Adaptive Control Design for Permanent Magnet Synchronous Motor with Uncertain Parameters: An LMI Approach

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Abstract—In this paper, an adaptive control scheme is proposed for the permanent magnet synchronous motor (PMSM) with uncertain parameters using an LMI approach. The adaptive control scheme is used to compensate some uncertain terms in the PMSM drive system. In this study, a novel formulation for the uncertain PMSM drive system is developed. By the formulation, the stability condition for the closed-loop uncertain PMSM drive system with the proposed controllers can be characterized in terms of some linear matrix inequalities (LMIs). The LMIs problem (LMIP) can be efficiently solved. Numerical solutions for the control gains and simulation results are provided to illustrate the design procedure and the performances. In opposition to try and error, the control gains in both speed loop and current loop can be obtained systematically by solving the corresponding LMIs. The proposed method is simple and is suitable for practical control design in motor drive.

I. INTRODUCTION

Permanent magnet synchronous motors (PMSM) have many applications in industrial due to its compact structure, high efficiency, high power density, and high torque to inertia ratio. To achieve satisfactory response in the presence of parameters variations, many robust vector-control strategies have been addressed in the design of the PMSM drive including $H_\infty$ control [1], sliding-mode control [2], [3], [4], adaptive control [5], [6], and fuzzy neural-network control [7], [8] or iterative learning control [9], [10]. For $H_\infty$ control, the order of the conventional $H_\infty$ controller is much higher than that of the plant, hence the high-order $H_\infty$ controller is not a welcome option for the most of the engineers. Chattering phenomenon and high heat loss in electrical power circuits are drawbacks for the sliding-mode control. Moreover, series chattering may excite unmodeled high-frequency dynamics, which degrades the performance of the system or leads to instability. For adaptive control, adaptive laws are often used to compensate the effect of parameters variations. In the development of the adaptive control algorithm, the use of the regressor matrix has become popular. In this situation, the uncertain terms can be expressed as a product of a regressor matrix and an uncertain parameter vector, which is also called linear parametrization form. And then, a parameter update law has been proposed to estimate the uncertain parameters which are assumed to be uncertain constants or slowly-varying uncertain parameters. However, sometimes, the transient response may not be satisfactory for the adaptive control. For the fuzzy neural-network control or iterative learning control, complex learning rules are often used to compensate the effect of the variations on parameters or plant uncertainties.

The PI, the most popular controller, plays an important role in the control system design of many industrial applications [11], [12]. However, the PI controller generally leads to large overshoot and long settling time. In this study, the proposed controller is slightly different from the conventional PI controller where a low-pass filter instead of a pure integrator is proposed. By this arrangement, the overshoot can be improved, however the steady-state error may not be satisfied. Fortunately, by a suitable adjustment for a design parameter, both the overshoot and the steady-state error can be improved simultaneously. The performance of the PMSM drive is affected by the variations of the plant parameters. To compensate the effect of the uncertain parameters, an adaptive control scheme is also proposed in this study.

In this study, by a suitable arrangement, a novel formulation for the uncertain PMSM drive system is developed. By the formulation, the stability condition for the closed-loop uncertain PMSM drive system with the proposed controllers can be characterized in terms of some linear matrix inequalities (LMIs) [13]. The LMIs problem (LMIP) can be efficiently solved by the LMI toolbox in Matlab [14]. For the most of the previous control schemes for the PMSM drive, the speed controller and current controller are designed separately. However, in this study, the speed controller and current controller are considered simultaneously. In opposition to try and error, the control gains in both speed loop and current loop can be obtained systematically by solving the corresponding LMIs. Numerical solutions of the control gains using the proposed LMI formulation and simulation examples are provided to illustrate the design procedure and the corresponding performances.

The main contributions of this study are stated as follows:

(1) A novel adaptive control scheme is proposed for the permanent magnet synchronous motor (PMSM) with uncertain parameters using an LMI approach. In opposition to try and error, the control gains in both speed loop and
current loop can be obtained systematically by solving the corresponding LMIs.

(2) Differently from the conventional PI controller, a novel controller is proposed where a low-pass filter instead of a pure integrator is proposed. By a suitable adjustment for a design parameter, both the overshoot and the steady-state error can be improved simultaneously.

(3) The speed controller and current controller are considered simultaneously using the proposed LMI approach.

