# Neutrino Oscillations: The Indefinite Mass Defined

Conceptual problems in neutrino oscillation theory are resolved by reference to the basic principles of quantum physics.

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# Abstract

This paper outlines the usual neutrino oscillation theory and points out the analogy between Lyman energy eigenstates and neutrino mass eigenstates. It then suggests several alternative interpretations of the neutrino oscillation theory in regard to determination of the rest mass of the neutrino.

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> *Nevertheless, I'm still far away from claiming the physical validity of the equations I have derived here. The reason for this is that I have not succeeded yet in deriving equations of motion for particles.*
>
> -- A. Einstein [1].

# Physical Insight and Inference

Physical insight into a new phenomenon can be achieved in three distinct ways:

1. By ***analogy***. This is the simplest and commonest way. In a trivial example, we assume a billiard ball to be analogous to one of Galileo's reputed cannon balls, and we then compute the rate of fall of the billiard ball the same way. Less trivially, we assume by extension that an atom is like a little Solar system, with electrons "orbitting" around a nucleus. Or, maybe atoms are like little walnuts, with a "shell" of electrons surrounding the nuclear "meat".

   Notice the quotes, a way in the English language of eliciting caution. The quotes here make explicit the action of analogizing, as opposed to deducing logically or calculating mathematically. The quotes reserve a little disbelief, because all analogies are wrong to some extent -- it's just that an analogy may be right enough to make it a completely correct foundation for calculations.

2. By ***mathematics***. Mathematics is no more than a sequence of substitutions. Nevertheless, we start with quantities of some kind, apply equations to them, and come out with perhaps unexpected solutions. For example, applying equations representing conservation of energy and momentum allows us to predict the velocities of elastic objects colliding with others.

3. By ***logic***. We state postulates and arrive at perhaps unexpected results by deduction. For example, the postulate that the speed of light is the same to all observers in every inertial frame can be used to deduce that the speed of light is the greatest speed possible.

Formal logic is mathematical, and mathematics generally depends on some logic. Physics depends on analogies. But, any of these ways will lead to wrong results if either the starting point or the process is incorrect.

In this little essay, we bring out, very explicitly, the ***analogy*** underlying the statement that *the mass of the neutrino is indefinite*; we then use ***logic*** to arrive at theoretical deductions generally ignored by oscillation theorists. The ***mathematics*** in the present work will be elaborative, only, and will not be used to propose any new conclusion.

Our main analogy will have its basis in the familiar Lyman series of the spectrum of hydrogen; the extension of this analogy will be to the neutrino mass spectrum.

# Hydrogen: The Lyman Eigenstates

Rarified, ionized hydrogen gas at high temperatures emits light because hydrogen nuclei capture electrons in certain energy levels (shells). When an electron briefly in such a shell moves to a shell closer to the nucleus, it emits a photon of energy exactly equal to the difference in shell energies. If we ignore Doppler shifts, the emission spectrum of this hydrogen consists of a set of discrete lines determined by the quantization of the energies of the nuclear shells.

The set of spectral lines is called the ***Lyman series*** of lines and was described in an empirical formula by Rydberg and later found to be deducible from a Bohr atom model, an analogy to a nucleus surrounded by planar electron "orbits" [2] or "shells".

The spectral lines represent energy ***eigenstates*** of this hydrogen gas, which are quantum-mechanical states. What this means is that, again ignoring Doppler shifts, although other energy values are possible computationally, the only energy of a photon emitted from this gas always will be an energy corresponding to an eigenstate. No other photon energy can be observed, because quantum theory forbids it.

However, if we are interested in extremely rare events, we could consider the brief formation of a covalent electron bond between two of these hydrogen atoms. The energy of such a bond can not be described in terms of Lyman eigenstates and ***is indefinite*** in the context of Lyman emission.

Notice the use of the word *indefinite* here: It does not mean physically undefined; it just means that no definite value can be measured in the context of (quantized) Lyman emission. The energy is *indefinite* in terms of hydrogen-nucleus energy shells. Although not absorbable by this kind of hydrogen gas, a covalently-emitted photon could be detected by a solid-state instrument.

Clearly, energy is a conserved quantity; and, if a photon was emitted because a covalent electron moved to a Lyman-defined shell, the photon's energy would have to have been a definite quantity. By energy conservation, the photon energy would have to be exactly equal to the difference in energies of the electron under consideration.

We now extend this analogy toward a better understanding of neutrino mass eigenstates.

