Six Degree of Freedom Dynamical Model of a Morphing Aircraft

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Morphing aircraft are envisioned to have multirole capability where the ability to change shape allows for adaptation to a changing mission environment. In order to calculate the properties of many wing configurations efficiently and rapidly, a model of a morphing aircraft is needed. This paper develops an aerodynamic model and a dynamic model of a morphing flying wing aircraft. The dynamic model includes realistic aerodynamic forces, consisting of lift, drag, and pitching moment about the leading edge, calculated using a constant strength source doublet panel method. The panel method allows for the calculation of aerodynamic forces due to large scale shape changing effects. The aerodynamic model allows for asymmetric configurations in order to generate rolling and yawing moments. The dynamic model calculates state information for the morphing wing based on the aerodynamic forces from the panel method. The model allows for multiple shape changing degrees-of-freedom for the wing, including thickness, sweep, dihedral angle, and chord length. Results show the model provides a versatile and computationally efficient tool for calculating the aerodynamic forces on the morphing aircraft and using these forces to show the associated states.

Nomenclature

\( r \) Coordinate vector of any point \((x,y,z)\)
\( n \) Normal vector
\( A \) Doublet influence coefficient
\( B \) Source influence coefficient

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\( q \)  Local velocity
\( Q \)  Total Velocity
\( C \)  Aerodynamic coefficient
\( S \)  Wing Area
\( m \)  Mass
\( v \)  Velocity vector
\( u \)  Body x-axis velocity
\( v \)  Body y-axis velocity
\( w \)  Body z-axis velocity
\( p \)  Body axis roll rate
\( q \)  Body axis pitch rate
\( r \)  Body axis yaw rate
\( X \)  Body x-axis aerodynamic force
\( Y \)  Body y-axis aerodynamic force
\( Z \)  Body z-axis aerodynamic force
\( T \)  Thrust force
\( G \)  Gravitational force
\( g \)  Acceleration due to gravity
\( D \)  Drag Force
\( L \)  Lift Force
\( \mathbf{L} \)  Moment vector
\( h \)  Angular momentum vector
\( \mathbf{I} \)  Inertia matrix
\( I_{xx} \)  Airplane moment of inertia about X-axis
\( I_{xz} \)  Airplane moment of inertia about XZ-plane
\( I_{yy} \)  Airplane moment of inertia about Y-axis
\( I_{zz} \)  Airplane moment of inertia about Z-axis
\( L_A \)  Aerodynamic rolling moment
\( M_A \)  Aerodynamic pitching moment
\( N_A \)  Aerodynamic yawing moment
\( V \)  Total velocity

**Greek**
\( \alpha \)  Angle-of-attack
\( \beta \)  Sideslip angle
\( \theta \)  Pitch attitude angle
I. Introduction

Recent interest in morphing air vehicles has been increased due to advances in smart materials and their associated electronic components. While several definitions for morphing exist, it is generally accepted that morphing refers to a large scale change in the original shape of the body. The National Aeronautics and Space Administration’s (NASA) Morphing Project defines morphing as an efficient, multi-point adaptability that includes macro, micro, structural and/or fluidic approaches.¹ The Defense Advanced Research Projects Agency (DARPA) defines morphing to be a platform that is able to change its shape substantially (approximately 50 per cent or more) in order to adapt to a changing environment, making the vehicle superior in the
new environment than before the change. In this paper, the authors use the DARPA definition of morphing.

There have been other attempts at calculating different aspects of a morphing air vehicle. In Reference 2, the authors examine the aerodynamic and aeroelastic characteristics of a morphing wing. The authors use variable span method in order to allow the wing to change its shape, thereby increasing its aspect ratio and wing area, which improves overall performance at different flight conditions. Reference 3 discusses the design, development and testing of an inflatable telescopic wing that permits a change in the aspect ratio while simultaneously supporting structural wing loads. Reference 4 studies a hyper elliptic cambered span wing that is allowed to morph and creates a nonlinear, CFD based, dynamic vehicle simulation. In Reference 5, the authors discuss the use of a CFD model which is used to help find the optimal airfoil shape for numerous flight conditions. In Reference 6 the authors use a linear parameter-varying model to capture the aeroelastic morphing dynamics of a morphing UAV. The authors assume a variable geometry rigid body but use quasi-steady aeroelastic effects to correct the dynamic coefficients.

