, the readers can refer to the papers ([1], [2], [3]) if interested in the details.

3.1 Model 1 (model of arbitrary networks in [1])

<u>Result 1 (main result 1 in [1])</u>
With the transmission model of Protocol Model, the transport capacity of networks in Model 1 is $\Theta\left(W\sqrt{n}\right)$ *bit-meters per second given that the nodes are optimally placed, the traffic pattern is optimally allocated, and the range of transmission is optimally chosen.*

<u>Result 2 (main result 2 in [1])</u>
With the transmission model of Physical Model, $cW\sqrt{n}$ *bit-meters per second is feasible, while* $c'Wn^{\alpha-1/\alpha}$ *bit-meters per seconds is not feasible for appropriate* c, c'.
Specifically, $\dfrac{1}{\left[16\beta\left(2^{\alpha/2}+\dfrac{6^{\alpha-2}}{\alpha-2}\right)\right]^{\frac{1}{\alpha}}}\dfrac{Wn}{\sqrt{n}+\sqrt{8\pi}}$ *bit-meters per second (n a multiple of 4) is feasible when the network is appropriately designed, with an upper bound of* $\dfrac{1}{\sqrt{\pi}}\left(\dfrac{2\beta+2}{\beta}\right)^{\frac{1}{\alpha}}Wn^{\frac{\alpha-1}{\alpha}}$ *bit-meters per second.*

From these results, we can directly conclude that for arbitrary network model, the capacity of wireless ad hoc network is in the order of $\Theta\left(W\sqrt{n}\right)$. If the total capacity is equally divided among all the nodes, then each node can achieve the capacity of $\Theta\left(\dfrac{W}{\sqrt{n}}\right)$ bit-meters per second. Furthermore, consider each source node transmits to the destination about the same distance of 1*m* apart, each node can obtain a capacity of $\Theta\left(\frac{W}{\sqrt{n}}\right)$ bits per second.

3.2 Model 2 (model of random networks in [1])

Result 3 (main result 3 in [1])

In case of both the surface of the sphere and a planar disk, the order of the throughput capacity is $\lambda(n) = \Theta\left(\frac{W}{\sqrt{n \log n}}\right)$ *bits per second for the Protocol Model. The upper bound can be indicated by the fact that for some* $c^{'}$*,*

$$\lim_{n\to\infty} \Pr ob\left(\lambda(n) = c^{'} \frac{W}{\sqrt{n \log n}} \quad \textit{is} \quad \textit{feasible}\right) = 0 .$$

Specifically, there exists constants $c^{''}$ *,* $c^{'''}$ *independent on* n *,* Δ *or* W *, such that* $\lambda(n) = \frac{c^{''}W}{(1+\Delta)^2\sqrt{n \log n}}$ *bits per second is feasible, and* $\lambda(n) = \frac{c^{'''}W}{\Delta^2\sqrt{n \log n}}$ *bits per second is not feasible, with the probability approaching one as* $n \to \infty$.

Result 4 (main result 4 in [1])

With the transmission model of Physical Model, a throughput of $\lambda(n) = \Theta\left(\frac{cW}{\sqrt{n \log n}}\right)$ *bits per second is feasible, while* $\lambda(n) = \frac{c^{'}W}{\sqrt{n}}$ *bits per second is not feasible, for appropriate* c*,* $c^{'}$*, with probability approaching one as* $n \to \infty$*. Specifically, there exists constants* $c^{''}$ *and* $c^{'''}$ *independent on* n *,* N *,* α *,* β *or* W *, such that*

$$\lambda(n) = \frac{c^{''}}{\left[2\left(c^{'''}\beta\left(3+\frac{1}{\alpha-1}+\frac{2}{\alpha-2}\right)\right)^{\frac{1}{\alpha}}-1\right]^2}\frac{W}{\sqrt{n \log n}}$$

bits per second is feasible with probability approaching one as $n \to \infty$*. If* $\overline{L}$ *is the mean distance between two points independently and uniformly distributed in the domain (either surface of sphere or planar disk of unit area), then there is a deterministic sequence* $\varepsilon(n) \to 0$ *, independent on* N *,* α *,* β *or* W *, such that* $\sqrt{\frac{8}{\pi}}\frac{W}{\overline{L}\left(\beta^{1/\alpha}-1\right)}\frac{1+\varepsilon(n)}{\sqrt{n}}$ *bit-meters per second is not feasible with probability approaching one as* $n \to \infty$.

From these results, we can see for random network model, the capacity is in the order of $\lambda(n) = \Theta\left(\frac{W}{\sqrt{n \log n}}\right)$, less than the capacity in the arbitrary network mode. That is because we add some limitations on the traffic pattern. Furthermore, from result 3, we can get some insights on what limits the capacity. In the case of a disk on the plane, the nodes lying in the center will have more possibilities to relay packets, so-called hot spots, but the order of throughput capacity is the same as on the surface of the sphere. That shows the cause of the throughput constriction is not the formation of hot spots, but is the pervasive need for all nodes to share the channel locally with other nodes.

