An Integrated Mathematical Model for Supplier Selection

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ABSTRACT
Extensive research has been conducted on supplier evaluation and selection as a strategic and crucial component of supply chain management in recent years. However, few articles in the previous literature have been dedicated to the use of fuzzy inference systems as an aid in decision-making. Therefore, this essay attempts to demonstrate the application of this method in evaluating suppliers, based on a comprehensive framework of qualitative and quantitative factors besides the effect of gradual coverage distance. The purpose of this study is to investigate the applicability of the numerous measures and metrics in a multi-objective optimization problem of the supply chain network design with the aim of managing the allocation of orders by coordinating the production lines to satisfy customers’ demand. This work presents a dynamic non-linear programming model that examines the important aspects of the strategic planning of the manufacturing in supply chain. The effectiveness of the configured network is illustrated using a sample, following which an exact method is used to solve this multi-objective problem and confirm the validity of the model, and finally the results will be discussed and analyzed.

Keywords: Supplier Selection, Fuzzy Inference System, Coverage Distance, Multi-objective Programming

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1. INTRODUCTION
Supply chain management (SCM) is a set of approaches used to efficiently integrate suppliers, manufactures, warehouses and stores (Simchi-Levi et al., 2000), which has made the evaluation, selection and improvement of suppliers to undergo a process required to be implemented in most organizations today. A large body of literature on the issue of vendor selection has appeared in the last several decades; see Deshmukh and Chaudhari (2011) for an extensive review.

Procurement costs constitute more than 50% of the companies’ costs (Aissaoui et al., 2007). Although the price of products or services is considered as an important parameter for choosing a supplier, other different criteria play a prominent role. For example, purchased products with low quality may increase the deficiency rate and eventually lead to customer dissatisfaction; delay leads to the cessation of production lines or increases the cost, which is caused by keeping more buffer stock. As a result, it is necessary to evaluate suppliers with a comprehensive framework of various criteria.

The objective of supplier selection is to identify suppliers with the highest potential for meeting a company’s needs consistently (Araz and Ozkarahan, 2007). Besides, supplier selection is considered to be a multi-criteria decision making (MCDM) problem that includes both quantitative and qualitative factors. To solve this problem, it is necessary to make a tradeoff between both tangible and intangible factors for selecting the best.

In the last two decades, several studies have been conducted on the issue, most of which have used MCDM methods such as analytic hierarchy process (AHP), data envelopment analysis (DEA), and the statistic method (Guo et al., 2014). The same factors like quality, delivery, capacity and price in the evaluation of supplier have
always been taken into consideration (Weber et al., 1991). The present study provides a new classification of a more comprehensive framework in comparison to the one presented in the past.

Being a traditional approach, MCDM faces the obstacle to handle the uncertainties of the real world. In addition, human preferences vary and decision-makers might be reluctant or unable to assign exact numerical values (Chan and Kumar, 2007). This is why some papers have investigated the use of fuzzy sets in the supplier selection.

This study presents a framework for assessing the competence of suppliers, which is followed by the application of a non-conventional technique, a method of handling both qualitative and quantitative factors to appraise providers in SCM practices besides a new approach that can evaluate them on the basis of their distance from the manufacturers.

Attention to risk management and related techniques is growing in supply chain (SC), especially after the recent financial crisis. Therefore, in this paper we illustrate risk equation as an objective. Several studies in the past have had a thorough review of the effects of SC disruption on the operation performance and its probable damage to the company's benefits. For instance, Singhal et al. (2011) provided a recent and comprehensive review of the same issue.

This study suggests and develops an integrated procedure that will aid decision-makers to appraise the performance of suppliers in a supply network with numerous measures and metrics. In this paper, decision-makers evaluate suppliers with qualitative criteria such as safety and quantitative criteria like part price appraised by MATLAB fuzzy inference system (FIS) editor. The present work uses the editor to define rules and determine the weights. All the features of this approach, such as applying various and occasionally different rules and the simultaneous use of a wide range of multiple conflicting factors in a comprehensive system, can be the reasons for using this methodology in the provider evaluation.

However, the focus of the study is on evaluating and selecting the suppliers, and a mathematical model is utilized to display the application of the discussed elements. The SC network is formulated as a multi-objective mixed integer non-linear programming (MINLP) model that is designed for the multiple considerations of the estimated supplier weight, coverage distance and risk. Afterwards, a compromising programming method is adopted to determine the weight of each objective function, and the Lingo software is used to solve the final model. Eventually, an example is selected to test the validity of the model.

This paper does not include any comparison because there has not been the same approach to determining the providers’ weight and using them simultaneously in a mathematical formulation. Moreover, it should be stated that the model is only an admirable alternative to providing a comprehensive review of the numerous factors, distance and the importance of weight in appraising the suppliers, and there is no reason for gaining better numeric result compared with the other method.

The remainder of this paper is organized as follows. After a brief review of the relevant literature in Section 2, we define the problem and introduce the fuzzy rule-based system and cover distance equation in detail in Section 3. Next, the mathematical formulation is developed. In Section 4, the solution method is presented and a numerical example of its occurrence is used to show the applicability of the model, after which the computational testing of its results is reported. Finally, the concluding remarks and directions for further development are provided in the last section.

2. LITERATURE REVIEW

Performance evaluation of the supplying firms is being recognized as one of the critical indicators (Sharma, 2010). The literature on supplier evaluation includes some surveys that 1) focused on problem criteria, and 2) proposed methods for the selection process. In identifying the qualitative and quantitative criteria after Dickson (1996), Dempsey (1978) described 18 criteria. Weber et al. (1991) extensively reviewed, annotated and classified 74 related articles, which have appeared since 1966, and gave specific attention to the criteria and analytical methods. They concluded that the quality, cost, and on-time delivery are the three most important supplier selection criteria commonly used.