Reference 7 investigates the dynamics associated with large scale morphing aircraft. Specifically, the authors investigate the inertial forces and moments not apparent in the standard rigid body equations. In Reference 8, the authors examine the longitudinal dynamics of a perching aircraft concept. A generalized lifting line method is used to calculate the aerodynamic forces on the perching aircraft. Reference 9 investigates the dynamic aeroelastic stability of a morphing wing structure, specifically by investigating the migration of the wing flutter boundary in the case of a morphing aircraft. The wing is able to change span via telescoping. In Reference 10, the authors investigated the applicability of existing simulation techniques in the analysis of a non-traditional morphing aircraft design.

Reference 11 discusses the application of various bio-inspired morphing concepts to unmanned aerial vehicles. The authors used a six degree-of-freedom simulation to evaluate the stability and dynamics of a morphing vehicle to various shape changes. Reference 12 focuses on a morphing airfoil concept. The focus is on the physical shape change of the airfoil modeled by a space/time transform parameterization. Reference 13 studies the effects of variable sweep, wing area, and aspect ratio on low speed performance. Specifically, range and endurance are addressed for speeds of up to Mach 0.8.

This research is a continuation of previous work, which was first published in 2004. In Reference 14, the authors began by modeling an ellipsoid that was assumed to have constant volume. The y and z dimensions were allowed to change freely, while the x dimension was determined by the constant volume assumption. The equations of motion were derived, with time varying inertias to account for the shape change effect. The next step in the research was to create a more sophisticated aerodynamic model. This was done by creating a morphing two dimensional airfoil section, which allowed for the thickness and camber of the airfoil to change shape. The aerodynamic effects on the airfoil were modeled using a constant strength doublet panel method. No dynamics were associated with the morphing airfoil problem. The next step in the research was to create a finite wing model. In Reference 16, the authors used a source-doublet panel
method to model the aerodynamics and the longitudinal equations of motion to model the dynamics of the aircraft.

This research expands upon previous research done in Reference 14 and Reference 16. The scope of this research includes the inviscid, incompressible flow regime. Thus, only the linear region of the aerodynamics is considered. A computational model of a morphing aircraft is developed which includes accurate aerodynamics for the inviscid, incompressible flow regime and the ability to handle both small scale and large scale shape changes. Some of the issues which arise from creating this model are creating an accurate yet computationally efficient aerodynamic model and combining the aerodynamic model with a dynamical model in order to calculate state information. The main objective of this research is to create a computational model for a morphing air vehicle which includes:

- Accurate aerodynamics for inviscid, incompressible flow
- Nonlinear flight dynamics
- Multiple shape changing degrees-of-freedom
- Run in a MATLAB environment
- The ability to be accessed by other subroutines for future research

This paper is organized as follows. Section II describes the aerodynamic representation and discusses how the aerodynamic properties are calculated by the panel method. Section II also mentions how a grid is fitted to the shape of the morphing wing. Section III describes the dynamic model of the morphing airfoil, including a development of the six degree of freedom equations of motion. Section IV is used to verify the aerodynamics of the morphing wing by comparing the results with results for known configurations. Section V is used to present the results of the dynamical model of the morphing flying wing. In Section VI, conclusions are drawn from the results that are presented Section IV and Section V.