The rest of this paper is organized as follows: The model description is presented in Section II. In Section III, problem formulation and its solution are presented. In Section IV, simulation examples are provided. Finally, concluding remarks are made in Section VI.

II. MODEL DESCRIPTION

The dynamic model of a typical sinusoidal permanent magnet synchronous motor (PMSM) with uncertain parameters in the synchronously rotating reference frame is given by (the time variable $t$ is suppressed for simplicity)

$$
p \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_s} & \omega_r \\ -\omega_r & -\frac{R_s}{L_s} \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \frac{1}{L_s} \begin{bmatrix} v_{ds} \\ v_{qs} - \omega_r \lambda_f \end{bmatrix}
$$

$$
T_e = \left( \frac{3}{2} \right) \left( \frac{P}{2} \right) \lambda_f i_{qs}
$$

where $i_{ds}$, $i_{qs}$: the $d$-axis and $q$-axis currents; $v_{ds}$, $v_{qs}$: the $d$-axis and $q$-axis voltages; $R_s$: the uncertain stator resistance with $R^-_s \leq R_s \leq R^+_s$; $L_s$: the uncertain stator inductance with $L^-_s \leq L_s \leq L^+_s$; $\lambda_f$: the uncertain flux linkage due to the permanent magnet with $\lambda^-_f \leq \lambda_f \leq \lambda^+_f$; $p$: differential operator; $\omega_r$: the rotor speed; $T_e$: the electromagnetic torque; $P$: the number of poles.

The electromechanical equation of the PMSM is

$$
\ddot{J}_m \frac{d\omega_r}{dt} + B_m \omega_r = T_e = \left( \frac{3P}{4} \right) \lambda_f i_{qs}
$$

where $\omega_r$: the rotor mechanical speed with $\omega_r = \frac{P}{2} \omega_r$; $\dot{J}_m$: the uncertain inertia of moment with $J^-_m \leq \dot{J}_m \leq J^+_m$; $B_m$: the uncertain viscous friction coefficient with $B^-_m \leq B_m \leq B^+_m$.

Remark 1: In this study, it is assumed that the parameters $R_s$, $L_s$, $\lambda_f$, $\dot{J}_m$, and $B_m$ are uncertain constants or slowly-varying uncertain parameters, but their upper bound ( $R^+_s$, $L^+_s$, $\lambda^+_f$, $J^+_m$, and $B^+_m$ ) and lower bound ( $R^-_s$, $L^-_s$, $\lambda^-_f$, $J^-_m$, and $B^-_m$ ) are known. This assumption satisfies certain practical situations for the PMSM drive system at least at the steady-state.

Note that (1) can be written as follows

$$
\begin{bmatrix} \dot{L}_s & 0 \\ 0 & \dot{L}_s \end{bmatrix} p \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} = \begin{bmatrix} -\dot{R}_s & 0 \\ 0 & -\dot{R}_s \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \begin{bmatrix} v_{ds} \\ v_{qs} \end{bmatrix}
$$

Three new error states $e_{ds}$, $e_{qs}$, and $e_l$ with the following dynamic equations

$$
\dot{e}_{ds} = -\kappa_1 e_{ds} + e_{ds} \\
\dot{e}_{qs} = -\kappa_1 e_{qs} + e_{qs} \\
\dot{e}_l = -\kappa_2 e_l + e
$$

where $\kappa_1$ and $\kappa_2$ are some positive constants which are design parameters and three error notations $e_{ds}, e_{qs}$, and $e_l$ are defined as follows

$$
e_{ds} = i^*_{ds} - i_{ds} \\
e_{qs} = i^*_{qs} - i_{qs} \\
e = \omega_{rm}^* - \omega_{rm}
$$

are introduced where reference settings $i^*_{ds}$, $i^*_{qs}$, and $\omega_{rm}^*$ are determined later.

Remark 2: For regulation problem, the reference setting $\omega_{rm} = 0$ can be assumed without loss of generality.

Remark 3: In this study, for simplicity of design, we let $\kappa = \kappa_1 = \kappa_2$.

Remark 4: The transfer function from $e_{ds}$ (or $e_{qs}$ or $e$) to $e_{ds}$ (or $e_{qs}$ or $e_l$) is $\frac{1}{\alpha_x}$ which is a low-pass filter. The behavior of a low-pass filter is similar to a pure integrator at high frequency. Moreover, the behavior of a low-pass filter $\frac{1}{\alpha_x}$ is like a pure integrator if $\alpha \rightarrow 0$.