De Boer et al. (2001) offer a literature review of supplier selection in all phases, including the initial problem definition, the formulation of the criteria, determining the eligibility of potential suppliers and ultimately choosing the qualified suppliers, which cover various methods of supplier selection. Some of these methods used a single model, such as linear programming, AHP, fuzzy set theory (Zadeh, 1973), etc., and others used a combined model, such as integrated AHP and DEA, fuzzy and multi-objective programming, etc.

According to Wang and Yang (2009), the quantitative decision methods for solving the supplier selection problem can be classified into three categories: 1) multi-attribute decision-making, which includes the linear weighting method and the AHP, 2) mathematical programming models, which include the linear programming models, mixed integer programming, multi-objective programming and data envelopment analysis, and 3) intelligent approaches, which consist of neural network based methods, expert systems, fuzzy decision-making and hybrid approaches.

In the last two decades, various decision-making approaches have been proposed to tackle the problem of supplier evaluation and selection; please refer to recent reviews by Ho et al. (2010) and Mafakheri et al. (2011),...
which show the popular applications of AHP and fuzzy AHP. This methodology has been used to rank potential suppliers in a hierarchical manner. For instance, Barbarosoglu and Yazgac (1997) applied a five-level AHP structure to the supplier selection problem.

Mathematical programming models, as the techniques used for evaluation, have significant problems in considering qualitative factors, which are very important (Mak et al., 2012). In this area, multi-objective optimization models have been proposed to identify appealing tradeoffs between two or more conflicting criteria that are involved in the order allocation process (see, Amid et al., 2006; Kannan et al., 2013; Nazari-Shirkouhi et al., 2013; Ware et al., 2014; Wu et al., 2010).

The use of the fuzzy set theory for modeling and analyzing decision systems is of particular interest in production management. This is because of the fuzzy theory’s ability to quantitatively and qualitatively model problems which involved vagueness and imprecision (Carrera and Mayorga, 2008). Some recent articles that used the fuzzy techniques for solving the selection problems are: Amin and Razmi (2009), Arikan (2013), Carrera and Mayorga (2008), Garcia et al. (2013).

In recent years, a number of researchers have taken advantage of FIS in their articles. However, these uses have been quite limited. For example, Liu and Wang (2009) developed an integrated fuzzy approach for the problem of provider selection that applies the fuzzy set theory, fuzzy Delphi, fuzzy inference, and fuzzy linear assignment, in order to deal with this problem. Guneri et al. (2011) proposed an analytical technique based on adaptive neuro-fuzzy inference system (ANFIS) model for supplier selection decision-making, which consists of two main stages: first, selecting inputs by ANFIS, and second, building the final model using the selected inputs. Amindoust et al. (2012) introduced a fuzzy ranking model on the basis of FIS, which considers sustainable criteria in the supplier selection problem.

In another study, Carrera and Mayorga (2008) applied the FIS to a supplier selection problem for a new product development. Their model includes 16 variables categorized in four groups and each group has an individual output. They did not assign the importance of weights for the selected indicators (criteria and sub-criteria). In their model, the fuzzy rules for each FIS did not envelop all possible characteristics of suppliers.

As a result, most of the discussed studies took two to three variables at a time, which are not sufficiently practical. Moreover, a proper design and management requires planning and decision-making with respect to a comprehensive consideration that involves linguistic variables.

3. PROBLEM DEFINITION

Determining an appropriate strategy for supplying raw materials is known as one of the basic principles to achieve an integrated configuration of SC network. In this section, a supply and production procedure is investigated. The representation of network is illustrated in Figure 1. Based on the figure, after assessing the sources

![Figure 1. Framework of proposed approach.](image-url)
for providing the required parts, the manufacturing strategies will be planned by production centers to cover different clients’ demands. First, we describe a structure for aiding decision-makers in the performance assessment of potential suppliers based on an evaluation framework as is shown in Figure 2. The results of this phase make the suppliers important through the application of a fuzzy rule-based system and a new equation for considering the distance effect. Following that, in the next phase, the best suppliers among the candidates are selected by the designed model.

3.1 Fuzzy Inference System

The Method adopted to obtain a relevant conclusion of some rules involves the use of Mamdani’s fuzzy inference method—proposed by Mamdani and Assilian (1975), which is the most commonly seen fuzzy meth-

![Figure 2. Some of the most important factors that influence supplier performance evaluation.](image-url)
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Odology and was among the first control systems built using the fuzzy set theory. This method attempts to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators. Mamdani’s effort was based on Zadeh (1973) on the fuzzy set theory as a methodology, which incorporates imprecision and subjectively into the model formulation and solution process in complex systems and decision processes. Although the inference process described in this section differs somewhat from the methods described in the original paper, the basic idea is much the same.

Fuzzy inference process comprises of five parts: fuzzification of the input variables, application of the fuzzy operator (AND or OR) in the antecedent, implication from the antecedent to the consequent, aggregation of the consequents across the rules, and defuzzification. These odd names have very specific meanings that are defined as follows.

Here we have used two membership functions with different fuzzy linguistic sets that are built on the Gaussian distribution curve and the Triangular for inputs and conclusions, respectively (see Figure 3). In this manner, each input is fuzzified over all the qualifying membership functions required by the rules. The basic structure of an example with the two-inputs, one-output, and three-rules is shown in Figure 4.