II. Aerodynamic Model Representation

To calculate the aerodynamic properties of many wing configurations efficiently and rapidly, a numerical model of the wing aerodynamics is developed. A constant strength doublet-source panel method is used to model the aerodynamics of the wing. This method was chosen over other CFD methods due to the success using a panel method in predicting the aerodynamic effects on a morphing airfoil. By using a source-doublet method, the effect of thickness is captured, which is often an effect neglected by other types of panel methods. The main assumption is that the flow is incompressible; otherwise a much more complex model is necessary. This assumption is valid because current interests lie in the realm of air vehicles which fly at speeds less than Mach 0.3. One other assumption is that the flow is inviscid. Thus, the model is only valid for the linear range of angle-of-attack. A method provided by DATCOM is used to estimate the profile drag.
The versatility of this type of model allows the author’s reinforcement learning algorithm developed in Reference\textsuperscript{15} to manipulate multiple morphing degrees of freedom and flight condition parameters. The morphing degrees of freedom and flight condition parameters are:

- wing root airfoil thickness
- left wing tip airfoil thickness
- right wing tip airfoil thickness
- wing root airfoil camber
- left wing tip airfoil camber
- right wing tip airfoil camber
- wing root airfoil location of maximum camber
- left wing tip airfoil location of maximum camber
- right wing tip airfoil location of maximum camber
- wing span
- aspect ratio
- leading edge sweep angle
- dihedral angle
- taper ratio
- twist

Yet, given this versatility, there are some limitations to the model. Since the model uses a panel method to determine the aerodynamics, it is very sensitive to the grid size, location of the panels, and the number of panels created. The grid uses cosine spacing for both chordwise and spanwise panels. By utilizing cosine spacing, more panels are placed near the leading and trailing edges of the wing as well as at the root and tip. This type of grid is necessary because many aerodynamic changes occur near the leading and trailing edges of the wing as well as at the tips of the wing. As the number of panels decrease, the accuracy of the model also decreases. However, as the number of panels is increased, the computational time of the model increases as well. Thus, a balance is needed between accuracy and computational time. This balance is achieved by defining a set number of panels for which any increase from that number of panels yields a minimal increase in accuracy. For example, if the number of panels were doubled and the accuracy of the model increased by 10 per cent, this increase in the number of panels would be deemed acceptable. However, if this set of panels
were increased by 50 per cent and the accuracy increased by less than 1 per cent, then this increase in the number of panels would be deemed unnecessary.

Also, the model has a limitation on the different types of airfoil sections which can be used. Only NACA 4-Digit Series airfoils are considered because there are explicit equations which easily describe the upper surface and the lower surface geometries. These equations have defined thickness and camber variables, which makes them easy to examine and optimize to achieve the best possible wing shape.

The aerodynamic model uses 130 chordwise panels, with 65 on the upper surface and 65 on the lower surface, and 25 spanwise panels for the half-span. Thus, 3250 total panels are used to model the half-span. The aerodynamic model does not assume a plane of symmetry, so each half of the wing is modeled separately. The control points are placed at the center of each panel. The outputs of the aerodynamic model are lift, drag, pitching moment about the apex, rolling moment, and yawing moment. The lateral and directional outputs are only shown if the morphing wing is asymmetric.

A. Aerodynamic Modeling

The wing is modeled using a constant strength doublet-source panel method. In order to obtain the equations for aerodynamic forces on a wing, basic potential flow theory is used. Equation 1 is the basic equation of potential flow theory.\(^\text{17}\)

\[
\nabla^2 \Phi = 0 \tag{1}
\]

Equation 1 satisfies the inviscid and incompressible flow assumptions to the general conservation equations. Using Green’s identity, a solution to Equation 1 is formed with a sum of source \(\sigma\) and doublet \(\mu\) distributions along the boundary, SB.

\[
\Phi = -\frac{1}{4\pi} \int_{SB} \left[ \sigma \left(\frac{1}{r}\right) - \mu \mathbf{n} \cdot \nabla \left(\frac{1}{r}\right) \right] dS + \Phi_\infty \tag{2}
\]

Assuming the wake convects at the trailing edge of the wing as a set of by thin doublets, Equation 2 may be rewritten as

\[
\Phi = \frac{1}{4\pi} \int_{Body+Wake} \mu \mathbf{n} \cdot \nabla \left(\frac{1}{r}\right) dS - \frac{1}{4\pi} \int_{Body} \sigma \left(\frac{1}{r}\right) dS + \Phi_\infty \tag{3}
\]