From the definition above, the following error dynamics can be obtained

$$
\ddot{L}_s e_{ds} = -\dot{R}_s e_{ds} - v_{ds} - \omega_r i_{qs} \dot{L}_s \\
\ddot{L}_s e_{qs} = -\dot{R}_s e_{qs} + \dot{R}_s K_2 e_l - v_{qs} + \omega_r i_{ds} \dot{L}_s + \omega_r \dot{\lambda}_f - \dot{L}_s K_2 e_l + \dot{L}_s K_2 e \\
\ddot{e}_l = -\dot{B}_m e + \left( \frac{3P}{4} \right) \dot{\lambda}_f e_{qs} - \left( \frac{3P}{4} \right) \dot{\lambda}_f K_2 e_l
$$

Let the voltage commands $v_{ds}$ and $v_{qs}$ be chosen as follows

$$
v_{ds} = K_3 e_{ds} + K_4 e_{ds} + v^a_{ds} \\
v_{qs} = K_5 e_{qs} + K_6 e_{qs} + K_1 e + v^a_{qs}
$$

where $K_1$, $K_3$, $K_4$, $K_5$, and $K_6$ are some positive constants (control gains) and $v^a_{ds}$ and $v^a_{qs}$ are auxiliary (adaptive) control signals, in which all of them can be determined later.
Therefore, by (15), the error dynamics in (12)-(14) can be rewritten as
\[
\begin{align*}
\dot{L}_s &= -\tilde{R}_s e_{ds} - K_3 e_{ds} - K_4 e_{dsl} - v_{i ds}^a - \omega_r i_{qs} \tilde{L}_s, \\
\dot{L}_s &= -\tilde{R}_s e_{qs} + R_s K_2 e_{l} - K_5 e_{qs} - K_6 e_{qsI} - K_1 e, \\
\dot{J}_m &= -\tilde{B}_m e + \left( \frac{3P}{4} \right) \tilde{\lambda}_f K_2 \dot{e}_l
\end{align*}
\]
and consequently the augmented error dynamics can be expressed as the following form
\[
\begin{bmatrix}
\dot{e}_{dsl} \\
\dot{e}_{ds} \\
\dot{e}_{qsI} \\
\dot{e}_{qs} \\
\dot{e}_l \\
\dot{\tilde{e}}
\end{bmatrix}
= \tilde{A} \begin{bmatrix}
e_{dsl} \\
e_{ds} \\
e_{qsI} \\
e_{qs} \\
e_l \\
e
\end{bmatrix}
+ \begin{bmatrix}
0 \\
-\omega_r i_{qs} \tilde{L}_s \\
0 \\
\omega_r i_{ds} \tilde{L}_s + \omega_r \tilde{\lambda}_f \\
0 \\
0
\end{bmatrix}
\]
where
\[
\tilde{A} = \begin{bmatrix}
-\kappa & 1 & 0 & 0 & 0 & 0 \\
-K_4 & M_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & -\kappa & 1 & 0 & 0 \\
0 & 0 & -K_6 & M_{44} & M_{45} & M_{46} \\
0 & 0 & 0 & 0 & -\kappa & 1 \\
0 & 0 & 0 & M_{64} & M_{65} & -\tilde{B}_m
\end{bmatrix}
\]
and
\[
\begin{align*}
M_{22} &= -\tilde{R}_s - K_3 \\
M_{44} &= -\tilde{R}_s - K_5 \\
M_{45} &= \tilde{R}_s K_2 - \tilde{L}_s K_2 \kappa \\
M_{46} &= -K_1 + \tilde{L}_s K_2 \\
M_{64} &= \left( \frac{3P}{4} \right) \tilde{\lambda}_f \\
M_{65} &= -\left( \frac{3P}{4} \right) \tilde{\lambda}_f K_2
\end{align*}
\]
Let the auxiliary (adaptive) control signals be defined as follows
\[
\begin{align*}
v_{i ds}^a &= -\omega_r i_{qs} \tilde{L}_s \quad (17) \\
v_{i qs}^a &= \omega_r i_{ds} \tilde{L}_s + \omega_r \tilde{\lambda}_f \quad (18)
\end{align*}
\]
with the following adaptive laws
\[
\begin{align*}
\dot{\tilde{L}}_s &= \frac{-(\omega_r i_{qs} e_{ds} + \omega_r i_{ds} e_{qs})}{\rho} \quad (19) \\
\dot{\tilde{\lambda}}_f &= \frac{\omega_r e_{qs}}{\rho} \quad (20)
\end{align*}
\]
where \(\rho\) is some positive adaptation gain then the augmented error dynamic in (16) can be rewritten as follows
\[
\begin{bmatrix}
\dot{e}_{dsl} \\
\dot{e}_{ds} \\
\dot{e}_{qsI} \\
\dot{e}_{qs} \\
\dot{e}_l \\
\dot{\tilde{e}}
\end{bmatrix}
= \tilde{A} \begin{bmatrix}
e_{dsl} \\
e_{ds} \\
e_{qsI} \\
e_{qs} \\
e_l \\
e
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]
where the estimated errors \(e_{L_s}\) and \(e_{\lambda_f}\) are denoted as follows
\[
\begin{align*}
e_{L_s} &= \tilde{L}_s - \hat{L}_s \\
e_{\lambda_f} &= \tilde{\lambda}_f - \hat{\lambda}_f
\end{align*}
\]
Remark 5: Obviously, the adaptive control signals \(v_{i ds}^a\) and \(v_{i qs}^a\) are used to cancel the uncertain terms \(-\omega_r i_{qs} \tilde{L}_s\) and \(\omega_r i_{ds} \tilde{L}_s + \omega_r \tilde{\lambda}_f\) in (16), respectively.