3.3 Model 3 (model with mobile nodes in [2])

<u>Result 5 (Theorem III-3 in [2])</u>

Consider a scheduling policy that is only allowed to schedule direct transmission between the source and destination nodes. Say, no relaying is permitted. If c is any constant satisfying $c > \left[2^{\alpha}\left(1+\frac{2}{\alpha}\right)\pi^{-\alpha/2}\frac{\beta+L}{\beta}\right]^{1/(1+\alpha/2)}$,

then $\Pr\{\lambda(n) = cn^{-1/(1=\alpha/2)}R \quad \text{is} \quad \text{feasible}\} = 0$ *for sufficiently large n.*

From this result, we can see the capacity per S-D pair goes to 0 as $n \to \infty$ if no relaying is permitted in the networks. That is because in each source node, high power is required to transmit the packets directly to the destination node, which leads to high interference and limits the capacity. It is possible to gain higher capacity if we schedule nodes to communicate only with close neighbors and relay packets for destination nodes far away.

<u>Result 6 (Theorem III-4 & Theorem III-5 in [2])</u>

Consider a scheduling policy π *allowing relaying nodes. For a given S-D pair, there is one direct route and n-2 two-hop routes that go through one relay node. The network can achieve a throughput of* $\Theta(1)$ *per S-D pair, i.e., there exists a constant* $c > 0$ *such that* $\lim_{n\to\infty} \Pr\{\lambda(n) = cR \quad \text{is} \quad \text{feasible}\} = 1$.

Comparing Result 5, 6 with Result 1, 2 and Result 3, 4, we can see immediately that if relaying is permitted in the networks, mobility can dramatically improve the capacity from $\Theta(\sqrt{n})$ or $\Theta\left(\sqrt{\frac{n}{\lg n}}\right)$ to $\Theta(n)$.

3.4 Model 4 (delay model in [3])

Result 7 (main result in [3])

In the wireless ad hoc network with n static nodes and m mobile nodes, which are characterized in Model 4, there exists a constant $c > 0$ *, such that each sender can achieve an average throughput of* $c\frac{W\min(m,n)}{n\log^3 n}$*, where W is the maximum available bandwidth, while the packet delay is at most* $\frac{2d}{v}$*, where d is the diameter of the network and v is the velocity of the mobile nodes.*

This result solve our problems proposed at the beginning, in the following a routing algorithm is described to achieve this objective.

Result 8 (routing algorithm in [3])

Step 1. Local leader election
A local lead is elected among the static nodes within each region of size $1/\sqrt{m}\times 1/\sqrt{m}$. This leader will be responsible for communicating all the messages of the static nodes in its region with the mobile nodes.

Step 2. Static to mobile phase
A static node S1 wanting to send messages to destination R first transfers its message to its local leader S. S stores the message and waits for a mobile node M1 such that M1 is close enough to S and moving approximately along the direction of R. when such a node is available S hands over the data from S1 to M1.

Step 3. Mobile to mobile phase
The mobile nodes relay the packets towards R amongst all possible mobile nodes such that the packet moves closer and closer to the destination.

Step 4. Static to static phase
When the mobile node carrying the packet is close enough to the destination, it hands off the packet to some leader node. This packet is then relayed among the static leader nodes towards the correct leader node, which can transmit the packet to the destination node directly.

With this routing algorithm, the wireless ad hoc network can achieve close-to optimal capacity while keeping the packet delay small. This algorithm exploits the mobility patterns of the nodes to provide guarantees on the packet delay. The readers can refer to [3] if interested in the detailed operations and arguments of the algorithm.

4. Conclusion

In this project, we explore the throughput and delay in wireless ad hoc networks. Our objective is to achieve high throughput while keeping the packet delay relatively small. In order to solve this problem, we start from the simplest model, compute the capacity only, then add more assumptions step by step, and finally find out a routing algorithm which can achieve our objectives. This is a very important methodology for any kind of research.

For wireless ad hoc networks with only static nodes, the capacity per node is $\Theta\left(\frac{W}{\sqrt{n}}\right)$ bits per second for Arbitrary Network model, and $\Theta\left(\frac{W}{\sqrt{n \lg n}}\right)$ for Random Network model. If mobility is considered in the network, the capacity can be dramatically improved to $\Theta(1)$ per S-D pair. Furthermore, if more assumptions on the traffic pattern and mobility pattern are introduced, the proposed routing algorithm can guarantee the packet delay and achieve a close-to optimal capacity, which is only a poly-logarithmic factor off from the optimal algorithm. Note that we have no considerations on the energy limitation of the nodes in the network, which is another important constraint actually existing in the wireless ad hoc networks.

References:

[1] Piyush Gupta and P. R. Kumar, "*The Capacity of Wireless Networks*", IEEE Transactions on Information Theory, Vol. 46, No. 2, March 2000.
[2] Matthias Grossglauser and David Tse, "*Mobility Increases the Capacity of Ad Hoc Wireless Networks*", IEEE/ACM Transactions on Networking, Vol. 10, No. 4, August 2002.
[3] Nikhil Bansal and Zhen Liu, "*Capacity, Delay and Mobility in Wireless Ad-Hoc Networks*" INFOCOM 2003; Twenty-Second Annual Joint Conference of the IEEE Computer and Communications Societies, IEEE , Volume: 2 , 30 March - 3 April 2003.