The boundary condition for Equation 1 is the no penetration condition, which requires the normal velocity of the flow at the surface to equal to zero. This boundary condition must be specified by either a direct or indirect formulation. The direct formulation forces the normal velocity component to be zero and is defined to be the Neumann problem. The indirect formulation specifies a value for the potential function on the boundary and, by doing so, the zero normal flow condition is indirectly satisfied. This method is defined to be the Dirichlet problem. The morphing wing model uses the Dirichlet problem to enforce the zero normal flow boundary condition. Using the Dirichlet boundary condition, the potential must be specified at all boundaries.
points on the boundary. If a point is placed inside the surface, the inner potential, $\Phi_i$, is defined by the singularity distributions along the surface.

$$
\Phi_i = \frac{1}{4\pi} \int_{\text{Body} + \text{Wake}} \mu \frac{\partial}{\partial n} \left( \frac{1}{r} \right) dS - \frac{1}{4\pi} \int_{\text{Body}} \sigma \left( \frac{1}{r} \right) dS + \Phi_\infty
$$

These integrals become singular as $r$ approaches zero. In order to evaluate the integrals near this point, the no penetration boundary condition must be enforced. In order to enforce this boundary condition, $\Phi_i$ is set to a constant value. If the direct formulation is used, it may be shown that $\Phi_i$ is a constant. Since $\Phi_i$ is a constant, Equation 4 is equal to a constant as well. The constant value of $\Phi_i$ is chosen to be $\Phi_\infty$. Thus, Equation 4 is reduced to a simpler form.

$$
\frac{1}{4\pi} \int_{\text{Body} + \text{Wake}} \mu \frac{\partial}{\partial n} \left( \frac{1}{r} \right) dS - \frac{1}{4\pi} \int_{\text{Body}} \sigma \left( \frac{1}{r} \right) dS = 0
$$

Next, one reduces these integral equations to a set of linear algebraic equations. Let the system be divided into $N$ panels for the surface and $N_w$ panels for the wake, as shown in Figure 1. The boundary condition is specified at a collocation point, which for the Dirichlet boundary condition is inside the body and at the center of the panel. Equation 5 is rewritten for $N$ collocation points for $N$ panels. The integrands shown below only depend on the geometry of the respective panel, and thus can be evaluated.

$$
\sum_{k=1}^{N} \frac{1}{4\pi} \int_{\text{BodyPanel}} \mu \mathbf{n} \cdot \nabla \left( \frac{1}{r} \right) dS + \sum_{l=1}^{N} \frac{1}{4\pi} \int_{\text{WakePanel}} \mu \mathbf{n} \cdot \nabla \left( \frac{1}{r} \right) dS - \sum_{k=1}^{N} \frac{1}{4\pi} \int_{\text{BodyPanel}} \sigma \left( \frac{1}{r} \right) dS = 0
$$

Figure 1. Paneling on a Wing
Since the singularity elements $\mu$ and $\sigma$, of each panel influence every other panel on the body, Equation 6 may be rewritten for each collocation point inside the body. Note that the variables preceding the singularity elements are the respective integrals evaluated for a particular panel and the respective collocation point. Thus,

$$\sum_{k=1}^{N} A_k \mu_k + \sum_{l=1}^{N} A_l \mu_l - \sum_{k=1}^{N} B_k \sigma_K = 0$$  \hspace{1cm} (7)$$

In order to eliminate the wake strength from Equation 7, a new relationship is introduced. The Kutta condition states there is no circulation at the trailing edge of a wing section. By using the Kutta condition, one will find the doublet strength of the wake panel is equivalent to the difference between the trailing edge panels on the upper surface and the lower surface. Figure 2 shows this relationship. By exploiting this relationship, the wake contribution may be eliminated from Equation 7 by substitution at the trailing edge panels only. Since the source strengths are known, Equation 7 reduces to a set of $N$ equations with $N$ unknown doublet strengths which may be solved by matrix inversion.