Note that each rule can have different weights that are generally considered as one and thus have no effect at all on the implication process. The input for the implication process is a single number given by the antecedent, and the output is a fuzzy set. The implication is implemented for each rule. Min (minimum) is applied as a built-in method, which truncates the output fuzzy set, and is used by the AND method.

Aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set. Aggregation only occurs once for each output variable. The input of the aggregation process is the list of truncated output functions returned by the implication process for each rule. The output of the aggregation process is one fuzzy set for each output variable that will eventually be defuzzification.

There are various built-in methods for the aggregation, such as max (maximum), probor (probabilistic OR value), and sum (simply the sum of each rule’s output set) to combine the output of each rule into a single fuzzy set whose membership function assigns a weighting for every output value. We used the centroid calculation for defuzzification, which is the most popular method and would restore the center of the area under the curve. There are other methods, such as bisector, middle of maximum, largest of maximum, and smallest of maximum.

Figure 5 shows the fuzzy inference diagram for an example with the two-input, one-output, and three-rule. In this figure, the flow shows everything at once, from linguistic variable fuzzification all the way through defuzzification of the aggregate output. Decision-makers use a crisp number between 0 and 10 for showing the importance of the criterion.

When the values for each class are obtained by FIS, the next step is establishing the final weight for the overall performance of the suppliers. The decision-makers make use of the linguistic variables with triangular fuzzy numbers for weight to show how well each class qualifies (seven classes include time, cost, quality, etc.). These weights for which a sample is presented in Table

![Figure 3](image-url) The membership function for (A) the input variable, (B) the output variable.

![Figure 4](image-url) The basic structure of fuzzy inference process.
I are considered the same for all suppliers.

According to this table, the linguistic variables used in the rating of each class were limited to seven scales including, extremely unimportant, strongly unimportant, unimportant, moderately important, important, strongly important, and extremely important, to form the linguistic set used to express opinions on the classes for the final performance evaluation of the supplier.

This methodology is used in the weighting of the importance of all classes and the next final weight of the supplier is calculated by the following equation.

\[
W_i^\text{f} = \frac{\mathbf{W}_i^\text{c}}{\mathbf{W}_i^\text{r}} \times \sum_{c=1}^{n} \mathbf{I}_{ct} \times \mathbf{P}_{ct} \sum_{c=1}^{n} \mathbf{I}_{ct}
\]

where \( W_i^\text{f} \) is the final weight of supplier \( i \) at time \( t \), \( \mathbf{I}_{ct} \) and \( \mathbf{P}_{ct} \) are the fuzzy importance number and the value calculated by FIS for each class, respectively, and \( n \) is the number of classes.

Now, this weight must be defuzzified. In this paper, a linear ranking function is applied for converting the calculated fuzzy weight into the crisp equivalent number, with the first index of Yager (1978, 1981). Thus, by applying the first index of Yager and by considering the triangular fuzzy numbers, a defuzzified number of \( \hat{W}_i^\text{f} \) is calculated by Eq. (2).

\[
W_i^\text{f} = \mathbf{w}_i^\text{c} + \frac{d_i^R - d_i^L}{3} \quad \forall i \in I, t \in T,
\]

where \( d_i^R \) and \( d_i^L \) are the lateral margins (right and left, respectively) of the central point \( \mathbf{w}_i^\text{c} \).

3.2 Coverage Distance

One of the assumptions in the cover location problem is the “abrupt” termination of coverage; critical points in a distance are fully covered and do not receive any coverage outside of that distance. This approach seems to be unrealistic in practice. Berman et al. (2003) developed a generalization of the cover location problem where two coverage radii have been replaced with this distance.

Based on the above content, the supplier selection decisions could be modeled by using the notion of gradual covering. The percentage of the final importance calculated for the supplier \( i \) when the distance to the nearest manufacturer is less than of \( u \), maximum distance that the manufacture is willing to purchase its needed materials from this supplier.

If we assume that there is a same pair of radii \((l, u)\) for all manufactures, to provide raw materials for factory \( j \), supplier \( i \) is evaluated by the calculated importance if it is fully covered by it \((d_{ij} \leq l)\). Eventually, if it is not covered by any manufacture, none of them is considered \((d_{ij} \geq u)\).

Finally, the percentage of the final importance calculated for supplier \( i \) is placed in the model for selecting

Table 1. Linguistic variable values for weighting of each class

<table>
<thead>
<tr>
<th>Linguistic data</th>
<th>Triangular fuzzy number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extremely unimportant</td>
<td>(0.0, 0.0, 0.1)</td>
</tr>
<tr>
<td>Strongly unimportant</td>
<td>(0.0, 0.1, 0.3)</td>
</tr>
<tr>
<td>Unimportant</td>
<td>(0.1, 0.3, 0.5)</td>
</tr>
<tr>
<td>Moderately</td>
<td>(0.3, 0.5, 0.7)</td>
</tr>
<tr>
<td>Important</td>
<td>(0.5, 0.7, 0.9)</td>
</tr>
<tr>
<td>Strongly important</td>
<td>(0.7, 0.9, 1.0)</td>
</tr>
<tr>
<td>Extremely important</td>
<td>(0.9, 1.0, 1.0)</td>
</tr>
</tbody>
</table>
the most qualified if the supplier is partially covered \((l < ds_{ij} < u)\). In this case, the coverage percentage can be expressed as follows:

\[
p(c_{ij}) = \begin{cases} 
1 & \text{if } ds_{ij} \leq l \\
\frac{u - ds_{ij}}{u - l} & \text{if } l < ds_{ij} < u \\
0 & \text{if } ds_{ij} \geq u
\end{cases}
\]

(3)

It is noted that the producers could be assumed to have different coverage intervals and as illustrated in Figure 1. Figure 6 shows the weight percentage of supplier \(i\), which is evaluated to provide raw materials for manufacture \(j\) when located in distance \(d_{ij}\).