# The Neutrino

## *Properties*

Neutrinos are electrically-neutral, massive particles with rest mass of no more than a couple of eV/$c^2$. By contrast, the mass of an electron is about 511,000 eV/$c^2$. Neutrinos do not interact electromagnetically; also, they are leptons and do not interact by the strong force; they are not believed to interact at all, except, rarely, by the weak force.

Recall that the total energy of a particle which is propagated freely in a vacuum can be represented by Einstein's formula,

$$E^2 = (pc)^2 + (mc^2)^2 .$$

For any experimentally reasonable $E$, the tiny value of the neutrino $m$ is dwarfed by the term in momentum $p$, and the measurement uncertainty in $m$ is amplified by powers of the huge number, $c$.

Currently, neutrinos can be studied in accelerator experiments if they are extremely numerous and if their energies are high enough, above maybe 50 keV and more commonly in the MeV or GeV range. The enormous difference between the energy in the neutrino mass and its total energy makes direct measurement of the neutrino mass exceedingly difficult and perhaps practically impossible. However, as we shall see, this kind of indefiniteness of the neutrino mass is not the meaning of the word in oscillation theory.

Three types (*flavors*) of neutrino are known to exist, depending on the charged particle interactions involved with the neutrino initial or final interaction: ***electron neutrinos***, corresponding to electrons; ***muon neutrinos***, corresponding to muons; and, ***tauon neutrinos***, corresponding to tauons. To each of these neutrinos there also corresponds an antineutrino of opposite chirality, but we shall use the word "neutrino" here to refer to either. For example, ignoring chirality, electron neutrinos are produced along with electrons during beta-decay; muon neutrinos are produced along with muons by pion decay, and also during muon decay. Tauons and their neutrinos have not much been studied and will be ignored here. It is believed that particles of different flavor can not interact with each other by the weak force. Thus, it is believed that a muon neutrino can not interact with an electron.

An electron-flavor neutrino can be detected because, with small but finite probability, it will interact ("collide") with an electron, transferring much of its energy to the electron. The fast-moving electron then can be detected easily because of its electric charge. Similarly, fast-moving, charged muons from muon neutrinos also can be detected easily. Electrons or muons can exist briefly in virtual states in atomic nuclei, so electron or muon neutrinos can interact with nuclear matter.

## *Neutrino Oscillations*

Neutrino oscillation theory was formulated to explain problems in detecting neutrinos [3,4]. Basically, fewer neutrinos were being detected than was expected; and, the farther the neutrinos travelled, the fewer of them seemed to be detected [5,6]. It was postulated that neutrino flavor varied as a function of distance and that the neutrinos were oscillating among the three possible flavors. According to this idea, neutrinos seemed to disappear because detectors were specialized to detect just one flavor, and thus the predicted flux at any given flavor would decrease with distance because of oscillation to one or both of the other, less-well detected flavors.

Because massless particles must travel at speed $c$ and can not change during propagation because they lack proper time, the observed flavor oscillations were the first proof that neutrinos were massive.

Most of the present experiments have shown just a loss, never a gain, in flavor; they have not shown an appearance of neutrinos of a new flavor. One experiment, LSND [7], did seem to show an appearance, but a later replication [8] failed to confirm it. Very recently, an experiment [12] seems to have shown appearance of electron-flavor antineutrinos.

# Neutrino Mass Eigenstates

To understand the importance of neutrino mass eigenstates, we should look into the basics of the neutrino oscillation theory.

The mathematical approach was analogous to that of a nuclear theory describing quark mixing [9,10]. In the neutrino oscillation theory [6,11], a 3-component neutrino flavor vector is assumed transformable from a corresponding 3-component mass vector $\vec{m}$. The transformation is by a 3x3 mixing matrix $\mathbf{V}$. The mass state varies with distance of neutrino propagation $L$, so that, the resulting flavor vector $\vec{f}(L)$ is such that,

$$\vec{f}(L) = \vec{\mathbf{V}} \cdot \vec{m}(L),$$

which may be written out as,

$$\begin{bmatrix} f_e \\ f_\mu \\ f_\tau \end{bmatrix}(L) = \begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{bmatrix} \cdot \begin{bmatrix} m_e \\ m_\mu \\ m\tau \end{bmatrix}(L),$$