\[
\begin{pmatrix}
  a_{11} & \ldots & a_{1N} \\
  \vdots & \ddots & \vdots \\
  a_{N1} & \ldots & a_{NN}
\end{pmatrix}
\begin{pmatrix}
  \mu_1 \\
  \vdots \\
  \mu_N
\end{pmatrix}
= -
\begin{pmatrix}
  b_{11} & \ldots & b_{1N} \\
  \vdots & \ddots & \vdots \\
  b_{N1} & \ldots & b_{NN}
\end{pmatrix}
\begin{pmatrix}
  \sigma_1 \\
  \vdots \\
  \sigma_N
\end{pmatrix}
\] \hspace{1cm} (8)

Once the doublet strengths are found, it is possible to find the aerodynamic forces acting on each panel. The first step is to determine the tangential and normal perturbation velocity components for each of the panels. Note that the partial derivatives are taken with respect to the local panel coordinate system, which is shown in Figure 3. By exploiting

$$q_t = -\frac{\partial \mu}{\partial l}, q_n = -\frac{\partial \mu}{\partial n}, q_n = -\sigma$$  \hspace{1cm} (9)$$

With these velocities, the total velocity of each panel may be computed.
Using the velocities at each panel, the pressure coefficient at each panel is found using a modified form of Bernoulli’s Equation.

\[ C_{p_k} = 1 - \frac{Q^2_k}{Q^2_{\infty}} \]  

(11)

Once the pressure coefficient has been determined, the non-dimensional aerodynamic forces are calculated for each panel. The total aerodynamic forces are found by summing the contributions from each panel.

\[ \Delta C_{F_k} = -\frac{C_{p_k} \Delta S}{S} \cdot n_k \]  

(12)

Reference\(^{18}\) shows a DATCOM method for calculating the value of parasite drag, \(C_{D_0}\), based on the wing area and the equivalent parasite area.

\[ C_{D_0} = \frac{f}{S} \]  

(13)

The equivalent parasite area is related to the wetted area of the morphing wing aircraft as

\[ \log_{10} f = a + b \log_{10} S_{\text{wet}} \]  

(14)

The terms \(a\) and \(b\) are correlation coefficients related to the equivalent skin friction of the aircraft. The equivalent skin friction coefficient is estimated by using data from similar aircraft.

The Oswald efficiency factor may be expressed as a function of the aspect ratio of the wing and is valid if the leading edge sweep angle of the wing is less than 30°. Equation 15 provides the Oswald efficiency factor, which was developed empirically in Reference.\(^{19}\)

\[ e = 1.78(1 - 0.045AR^{0.68}) - 0.64 \]  

(15)

Once the Oswald efficiency factor has been determined, an expression for the total drag acting on the morphing wing can be constructed. Equation 16 provides this relationship.

\[ C_D = C_{D_0} + \frac{C^2_D}{\pi e AR} \]  

(16)
B. Model Grid

One important aspect of this model is the ability to generate a grid which is easy to manipulate and easily allows for many morphing degrees of freedom. In order to accomplish this, the first step is to represent the airfoil sections that allow for camber and thickness to change and be suitable for flight conditions that are within the incompressible regime. There are multiple ways to accomplish this. One way is to create a table of known airfoil coordinates for look up whenever a reconfiguration occurs. The advantages of this type of representation are the wide variety of airfoils available for selection and the ease with which to transition from one type of airfoil to another. However, the major disadvantage of a table with multiple airfoil sections is not being able to have direct control over the thickness and camber of the airfoil. For this model, being able to have direct, quick control for a change in thickness and camber is vital. Therefore, the approach which was chosen is to use a set of airfoils for which there were equations that describe the upper and lower geometries of the airfoil section as a function of a thickness and camber. The class of airfoils chosen are the NACA 4-Digit series airfoils. Herein, only 4-Digit airfoils can be chosen. These airfoils have blunt leading edges with either thick or thin airfoil sections, which makes them ideal for subsonic speeds.