### 3.3 Mathematical Formulation

This section illustrates a nonlinear programming

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**Table 2. Nomenclature**

<table>
<thead>
<tr>
<th>Index Description</th>
<th></th>
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<tbody>
<tr>
<td><strong>Set</strong></td>
<td></td>
</tr>
<tr>
<td>(P) Set of product types</td>
<td></td>
</tr>
<tr>
<td>(Q) Set of part types</td>
<td></td>
</tr>
<tr>
<td>(T) Set of time periods</td>
<td></td>
</tr>
<tr>
<td>(I) Set of fixed supplier locations</td>
<td></td>
</tr>
<tr>
<td>(J) Set of production centers</td>
<td></td>
</tr>
<tr>
<td>(K) Set of customer zones</td>
<td></td>
</tr>
<tr>
<td>(L) Set of production lines</td>
<td></td>
</tr>
<tr>
<td>(N) Set of capacity levels for lines</td>
<td></td>
</tr>
<tr>
<td><strong>Parameter</strong></td>
<td></td>
</tr>
<tr>
<td>(s_{l}^{n}) Setup cost for installation of line (l) with capacity level (n) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(f_{l}^{n}) Operating fixed cost for line (l) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(v_{p}) Variable operating cost of per unit manufactured product by line (l) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(w_{pj}) Inventory capacity of manufactured products at center (j) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(h_{pj}) Inventory holding cost at center (j) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(pc_{lj}) Penalty cost per unit of non-utilized capacity at line (l) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(ca_{lj}^{p}) Capacity with level (n) for line (l) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(pr_{pt}) Unit profit of product (p) selling in period (t)</td>
<td></td>
</tr>
<tr>
<td>(d_{pqk}) Quantity of demand for product (p) by customer zone (k) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(r_{pq}) Units of part (q) which is required to produce one unit of product (p)</td>
<td></td>
</tr>
<tr>
<td>(z_{pil}) Binary parameter, it takes 1 if line (l) be able to manufacture product (p) and 0 otherwise</td>
<td></td>
</tr>
<tr>
<td>(p(c_{ij})) Covering percent of supplier (i) by center (j) (calculated by Eq. (3))</td>
<td></td>
</tr>
<tr>
<td>(w(s_{ij})) Calculated weight by FIS for supplier (i) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(p(s_{ij})) The risk of supplier (i) in the provision of part (q) for using at center (j) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(ca(s_{ij})) The capacity limits for supplying part (q) by supplier (i) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(\rho_{ij}) The productivity factor of supplier (i) in the provision of part (q) in period (t)</td>
<td></td>
</tr>
<tr>
<td><strong>Decision variable</strong></td>
<td></td>
</tr>
<tr>
<td>(Q_{1q}^{ij}) Quantity of part (q) purchased of supplier (i) for line (l) at center (j) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(Q_{2}^{ij}) Quantity of manufactured products by line (l) in center (j) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(Q_{3}^{ij}) Quantity of product (p) sold by center (j) to customer zone (k) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(\beta_{qij}) Proportion of supplier (i) within the suppliers network in the provision of part (q) for using at center (j) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(I_{p}^{n}) Inventory level of product (p) at center (j) in period (t)</td>
<td></td>
</tr>
<tr>
<td>(X_{j}^{n}) Binary variable, it takes 1 if line (l) with capacity level (n) at center (j) is installed in period (t) and 0 otherwise</td>
<td></td>
</tr>
<tr>
<td>(\varphi_{nl}^{i}) Binary variable, it takes 1 if line (l) with capacity level (n) at center (j) has installed in the period before of (t) and 0 otherwise</td>
<td></td>
</tr>
</tbody>
</table>

FIS: fuzzy inference system.
model to apply the marked factors, supplier weight and distance effect, in designing and strategic planning of SC. This multi-objective programming problem is presented as a methodology to offer a set of the best various decisions that ensure a tradeoff between different objectives with respect to restrictions.

3.3.1 Assumptions and limitations
The following are the assumptions considered in the model:
1) The model is a multi-period.
2) The location of suppliers and producers are known.
3) Cost parameters (setup, fixed variable, non-utilized capacity, non-satisfied demands and holding costs) are known for each center, product and time period.
4) The holding costs depend on the residual inventory at the end of each period.
5) Each production line manufactures one product type.

3.3.2 Parameters and model formulation
The used nomenclature in the model is shown in Table 2.

Based on the aforementioned parameters and indices, the MINLP model is developed as follows.

\[ \sum_{p} Q_{3,pkt} = d_{pl}, \quad \forall p \in P, k \in K, t \in T, \]  \hspace{1cm} (13)

\[ \beta_{ij} = \sum_{q} Q_{ijqt} \bigg/ \left( \sum_{q} \sum_{i} Q_{ijqt} \right), \quad \forall q \in Q, \forall i \in I, \forall j \in J, t \in T, \]  \hspace{1cm} (14)

\[ \sum_{j} \sum_{t} Q_{ijqt} \leq ca(s_{ij}), \quad \forall q \in Q, \forall i \in I, t \in T, \]  \hspace{1cm} (15)

\[ \sum_{i}^{n} X_{ijt} = \phi_{ij}, \quad \forall j \in J, \forall i \in I, \forall t \in T, \]  \hspace{1cm} (16)

\[ X_{ij} \cdot \phi_{ij} \in [0, 1], \quad \forall j \in J, \forall i \in I, n \in N, t \in T, \]  \hspace{1cm} (17)

\[ I_{prt}, \forall q \in Q, \forall i \in I, \forall j \in J, \forall k \in K, \forall l \in L, t \in T, \]  \hspace{1cm} (18)