Remark 6: In this study, the parameters \(\tilde{L}_s\) and \(\tilde{\lambda}_f\) are assumed to be uncertain constants which imply
\[
\begin{align*}
\dot{\tilde{L}}_s &= \dot{\hat{L}}_s \\
\dot{\tilde{\lambda}}_f &= \dot{\hat{\lambda}}_f
\end{align*}
\]
Note that the augmented error dynamics in (21) can be expressed as the following compact form
\[
\begin{bmatrix}
\dot{\tilde{e}} \\
\dot{\tilde{\eta}}
\end{bmatrix}
= \tilde{E} \begin{bmatrix}
\dot{e}_{L_s} \\
\dot{e}_{\lambda_f}
\end{bmatrix}
\]
where
\[
\tilde{E} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & \tilde{L}_s & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \tilde{L}_s & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \tilde{J}_m
\end{bmatrix}
\]
and
\[
\begin{bmatrix}
\dot{e}_{L_s} \\
\dot{e}_{\lambda_f}
\end{bmatrix}
= \tilde{E}^{-1} \begin{bmatrix}
\dot{e}_{L_s} \\
\dot{e}_{\lambda_f}
\end{bmatrix}
\]
Remark 7: The matrix \(\tilde{E}\) is a symmetric positive definite matrix which is nonsingular and the matrix \(Y\) is usually called regressor matrix.
III. Problem Formulation and Its Solution

The main objective of this study is to determine the control gains $K_1 \sim K_6$ in (11) and (15) such that the closed-loop system in (25) with the adaptive laws in (19)-(20) is stable. Let us consider the following candidate of the Lyapunov function for the closed-loop system in (25)

$$V = \eta^T \dot{E} \eta + \rho \left[ \begin{array}{c} e_{L_s} \\ e_{\lambda_f} \end{array} \right]^T \left[ \begin{array}{c} e_{L_s} \\ e_{\lambda_f} \end{array} \right]$$

(26)

where $\rho$ is some positive constant then its derivative can be expressed as

$$\dot{V} = \eta^T (\dot{A}^T + \dot{A}) \eta + \left( \begin{array}{c} e_{L_s} \\ e_{\lambda_f} \end{array} \right)^T + \eta^T Y \left[ \begin{array}{c} e_{L_s} \\ e_{\lambda_f} \end{array} \right]$$

$$+ \rho \left[ \begin{array}{c} \dot{e}_{L_s} \\ \dot{e}_{\lambda_f} \end{array} \right]^T \left[ \begin{array}{c} e_{L_s} \\ e_{\lambda_f} \end{array} \right] + \rho \left[ \begin{array}{c} e_{L_s} \\ e_{\lambda_f} \end{array} \right]^T \left[ \begin{array}{c} \dot{e}_{L_s} \\ \dot{e}_{\lambda_f} \end{array} \right]$$