The method of spacing used to place the panels on the wing is also an important aspect of generating a grid. For this particular model, cosine spacing is utilized for both chordwise and spanwise paneling. The cosine spacing method is defined by the following equation.

\[ x_{v_i} = \frac{c(y)}{2} \left( 1 - \cos \left( \frac{i\pi}{N+1} \right) \right) \]

\[ i = 1 : N \]

By using cosine spacing in the chordwise direction, more panels are placed near the leading edges and trailing edges of the wing. Cosine spacing in the spanwise direction places more panels at the span stations which are near the tips of the wing. Figures 4, 5, and 6 show the effect of utilizing cosine spacing in both the spanwise and chordwise directions as well as some of the different shapes which the aerodynamic model is able to model. As can be seen from the figures, the four corner points on the wing have the most number of panels.

III. Dynamical Modeling

Starting with Newton’s Second Law, \( F = m\ddot{v} \), the translational equations of motion for the morphing wing may be developed. The body axis system can be seen in Figure 7. Note that the same convention is used in the following derivation as by Roskam. Consider a velocity vector \( \mathbf{v} \) and an angular velocity vector \( \omega \) expressed in the body axis system of the morphing wing.
Taking an inertial derivative of the velocity vector with respect to time yields

\[ \dot{\mathbf{v}}^N = \dot{\mathbf{v}}^B + \mathbf{\omega} \times \mathbf{v} = \begin{bmatrix} \dot{u} + qw - rv \\ \dot{v} + ru - pw \\ \dot{w} + pv - qu \end{bmatrix} \]
The forces acting on the morphing wing consist of gravitational, propulsive, and aerodynamic forces. The gravitational forces can be easily calculated using simple trigonometry. The propulsive force acting on the morphing wing is assumed to be constant. The only remaining forces are the aerodynamic forces, which are calculated using the constant strength source-doublet panel method mentioned in Chapter II. The forces acting on the morphing wing are

\[
\mathbf{F} = \begin{bmatrix}
X + T_x + G_x \\
Y + T_y + G_y \\
Z + T_z + G_z
\end{bmatrix}
\]
Substituting Equation 20 and Equation 21 into Newton’s Second Law, the translational equations of motion are obtained. These translational equations of motion are

\[ m (\ddot{u} + q \dot{w} - r \dot{v}) = -mg \sin (\theta) - D \cos (\alpha) \cos (\beta) + L \sin (\alpha) \cos (\beta) + T \]  

\[ m (\ddot{v} + r \dot{u} - p \dot{w}) = mg \sin (\phi) \cos (\theta) + D \cos (\alpha) \sin (\beta) - L \sin (\alpha) \sin (\beta) \]  

\[ m (\ddot{w} + p \dot{v} - q \dot{u}) = mg \cos (\phi) \cos (\theta) - D \sin (\alpha) - L \cos (\alpha) \]  

Using Euler’s Equations, \( \mathbf{L} = \dot{\mathbf{N}} \), one may obtain a relationship between the rotational motion and the moments acting on the morphing wing. Consider an angular momentum vector \( \mathbf{h} \), defined as

\[ \mathbf{h} = I \omega \]  

The inertia matrix, \( I \), may be written as

\[ I = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{pmatrix} \]  

These moments of inertia are functions of the configuration of the morphing wing. These inertias are time varying and represent the morphing wing due to the shape changes that occur during flight. Taking the time derivative of the angular momentum vector, the following relationship is obtained.

\[ \dot{\mathbf{h}}^N = \mathbf{I} \dot{\omega} + \mathbf{I}^B \omega + \omega \times \mathbf{I} \omega \]  

The moments that act on the morphing wing are due to the aerodynamic forces and the propulsive forces. The moments caused by the aerodynamic forces are calculated from the panel method mentioned in Chapter II. It is assumed for the purpose of this research that the thrust line is parallel to the body x-axis and acts at the center of gravity location, thus not producing a moment. The moments acting on the morphing wing
are

\[
\mathbf{L} = \begin{bmatrix}
L_A \\
M_A \\
N_A
\end{bmatrix}
\]  

(28)