In the model, Eqs. (4)–(6) are the objective functions. Eq. (4) namely minimizes the setup, total fix and variable costs for production lines, inventory-holding costs and unutilized capacity penalty at production facilities, and at the end of this equation, we maximize the total profit of new products’ sale. The objective function (5) minimizes the risk associated with the procurement of the needed parts from the suppliers. The applied risk equation is a modification of the problem studied by Kenyon and Neureuther (2012). Furthermore, the objective function (6) maximizes the importance of suppliers. Constraint (7) ensures that production lines can be installed for one product and with one capacity level during the entire periods. Eq. (8) represents the relationship between the amounts of purchasing raw materials from the suppliers and the number of needed products in the production line. Eq. (9) expresses the capacity limitation of different production lines. Constraints in set (10) assure the inventory balance of products at the centers’ warehouses across periods. Eq. (11) states that there is not any initial inventory of products at the centers’ warehouses. Eq. (12) formulates a capacity constraint for holding extra products in the warehouse. Eq. (13) relates the quantity of manufactured products to their potential quantity demanded in customer zones. On the other hand, this equation shows the necessity for satisfying customers’ demand. Eq. (14) relates the proportion of supplier to meet the needs of the producer to all suppliers. Eq. (15) formulates the capacity limit in supplying the needed parts. Eq. (16) shows opened production lines in previous periods. Finally, constraints in set (17) enforce the integrality restrictions on the binary variables and constraints in set (18) enforce the non-negativity restrictions on the corresponding decision variables.

4. SOLVING METHOD
This research aims to develop an integrated SC planning that uses the production modules. Therefore, illustrations of a numerical example will emphasize the selection and assignment of suppliers, as well as the strategic decisions about the commissioning of produc-
tion lines with different capacities. In addition, it will determine the optimal amount of production volume. Prior to demonstrating an example problem, an approach has been applied for solving the proposed multi-objective model, which will be discussed as follows.

4.1 Solving Multi-objective Programming

We use a general form of multi-objective programming that is a family of $L_p$-metrics and is adopted from Tzeng and Huang (2011). This method considers the minimum deviation from the ideal solution as follows:

$$\begin{align*}
\text{Max } & f_i(x), f_2(x), \ldots, f_n(x) \\
\text{Min } & g_1(x), g_2(x), \ldots, g_m(x) \\
\text{S.t. } & x \in X
\end{align*}$$

That $f_i(x), f_2(x), \ldots, f_n(x)$ and $g_1(x), g_2(x), \ldots, g_m(x)$ are the objective functions and $x$ is the feasible region. First, an ideal solution for each objective function is separately obtained by solving the following problems:

$$\begin{align*}
f_i^* &= \text{Max } f_i(x) \quad i \in (1, \ldots, n) \\
&\text{s.t. } x \in X \\
f_i' &= \text{Min } f_i(x) \quad i \in (1, \ldots, n) \\
&\text{s.t. } x \in X \\
g_j^* &= \text{Max } g_j(x) \quad j \in (1, \ldots, m) \\
&\text{s.t. } x \in X \\
g_j' &= \text{Min } g_j(x) \quad j \in (1, \ldots, m) \\
&\text{s.t. } x \in X
\end{align*}$$

Afterwards, a non-scale function is achieved by dividing each function in its optimal value. Thus, following the new objective function can satisfactorily solve the multi-objective programming problem:

$$\begin{align*}
\text{Min } F &= \left[ \sum_{i} \alpha^p \frac{f_i^* - f_i(x)}{f_i^*} \right]^\frac{1}{p} + \left[ \sum_{j} \beta^p \frac{g_j(x) - g_j^*}{g_j^* - g_j} \right]^\frac{1}{p} \\
\text{s.t. } & x \in X
\end{align*}$$

where each function is weighted using "$\alpha$" to denote the importance of objective functions. This adjustment weight is used for alimenting and balancing between functions that will be determined by decision-makers just as the following relationship can be established.

$$\sum_i \alpha_i + \sum_j \beta_j = 1$$

$$\alpha_i, \beta_j \geq 0 \quad i \in (1, \ldots, n), j \in (1, \ldots, m)$$

Obviously the result is dependent on the value of $p$. Generally, $p$ is 1 or 2. However, other values of $p$ can also be used. We applied the following auxiliary objective function in the new model:

$$\begin{align*}
\text{Min } F &= \left[ \sum_{i} \alpha^p \frac{f_i^* - f_i(x)}{F_i^* - F_i} \right] + \left[ \sum_{j} \beta^p \frac{g_j(x) - g_j^*}{g_j^* - g_j} \right]^\frac{1}{p} \\
\text{s.t. } & x \in X
\end{align*}$$

4.2 Numerical Example

A small illustrative example has been developed to evaluate the performance of the model, which included 3 products manufactured by 4 plants with their 5 required parts being supplied by 7 suppliers. Matched with the number of products, 3 production lines with 3 possible capacities for construction were intended. In this example, we examined 5 customer zones with different demands during 3 periods.