(27)

If the adaptive laws (already defined in (19)-(20)) for $\dot{L}_s$ and $\dot{\lambda}_f$ are chosen as follows

$$\left[ \begin{array}{c} \dot{e}_{L_s} \\ \dot{e}_{\lambda_f} \end{array} \right] = -\frac{1}{\rho} Y^T \eta = \left[ \begin{array}{c} (\omega_i q_i e_{q,i}, \omega_i i_i e_{i,i}) \\ \frac{\alpha_i \eta_i}{\rho} \end{array} \right]$$

(28)

where $\rho$ is also called adaptation gain, then

$$\dot{V} = \eta^T (\dot{A}^T + \dot{A}) \eta$$

(29)

If the control gains $K_1 \sim K_6$ are chosen properly such that

$$\dot{A}^T + \dot{A} < 0$$

(30)

then

$$\dot{V} \leq 0$$

(31)

which guarantees that the closed loop system in (25) with the adaptive laws in (19)-(20) is stable in the sense of Lyapunov.

Remark 8: Although, the stability of the closed-loop system is guaranteed for any adaptation gain $\rho$, the performance of the adaptive laws depends critically on $\rho$. If a small gain is chosen, the adaptation will be slow and the transient error will be large. Conversely, the magnitude of the gain should be limited because too large an adaptation gain will lead to very oscillatory parameters [15].

In the rest of this section, the procedure is developed to determine $K_1 \sim K_6$ in terms of LMI formulation such that the closed-loop system in (25) is stable. It is clear that the uncertain (interval) matrix $A$ in (30) is function of four uncertain parameters, i.e., $A = \hat{A}(\hat{R}_s, \hat{L}_s, \hat{\lambda}_f, \hat{B}_m)$, and can be characterized in terms of the following sixteen ($2^4 = 16$) vertex (or corner) matrices [16]

$$\hat{A} = \sum_{i=1}^{16} \alpha_i A_i$$

(32)

where $A_1 = \hat{A}(\hat{R}_s^+, \hat{L}_s^+, \hat{\lambda}_f^+, \hat{B}_m^+)$, $A_2 = \hat{A}(\hat{R}_s^-, \hat{L}_s^+, \hat{\lambda}_f^+, \hat{B}_m^+)$,

$$\ldots, A_{16} = \hat{A}(\hat{R}_s^-, \hat{L}_s^-, \hat{\lambda}_f^-, \hat{B}_m^-)$$

and $\sum_{i=1}^{16} \alpha_i = 1$ with $0 \leq \alpha_i \leq 1$ ($i = 1, \ldots, 16$).

Remark 9: In this study, the exact values of $\alpha_i$ are not required.

From the formulation for uncertain matrix $\hat{A}$ in (32), the stable condition in (30) is equivalent to

$$\sum_{i=1}^{16} \alpha_i (A_i^T + A_i) < 0$$

(33)

Obviously, a sufficient condition for (33) being satisfied is that the following linear matrix inequalities (LMIs)

$$A_i^T + A_i < 0$$

(34)

hold for all $i = 1, \ldots, 16$.

From the analysis above, the problem of determining the control gains $K_1 \sim K_6$ can be formulated in terms of solving the LMIs in (34) for all $i = 1, \ldots, 16$.

Remark 10: The LMIs problem (LMIP) in (34) can be efficiently solved by the LMI toolbox in Matlab [14].

Remark 11: A necessary condition for $A_i^T + A_i < 0$ being satisfied is that all diagonal entries of $A_i$ should be negative. This is the reason why three error dynamics for $e_{d,i}$, $e_{q,i}$, and $e_t$ are defined as that in (5)-(7), which are all low-pass filters instead of pure integrators. By the assertion in (5)-(7), the diagonal entries of $A_i$ at (1, 1), (3, 3), and (5, 5) are $-\kappa$ which is negative and thus some feasible solutions for the LMIs in (34) may be possibly obtained by this arrangement. In other words, if pure integrator scheme, $\kappa = 0$, is adopted for $e_{d,i}$, $e_{q,i}$, and $e_t$ defined in (5)-(7), then the diagonal entries of $A_i$ at (1, 1), (3, 3), and (5, 5) are all zero and obviously no feasible solutions for the LMIs in (34) can be found. Note that (34) is only a sufficient condition for the stability of the closed-loop system in (25). However, it does not guarantee that there always exist the control gains $K_1 \sim K_6$ such that LMIs in (34) hold.