By substituting Equation 27 and Equation 28 into Euler’s Equations, the rotational equations of motion for the morphing wing are

\[
I_{xx}\dot{p} - I_{xy}\dot{q} - I_{xz}\dot{r} + \dot{I}_{xx}p - \dot{I}_{xy}q - \dot{I}_{xz}r + p(-I_{xz}q + I_{xy}r) \\
+ q(-I_{yz}q - I_{yy}r) + r(I_{zz}q + I_{yz}r) = L_A
\]

(29)

\[
I_{xy}\dot{p} - I_{yy}\dot{q} - I_{yz}\dot{r} + \dot{I}_{xy}p - \dot{I}_{yy}q - \dot{I}_{yz}r + p(I_{xx}r + I_{xz}p) \\
+ q(-I_{xx}r + I_{yz}p) + r(-I_{zz}r - I_{zz}p) = M_A
\]

(30)

\[
I_{xz}\dot{p} - I_{yz}\dot{q} - I_{zz}\dot{r} + \dot{I}_{xz}p - \dot{I}_{yz}q - \dot{I}_{zz}r + p(-I_{xy}p - I_{xx}q) \\
+ q(I_{yy}p + I_{xy}q) + r(-I_{yz}p + I_{zz}q) = N_A
\]

(31)

The other states which need to be considered are the angle-of-attack and the side-slip angle, since the aerodynamic forces are directly dependent on them. From geometry, the relationship between angle-of-attack, side-slip angle, and the respective body axis velocities can be expressed as

\[
\tan(\alpha) = \frac{w}{u}
\]

(32)

\[
\tan(\beta) = \frac{v}{u}
\]

(33)

Differentiating Equation 32 and Equation 33 with respect to time yields

\[
\dot{\alpha} = \frac{1}{V}(\dot{w}\cos(\alpha) - \dot{u}\sin(\alpha))
\]

(34)

\[
\dot{\beta} = \frac{1}{V}(\dot{v}\cos(\beta) - \dot{u}\sin(\beta))
\]

(35)

IV. Aerodynamic Model Verification

This chapter presents the results of the aerodynamic model for the morphing wing and verifies it accurately represents the shape represented. The aerodynamic model is compared, for two cases, to a model which was developed by Dr. Leland Carlson at Texas A&M University called TRANS3DNS. TRANS3DNS solves the three-dimensional extended transonic small perturbation equations and can allow for wing sweep, taper, twist, and different airfoil sections.

A. Example 1: Rectangular Wing Planform

This example tests the aerodynamic model using a rectangular wing planform with a NACA 0012 airfoil section. The aerodynamic model produces the lift, drag, and moment data illustrated in Figures 8, 9, and 10, respectively.
Figure 9 shows the variation of the induced drag for the morphing wing in this example. The drag polar is plotted as a function of the lift coefficient.

Figure 10 shows the variation of the pitching moment about the leading edge of the wing root with respect to the angle-of-attack.

These results are accurate with respect to the physics involved and indicate the aerodynamic model produces reasonably accurate aerodynamics on a basic wing shape. However, due to the assumptions made and the method in which the aerodynamic forces are calculated, there is some error present when this data is compared with TRANS3DNS. Since TRANS3DNS uses a small perturbation method to calculate the aerodynamic forces, TRANS3DNS will have discrepancies when compared to any panel method. TRANS3DNS also does not take into account the thickness of the airfoil section, whereas the aerodynamic model does consider thickness. As the thickness of an airfoil tends to increase, the lift curve slope will increase as well. As a result, the panel method shows an increased value in lift curve slope.

B. Example 2: Nonrectangular Wing Planform

This example tests the aerodynamic model using a wing with a 15° leading edge sweep angle, a taper ratio of 0.7, and a NACA 2412 airfoil section. The aerodynamic model produces the lift, drag, and moment data in Figures 11, 12, and 13, respectively.

A drag polar is constructed using the aerodynamic model. The induced drag for the morphing wing, as a function of lift coefficient, is shown in Figure 12.