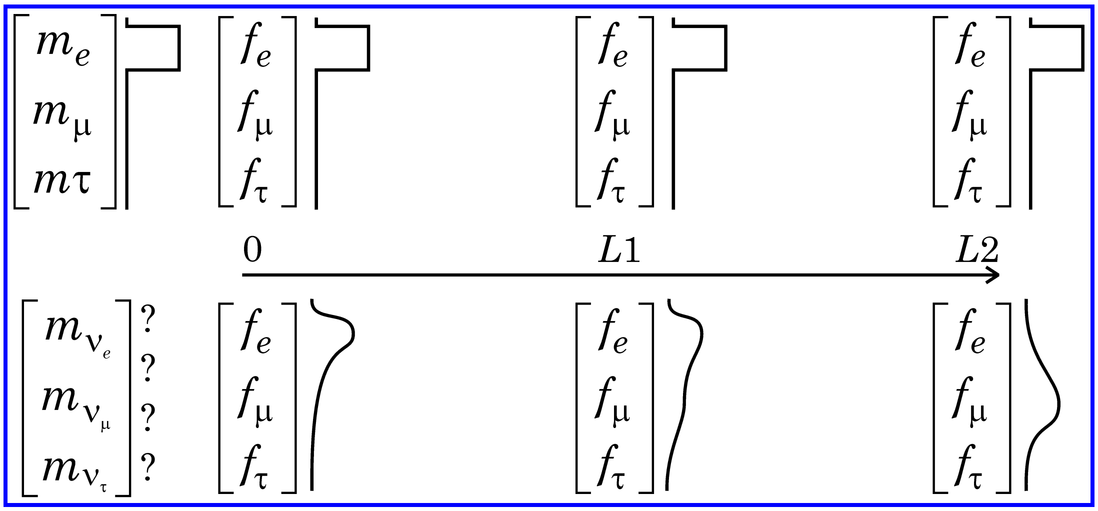
in which flavor changes as a function of distance because mass does.

Now, of course, the mass of a freely propagating particle can not change. The theory works because the mass components and the mixing matrix elements are assumed to be complex quantities; only the imaginary part of the mass components changes with distance, and this may be referred to as the *mass phase*. The vector components are associated with probability amplitudes, and the bases of the mass vector components are the ***mass eigenstates***. In the matrix equation above, the mass amplitudes on the right are determined by the initial (neutrino creation) interaction and by the distance $L$ from the point of that interaction; the flavor amplitudes on the left are calculated from the mass amplitudes at distance $L$ (to the final interaction) and the mixing matrix. The mixing matrix is assumed an invariant associated with the nature of the neutrino particle. The mass phase probably should be related to the phase of the quantum-mechanical neutrino position wavefunction, but this is not necessary to the theory.

The relatively massive charged leptons, electrons, muons, and tauons, may be described by a mixing matrix $\mathbf{V}$ which is equal to the identity matrix. Therefore, a charged lepton created with probability 1 of having the mass of an electron, will be created necessarily in an electron mass eigenstate and will arrive at its final interaction point with probability 1 of being of electron flavor, regardless of the mass phase shift. Likewise, muon or tauon particles, with masses of the muon or tauon, always will interact as muons or tauons, respectively. Therefore, the theory predicts no flavor changing in the interactions of the charged leptons (none is observed experimentally).

However, the very light uncharged leptons, the neutrinos, are assumed to be governed by a mixing matrix with nonzero off-diagonal elements. Therefore, although every neutrino will be created in a flavor eigenstate, its mass phase will rotate during propagation, causing it to enter into its final interaction with some probability of being in a different flavor, the probability depending on distance of propagation.

To summarize graphically, for electron-flavor creation, we have this:




Lepton flavor probability as a function of propagation distance $L$. Electron *vs*. electron neutrino. Probability weights are mapped to the right of each vector. The charged leptons (above) conserve flavor; the neutrinos are destroyed with flavor probably changed.

Because all matrix bases are orthonormalized, eigenstate masses are determined only in ratio to one another. The theory itself does not constrain the absolute eigenstate mass values.

# The Neutrino Mass

So far, this has been just a review for anyone already familiar with neutrino oscillations: This theory implies that the neutrino mass must be "indefinite".