Our study includes the typical input information for cost, capacity, demands, and the main related parameters to create the basis for model outputs. Other than the weight and risk for suppliers, which we used normal distribution, the parameters are generated using uniform distribution. The parameters' value of the investigated example is given in Tables 3–9. The problem is solved for $p = 1$ and the importance of objective functions is determined as $\alpha_1 = 0.42, \alpha_2 = 0.23$ and $\alpha_3 = 0.35$.

The model has been solved by the Lingo 13.0 solver. The experiments were run in an Intel Core i3 CPU, at 2.13 GHz and with 4.00 GB of RAM memory. The example involved 1,728 total variables, 108 binary variables and 966 constraints and the final solution took approximately 14 seconds.

### 4.2.1 Inputs

Table 3 presents basic information about the product lines considered at the start-up time. For example, the implementation of production line 1 with the production capacity of 4,120 units will cost $158. Table 4 shows fixed and variable costs of the production line operation and penalty for non-utilized capacity. Production line 1, for instance, charges $37 independent of its oper-

<table>
<thead>
<tr>
<th>Table 3. Fundamental data associated with the production lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production line</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>Capacity</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>Capacity</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>Capacity</td>
</tr>
</tbody>
</table>

The parameter values have been considered same in the different periods.
The parameters have been considered same in the different periods.

Importance weight, productivity factor and capacity limit in the provision of parts by suppliers.
coverage intervals that is intended for production center 1, for example, the cover percent is equal to 1. The provision risk of different parts for being used at production centers is another parameter in this table, which has been considered the same in the different periods. For supplying part 1 of supplier 1 to be used in production center 1, for example, the risk would be 0.35 in period 1.

Table 9 provides parameters associated with the sale of various products including customers’ demand and the benefits earned from the sale. The demand of customer zone 1 and the unit profit for product 1 in period 1, respectively are 3,680 units and $1.34. If the company has already set up the supply network as part of the business process, the demand for various products can be estimated on the basis of historical data of sales and customer locations. Otherwise, if the company is planning to set up a network for the first time, the demand may be estimated based on customer surveys.

The supplying risk has been considered same in the different periods.

The supplying risk of parts and covering percent of suppliers by production centers.

Table 8. Associated parameters with the suppliers that intended to distinguish between the different production centers

<table>
<thead>
<tr>
<th>Production center</th>
<th>Parameter</th>
<th>Supplier</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Covering percent</td>
<td>1.0</td>
<td>0.76</td>
<td>0.98</td>
<td>1.0</td>
<td>0.68</td>
<td>1.0</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Risk</td>
<td>0.35</td>
<td>0.13</td>
<td>0.00</td>
<td>0.42</td>
<td>0.20</td>
<td>0.39</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>0.51</td>
<td>0.44</td>
<td>0.27</td>
<td>0.01</td>
<td>0.37</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.00</td>
<td>0.52</td>
<td>0.43</td>
<td>0.24</td>
<td>0.01</td>
<td>0.08</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.42</td>
<td>0.19</td>
<td>0.33</td>
<td>0.03</td>
<td>0.18</td>
<td>0.04</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.48</td>
<td>0.19</td>
<td>0.39</td>
<td>0.40</td>
<td>0.42</td>
<td>0.02</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Covering percent</td>
<td>0.86</td>
<td>0.66</td>
<td>0.40</td>
<td>0.62</td>
<td>1.0</td>
<td>0.99</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Risk</td>
<td>0.34</td>
<td>0.16</td>
<td>0.25</td>
<td>0.40</td>
<td>0.40</td>
<td>0.22</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.15</td>
<td>0.46</td>
<td>0.26</td>
<td>0.53</td>
<td>0.14</td>
<td>0.40</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.21</td>
<td>0.42</td>
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<td>0.51</td>
<td>0.19</td>
<td>0.36</td>
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<td></td>
<td>0.00</td>
<td>0.42</td>
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<td>0.38</td>
<td>0.24</td>
<td>0.14</td>
<td>0.34</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>0.45</td>
<td>0.13</td>
<td>0.30</td>
<td>0.16</td>
<td>0.35</td>
<td>0.23</td>
<td>0.49</td>
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</tr>
<tr>
<td>3</td>
<td>Covering percent</td>
<td>0</td>
<td>0.32</td>
<td>1.0</td>
<td>1.0</td>
<td>0.12</td>
<td>0.43</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Risk</td>
<td>0.34</td>
<td>0.11</td>
<td>0.23</td>
<td>0.29</td>
<td>0.18</td>
<td>0.19</td>
<td>0.20</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>0.28</td>
<td>0.42</td>
<td>0.49</td>
<td>0.25</td>
<td>0.02</td>
<td>0.44</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.37</td>
<td>0.36</td>
<td>0.07</td>
<td>0.45</td>
<td>0.52</td>
<td>0.50</td>
<td>0.41</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>0.41</td>
<td>0.34</td>
<td>0.19</td>
<td>0.41</td>
<td>0.08</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.25</td>
<td>0.33</td>
<td>0.18</td>
<td>0.38</td>
<td>0.14</td>
<td>0.40</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Covering percent</td>
<td>0.46</td>
<td>1.0</td>
<td>0.82</td>
<td>0.06</td>
<td>0</td>
<td>0.08</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Risk</td>
<td>0.35</td>
<td>0.11</td>
<td>0.26</td>
<td>0.25</td>
<td>0.10</td>
<td>0.24</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.44</td>
<td>0.07</td>
<td>0.13</td>
<td>0.31</td>
<td>0.38</td>
<td>0.21</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18</td>
<td>0.07</td>
<td>0.48</td>
<td>0.21</td>
<td>0.29</td>
<td>0.25</td>
<td>0.17</td>
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<tr>
<td></td>
<td></td>
<td>0.53</td>
<td>0.10</td>
<td>0.49</td>
<td>0.21</td>
<td>0.04</td>
<td>0.53</td>
<td>0.07</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>0.33</td>
<td>0.07</td>
<td>0.12</td>
<td>0.26</td>
<td>0.46</td>
<td>0.24</td>
<td>0.04</td>
<td></td>
</tr>
</tbody>
</table>

The parameter values have been considered same in the different periods.