Remark 12: From the stability analysis in this section, it does not guarantee that all error states including estimated errors can approach zero asymptotically, but it does guarantee that all error states are bounded. However, the steady-state errors for $e$, $e_{d,i}$, and $e_{q,i}$ can approach zero by a suitable adjustment for the parameter $\kappa$, which is shown in the next section. Although, bounded estimated errors are guaranteed, as well-known, the convergence of the estimated parameters is guaranteed only for the driving signal $Y$ with persistent excitation condition in the adaptive law (28). However, the persistent excitation condition for the signal $Y$ is difficult to be verified in advance.

IV. Simulation Examples

In this section, some simulation examples are provided to illustrate the design procedure and the performances of the proposed control scheme in the motor drive. The parameters of the tested PMSM (Sinano 7CB30-2DE67) are listed as follows: number of poles $P = 8$, $R_s = 3.55\, \Omega$ with $R_s^+ = 0.5 * R_s$ and $R_s^- = 1.5 * R_s$, $L_s = 5.92\, mH$ with $L_s^- = 0.5 * L_s$ and $L_s^+ = 1.5 * L_s$, $\lambda_f = 0.0579\, \text{v/(rad/sec)}$ with $\lambda_f^- = 0.8 * \lambda_f$ and $\lambda_f^+ = 3650$.
1.00 \ast \dot{\lambda}_f, \dot{J}_m = 1.962 \times 10^{-4} kg \cdot m^2 with \dot{J}_m = 0.5 \ast \dot{J}_m and \dot{J}_m^+ = 5 \ast \dot{J}_m, \dot{B}_m = 2.4 \times 10^{-4} N \cdot m/(rad/sec) with \dot{B}_m = 0.5 \ast \dot{B}_m and \dot{B}_m^+ = 5 \ast \dot{B}_m.

**Example 1:** Solving the LMIP in (34) with \( \kappa = 50 \) given beforehand using LMI toolbox in Matlab, we obtain \( K_1 = 0.4985, K_2 = 3.0454, K_3 = 1.7454 \times 10^5, K_4 = 1.0122 \times 10^3, K_5 = 2.7601 \times 10^5, \) and \( K_6 = 1.0193 \times 10^5 \). Obviously, high gain solutions, especially for \( K_3 \) and \( K_5 \), are obtained in this case. There may exist more than one solution in this case. Therefore, we can search for a more satisfied solution by an alternative formulation which is shown in the next example.

**Example 2:** To avoid high gain problem, we can constrain the gains \( K_1 \sim K_6 \) by solving the following minimization problem

\[
\min \quad \beta_1 K_1 + \beta_2 K_2 + \beta_3 K_3 + \beta_4 K_4 + \beta_5 K_5 + \beta_6 K_6
\]

subject to \( A_i^T + A_i < 0, \forall i = 1, \ldots, 16 \) \( (35) \) which minimizes the sum of all gains \( K_1 \sim K_6 \) with semi-positive weightings \( \beta_1 \sim \beta_6 \). The minimization problem in \( (35) \) is also called eigenvalue problem (EVP) \( [13] \) which can also be efficiently solved by the LMI toolbox in Matlab. Solving the EVP in \( (35) \) with \( \kappa = 50 \) given beforehand and \( \beta_i = 1 \) \( (i = 1 \sim 6) \) for equal weightings, we obtain \( K_1 = 0.3416, K_2 = 3.0447, K_3 = 0.1279, K_4 = 0.1355, K_5 = 293.2949, \) and \( K_6 = 0.1339 \). Obviously, the high gain problem is overcome by the proposed formulation in \( (35) \). Fig. 1 shows the simulation response with the reference setting \( \omega_{\text{r}}^* = 100 \) (rad/sec) and the adaptation gain \( \rho = 1 \). From Fig. 1, large overshoot and small steady-state error (the steady-state error is small but not tends to zero) are observed. Moreover, by more simulations, one observes that the smaller \( \kappa \) is, the larger overshoot is. In other words, one can constrain the overshoot by a larger \( \kappa \), which is shown in the next example.