In what sense? Well, here is the arithmetic: If the final interaction of a neutrino is to be definitely of a specific flavor, then the final flavor vector must be of the form, $\left[f_e, f_\mu, f_\tau\right]$, in which one component has a probability weight of 1 and the others 0. The mixing matrix **V** (above) is nonsingular and therefore can be inverted; then, the final flavor vector can be multiplied by the inverse mixing matrix to obtain the final mass vector. However, only if the mixing matrix (and thus its inverse) had no off-diagonal element, could all the weight of the final mass vector be on one mass eigenstate. Therefore, the final mass calculated from the final flavor can not be in an eigenstate. Because the mass (real part) has not changed during propagation, the initial mass also must not have been in an eigenstate. The neutrino mass in general, according to this theory, must be indefinite.

This does not mean that the mass of a specific neutrino somehow is undefined; what it means is that neutrino mass can't be predicted by this theory. According to the theory, the three mass eigenstates do differ in mass (real part); although only the mass phase of a neutrino varies during propagation, the final mass phase may be weighted on a different mass eigenstate from the one in which the neutrino originally was created.

If so, what is the mass of a neutrino? Does this question make sense?

Theoretical treatments usually leave the discussion at this point and don't discuss this question. However, some papers have tried to show how a neutrino with "indefinite" mass might transfer something indefinite to other particles in the final detection context, and maybe how randomization of the mass of the neutrino population might be resolved by uncertainty in the state of the atoms in the final-interaction detector. This sort of question is interesting but unnecessary.

There are three alternative resolutions of the question of the atomic mechanics in neutrino interactions, all of which exclude the propagation of neutrino indefiniteness to atoms in the detector:

1. Most usually assumed: The mass of any specific neutrino is always of some well-defined, intermediate value, less than or equal to that of the heaviest mass eigenstate, and greater than or equal to that of the lightest one. The value varies randomly and unpredictably from neutrino to neutrino. The analogy is to the position probability of an electron near an electric potential well or barrier.

2. By analogy to the charged leptons: The three flavors of neutrino are created always in three exactly equal, distinct masses; however, a neutrino of a given flavor might have any of three different, eigenstate masses. The mass phase doesn't matter, but eigenstates do. Accepting the Lyman analogy, if there exist mass eigenstates, a neutrino can not be observed except in a mass eigenstate.

3. By analogy to a single charged lepton: Like a hydrogen atom, all neutrinos have exactly the same, well-defined mass which equals some value in the range of the heaviest to lightest mass eigenstates. The mass phase doesn't matter, and neither do the eigenstates.

We shall dismiss the last alternative and concentrate on the first two. How can these be possible? The answer is, by basic quantum theory: Neutrino oscillation calculations generally assume either that energy is conserved or momentum is conserved over the propagation interval $L$ of a neutrino. This causes the final results of a calculation to depend quantitatively on whether energy or momentum is postulated conserved. So, the nonconserved quantity is called indefinite, and the theorist thus infers an "indefinite" mass.

Now, one way that an interaction can violate energy or momentum conservation is if it is ***virtual***. When a virtual particle of mass $m$ is created, it exists briefly, in a small space-time interval, and then vanishes; if, by chance, enough energy to equal $mc^2$ is supplied at its instantaneous position, the virtual particle can become real. But, neutrinos, especially astronomical ones, cover too great an interval for virtualization to account for the "indefinite" quantity.

So, how can flavor oscillate and thus imply a change in mass during a propagation interval not accompanied by any intervening interaction? Simple, because the question is poorly posed. The proper question is, How can flavor oscillate and thus imply that the observed mass could vary from neutrino to neutrino?

Quantum theory requires that interactions not be based on classically definite properties but rather on probable values of those properties. Location of a specific electron near a potential barrier is the paradigmatic example of this principle: No matter what the value of the barrier, given that the electron is close to it, there will be some probability that the electron will be found by an interaction on the other side of the barrier. Sometimes, this is called electron "tunneling" through the barrier.

Because flavor oscillation implies an oscillation in mass, one can not predict with certainty the mass of a neutrino until that specific neutrino has interacted. One then may know, definitely and precisely, what the mass was when that specific neutrino was created. Nothing indefinite; merely something not definitely predictable.

# Summary

The assertion that "the mass is indefinite" merely means that the mass of a neutrino can not be determined by the initial interaction, the one creating the neutrino, according to the current oscillation theory.

This doesn't mean that the mass has to vary randomly from neutrino to neutrino; all it means is that some other way must be devised to measure the mass. If such other way never is devised, it still doesn't mean that the mass of a neutrino has to vary at all. However, if no other way is devised, then the theory is the only way, and the mass remains not definitely measurable.

There is no proof that the mass of the neutrino is indefinite, except in the sense that it is going to be a hard thing to measure any way other than by the oscillation theory.

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