Table 9. Associated parameters with the products sale

<table>
<thead>
<tr>
<th>Product</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit ($)</td>
<td>1.34</td>
<td>0.91</td>
<td>1.01</td>
</tr>
<tr>
<td>Demand-1</td>
<td>Customer zone 1</td>
<td>3,680</td>
<td>3,670</td>
</tr>
<tr>
<td></td>
<td>Customer zone 2</td>
<td>3,420</td>
<td>3,330</td>
</tr>
<tr>
<td></td>
<td>Customer zone 3</td>
<td>4,770</td>
<td>3,630</td>
</tr>
<tr>
<td></td>
<td>Customer zone 4</td>
<td>3,460</td>
<td>3,350</td>
</tr>
<tr>
<td></td>
<td>Customer zone 5</td>
<td>2,970</td>
<td>3,440</td>
</tr>
<tr>
<td>Demand-2</td>
<td>Customer zone 1</td>
<td>4,250</td>
<td>7,550</td>
</tr>
<tr>
<td></td>
<td>Customer zone 2</td>
<td>5,640</td>
<td>5,590</td>
</tr>
<tr>
<td></td>
<td>Customer zone 3</td>
<td>5,580</td>
<td>7,470</td>
</tr>
<tr>
<td></td>
<td>Customer zone 4</td>
<td>5,590</td>
<td>4,520</td>
</tr>
<tr>
<td></td>
<td>Customer zone 5</td>
<td>7,810</td>
<td>7,320</td>
</tr>
</tbody>
</table>

The parameter values have been considered same in the different periods.
4.2.2 Model outputs

We begin illustrating the model outcomes that select the best suppliers and design a proper planning for each product to be manufactured in a right way that fulfills the market demand. Based on the values of effective parameters given in Tables 3–9, the model determines appropriate suppliers for supplying the primary parts and calculates the optimal numbers of production lines to ensure the optimality of the objective function.

The design has been separately finalized for each objective function; the model estimated the best suppliers for planning and determining the optimum strategies for purchasing, production, and storage. Table 10 listed several main and significant variables in the model in different periods in the optimization of the integrated mode. In this table, the launched production lines, the number of different manufactured products, the average percentage of the lines’ used capacity in each center, stored inventory in the warehouse, and eventually the objective value have been reported. According to this table, the aggregated objective function enables us to consider all objective functions simultaneously.

Table 11 includes the percentage of supplying raw materials from different suppliers in three periods which is shown in comparison with the considered weight \( w(s_{ij}) \times p(c_{ij}) \) for each supplier. For example, it can be observed that the percentage of supplying needed parts for production center 1 at period 1 of suppliers 1 to 7 are \{0.07, 0, 0.73, 0.14, 0, 0.07, 0\}, respectively. This difference is justified according to the higher risk and lower weight of supplier 2 in comparison to supplier 1. Optimal planning results of computations show the effectiveness of the developed problem in a multi-period environment.

### Table 10. Obtained results of model by the commercial solver

<table>
<thead>
<tr>
<th>Time</th>
<th>Production center</th>
<th>Capacity levela)</th>
<th>Manufactured productb)</th>
<th>Average production capacity utilization (%)</th>
<th>Inventory level</th>
<th>Objective value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 3 3 3</td>
<td>7,350 6,570 7,370</td>
<td>100 3,793</td>
<td>0.0711</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 3 3 -</td>
<td>7,350 6,570 0</td>
<td>100 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 2 2 3</td>
<td>7,350 3,953 3,500</td>
<td>74 6,400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 3 3 2</td>
<td>7,350 6,570 5,980</td>
<td>100 7,600</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 3 3 3</td>
<td>5,918 6,570 7,370</td>
<td>93 7,270</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 3 3 3</td>
<td>4,528 7,370 56</td>
<td>0 6,400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 2 3 3</td>
<td>3,189 5,980 46</td>
<td>0.0711</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 3 3 2</td>
<td>1,442 6,570 5,980</td>
<td>70 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1 3 3 3</td>
<td>7,350 6,570 7,370</td>
<td>100 0</td>
<td></td>
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<td>2 3 3 3</td>
<td>0 4,528 7,370 56</td>
<td>0 6,400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 2 3 3</td>
<td>6,820 5,220 5,980</td>
<td>90 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 3 3 2</td>
<td>7,350 6,570 5,980</td>
<td>100 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1 3 3 3</td>
<td>7,350 6,570 7,370</td>
<td>100 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 3 3 3</td>
<td>6,820 5,220 5,980</td>
<td>90 0</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>3 2 3 3</td>
<td>6,820 5,220 5,980</td>
<td>90 0</td>
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<td>4 3 3 2</td>
<td>7,350 6,570 5,980</td>
<td>100 0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Capacity level of launched production lines, b) manufactured products by production lines.