**Example 3:** To decrease the overshoot, we solve the EVP in \( (35) \) with \( \kappa = 100 \) and \( \beta_i = 1 \) \( (i = 1 \sim 6) \) leading to the following solutions: \( K_1 = 0.3416, K_2 = 2.9985, K_3 = 0.0037, K_4 = 0.0038, K_5 = 11.1791, \) and \( K_6 = 0.0038 \). Note that the control gains in this case are as the same scale as that in **Example 2**. Fig. 2 shows the simulation response with the reference setting \( \omega_{\text{r}}^* = 100 \) (rad/sec) and the adaptation gain \( \rho = 1 \). From Fig. 2, smaller overshoot but larger steady-state error are observed. By the simulations of **Example 2** and **Example 3**, one observes that the overshoot can be improved for a larger \( \kappa \), however the steady-state error can be improved for a smaller \( \kappa \). Recall that from **Remark 2**, the behavior of the transfer function from \( e \) to \( e_L \) is like a pure integrator if \( \kappa \rightarrow 0 \), which can improve the steady-state error. From **Example 2** and **Example 3**, one notices that the overshoot and the steady-state error are contradictory performances for some fixed \( \kappa \). However, by a suitable arrangement for the parameter \( \kappa \), which is shown in the next example, both the overshoot and the steady-state error can be improved simultaneously.

**Example 4:** In this example, the same control gains \( K_1 \sim K_6 \) derived from **Example 3** are adopted, however, a time-varying \( \kappa = 100e^{-5t} \) is used in the simulation, to improve both the overshoot and the steady-state error. Fig. 3 shows the simulation response with a time-varying \( \kappa = 100e^{-5t} \), the reference setting \( \omega_{\text{r}}^* = 100 \) (rad/sec), and the adaptation gain \( \rho = 1 \). From Fig. 3, it shows that both the overshoot and the steady-state error (the steady state error approaches zero) can be improved dramatically by a time-varying \( \kappa \), in which \( \kappa = 100 \) at \( t = 0 \) and eventually \( \kappa \rightarrow 0 \) as \( t \rightarrow \infty \). This is because the overshoot can be improved by a larger \( \kappa \) in the transient and the steady-state error can be improved (approach zero) by setting \( \kappa \rightarrow 0 \) at the steady state. Fig. 4 shows the trajectories of the adaptive parameters \( \hat{L}_s \) and \( \dot{\lambda}_f \). From Fig. 4, one observes that the estimated parameters \( \hat{L}_s \) and \( \dot{\lambda}_f \) can approach to their true values of \( L_s \) and \( \dot{\lambda}_f \), respectively, by the proposed adaptive laws in \( (19)-(20) \). It also noted that the trajectories of the adaptive parameters \( \hat{L}_s \) and \( \dot{\lambda}_f \) can also approach to their true values of \( L_s \) and \( \dot{\lambda}_f \), respectively, in **Example 2** and **Example 3**.

**Remark 13:** By the scheme of decaying \( \kappa \) with time exponentially, the steady-state errors for \( e, e_{ds}, \) and \( e_{qs} \) can approach zero.

**Remark 14:** As well-known, the convergence of the estimated parameters is guaranteed only for the driving signal \( Y \) with persistent excitation condition in the adaptive law \( (28) \). By the simulation in the **Example 4**, it shows that the estimated parameters \( \hat{L}_s \) and \( \dot{\lambda}_f \) can approach to their true values of \( L_s \) and \( \dot{\lambda}_f \), respectively. However, the persistent excitation condition for the signal \( Y \) is difficult to be verified in advance.

**V. Conclusion**

In this study, a novel adaptive control scheme is proposed for the uncertain PMSM drive system using an LMI approach. The adaptive control scheme is used to compensate some uncertain terms in the PMSM drive system.
A novel formulation for the PMSM drive system is developed and by this formulation the stability condition for the closed-loop PMSM drive system with the proposed controllers can be characterized in terms of some linear matrix inequalities (LMIs). The LMIs problem (LMIP) can be efficiently solved. Numerical solutions of the control gains using the proposed LMI formulation and simulation examples are provided to illustrate the design procedure and corresponding performances. In opposition to try and error, the control gains can be obtained systematically in this study.

REFERENCES