### Table 11. Comparison between supply percentage and two effective factors (the weight and risk of each supplier)

<table>
<thead>
<tr>
<th>Time</th>
<th>Production center</th>
<th>The share of suppliers (risk, weight, supply percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 0.48, 0.59, 0.07</td>
<td>0.72, 0.54, 0.00 0.25, 0.90, 0.73 0.51, 0.77, 0.14 0.50, 0.46, 0.00 0.59, 0.88, 0.07 0.30, 0.59, 0.00</td>
</tr>
<tr>
<td></td>
<td>2 0.59, 0.51, 0.00</td>
<td>0.59, 0.47, 0.25 0.80, 0.37, 0.00 0.64, 0.48, 0.00 0.60, 0.68, 0.07 0.64, 0.87, 0.68 0.52, 0.32, 0.00</td>
</tr>
<tr>
<td></td>
<td>3 0.47, 0.00, 0.00</td>
<td>0.68, 0.23, 0.00 0.53, 0.92, 0.28 0.67, 0.77, 0.59 0.87, 0.08, 0.00 0.50, 0.38, 0.00 0.51, 0.22, 0.12</td>
</tr>
<tr>
<td></td>
<td>4 0.44, 0.27, 0.00</td>
<td>0.33, 0.71, 0.33 0.72, 0.75, 0.11 0.35, 0.05, 0.00 0.83, 0.00, 0.00 0.47, 0.07, 0.00 0.39, 0.77, 0.56</td>
</tr>
<tr>
<td>2</td>
<td>1 0.52, 0.66, 0.31</td>
<td>0.93, 0.56, 0.00 0.40, 0.90, 0.19 0.43, 0.79, 0.12 0.62, 0.45, 0.23 0.44, 0.64, 0.15 0.76, 0.46, 0.00</td>
</tr>
<tr>
<td></td>
<td>2 0.67, 0.57, 0.00</td>
<td>0.76, 0.49, 0.06 0.81, 0.37, 0.00 0.91, 0.49, 0.00 0.67, 0.66, 0.11 0.46, 0.63, 0.83 0.82, 0.25, 0.00</td>
</tr>
<tr>
<td></td>
<td>3 0.53, 0.00, 0.00</td>
<td>1.05, 0.24, 0.00 0.59, 0.92, 1.00 1.09, 0.79, 0.00 0.94, 0.08, 0.00 0.56, 0.28, 0.00 0.54, 0.17, 0.00</td>
</tr>
<tr>
<td></td>
<td>4 0.50, 0.30, 0.00</td>
<td>0.62, 0.74, 0.35 0.55, 0.75, 0.31 0.67, 0.05, 0.00 0.94, 0.00, 0.00 0.37, 0.05, 0.00 0.63, 0.60, 0.34</td>
</tr>
<tr>
<td>3</td>
<td>1 0.57, 0.76, 0.13</td>
<td>0.66, 0.72, 0.00 0.29, 0.81, 0.15 0.87, 0.89, 0.64 0.51, 0.39, 0.00 0.39, 0.82, 0.09 0.79, 0.49, 0.00</td>
</tr>
<tr>
<td></td>
<td>2 0.72, 0.65, 0.13</td>
<td>0.55, 0.63, 0.06 0.87, 0.33, 0.00 1.30, 0.55, 0.00 0.46, 0.57, 0.00 0.49, 0.81, 0.81 0.86, 0.27, 0.00</td>
</tr>
<tr>
<td></td>
<td>3 0.54, 0.00, 0.00</td>
<td>0.68, 0.30, 0.00 0.67, 0.83, 0.22 1.27, 0.89, 0.66 0.78, 0.07, 0.00 0.36, 0.35, 0.12 0.67, 0.18, 0.00</td>
</tr>
<tr>
<td></td>
<td>4 0.51, 0.35, 0.00</td>
<td>0.47, 0.95, 0.89 0.75, 0.68, 0.11 0.80, 0.05, 0.00 0.76, 0.00, 0.00 0.47, 0.07, 0.00 0.64, 0.65, 0.00</td>
</tr>
</tbody>
</table>

The share of suppliers in the supply of parts and assigned weight and risk to them in each period (risk, weight, supply percent).
5. CONCLUSION

In this paper, first, we investigated an approach for appraising potential providers based on FIS and the effect of gradual covering distance. Afterwards, these two factors along with other effective parameters were applied to configure a mixed integer non-linear programming model for supplier selection, order allocation and determining the production strategy in an integrated supply network. To solve the proposed multi-objective mathematical model, a method with the aim of minimizing the distance to the ideal vector was used. The final model was developed with the commercial optimization software, Lingo, which provided an illustrative example to analyze and validate the model. It should be noted that these findings may also be used for comparison in future works.

Some features of the proposed approach can be summarized as follows:
1) Decision-makers use the linguistic terms to define rules that may contain different types of importance from one to another.
2) Since there are not many articles that discuss sustainability issues in supplier evaluation, this paper has provided a review of the capability of the sustainable criteria in the assessment by FIS.
3) The proposed approach is applicable to any number of suppliers and criteria that are commonly used in large companies.
4) Even though defining rules in the FIS seems complicated and time consuming, the assessment costs are reduced dramatically due to the similar use in evaluating all supply centers, various parts and different periods.

In addition to the points of the FIS method, this paper has considered the extensive use of different criteria in assessing suppliers by a comprehensive system as well as considering the effect of cover distance in the decay form that has not been seen in supplier evaluation papers.

The present study has examined several issues that could be further investigated in future research. For example, the accuracy and efficiency of the proposed method could be improved. A number of verification and validation methods may be helpful in testing the accuracy and consistency of the process. Furthermore, it is supposed the centers have already been constructed. Since the computational time increases significantly when it is decided for constructing new facilities, developing a heuristic solution method is critical in overcoming the obstacle.

REFERENCES