Figure 13 shows the variation of the pitching moment about the leading edge of the wing root with
Again, these results are reasonably accurate with respect to the physics involved. By adding camber to the airfoil section, the lift curve slope is shifted. Adding a leading edge sweep angle causes the lift curve slope to decrease when compared with an unswept configuration. By adding taper to the wing, there is less lift produced at the wing tips, thus lowering the overall lift generated from the wing. As in the first example, there are sources of error within this method. A source of error for the more complex shape is the aerodynamic code uses a first order sweep correction method. Another source of error is from the thickness effect, as in the previous example. The effect of airfoil thickness appears to effect the lift coefficient at zero angle of attack. While slightly larger than the previous example, these errors have been deemed acceptable for this research.

V. Dynamic Model Results

This section presents the results of the dynamic model for the morphing wing. For the first example, the aircraft does not perform any shape changing. This is done in order to observe the natural movement of the aircraft. In the second example, the aircraft starts with an initial shape and performs two shape changes during the specified time frame. The initial conditions used in both examples are the same and are listed in Table 1. Note that the initial conditions for the angle-of-attack and sideslip angle states are found by using Equations 32 and 33.
A. Example 1: Longitudinal Motion with Immediate Shape Changes

In this example, the morphing aircraft will perform shape changes during the beginning portion of flight and will begin with the shape from Example 1. This example uses a different shape than the previous example and is used to show the model can handle shape changes at any point in time. The shape changes are presented in Figure 14.

Figure 15 shows the time history of the states for the morphing aircraft. Since the shape change occurs at the beginning of flight, the aircraft is able to establish positive lift while the pitching moment is slowly reduced in magnitude. The result is a slow sinusoidal type motion which resembles the phugoid mode of an aircraft. This motion is very lightly damped, though all states do eventually settle to constant values at a much later value of time. The pitch attitude angle settles at a positive value in this example and the pitch rate goes to zero. The angle-of-attack settles at a nonzero value which negates the aerodynamic pitching moment.

Figure 16 shows the time history of the $u$ velocity component. The airspeed of the aircraft also follows the lightly damped, sinusoidal motion the pitch attitude angle experiences. The airspeed does eventually settle to a constant value as well, which is lower in magnitude than the first example.

B. Example 2: Lateral/Directional Motion with Delayed Shape Changes

In this example, the morphing aircraft will begin with the shape from the first example and performs a shape change at a time of 15 seconds. Only lateral/directional motion is considered in this example. The shape changes are presented in Figure 17.
Figure 18 shows the time history of the states of the morphing aircraft. The roll and yaw Euler angles begin to increase in magnitude due to a steady increase in magnitude by the angular velocity rates. Once the shape change occurs, the angular velocity rates begin to damp out. The damping of the roll motion is not quite as good as in the previous example, but the damping in the yawing motion is significantly improved over the previous case. This allows the yaw attitude angle to settle to a constant nonzero value. The sideslip angle decreases in magnitude before the shape change and then appears to slowly move towards a nonzero value after the shape change occurs.

VI. Conclusions

This paper develops an aerodynamic model and a dynamic model of a morphing flying wing aircraft. The dynamic model includes accurate aerodynamic forces calculated using the aerodynamic model. The aerodynamic model calculates forces due to large scale shape changing effects. The dynamic model calculates state information for the morphing wing based on the aerodynamic forces from the aerodynamic model. The model allows for multiple shape changing degrees-of-freedom for the wing, including thickness, sweep, dihedral angle, and chord length. Results show the model provides a versatile tool for calculating the aerodynamic forces on the morphing aircraft and the use of these forces to determine the associated states. Two shapes are tested in order to demonstrate the effects of large scale shape changes.

Based on the results presented in this thesis it is concluded that:

1. A computational model for a morphing wing was developed and includes accurate aerodynamic results
in incompressible, inviscid flow for basic and complex wing shapes by using a constant strength source-doublet panel code. The dynamical model is able to provide state information based on the aerodynamic forces from the aerodynamic model.

2. The use of cosine spacing in the aerodynamic model reduces the computational time required by approximately 50 per cent. Since the panel method is sensitive to changes in the grid, especially near the leading and trailing edges of the wing, placing more panels in these areas allows for fewer overall panels to be placed on the wing.

3. The dynamic model shows that when the morphing wing does not have any twist or camber, the aircraft is unable to fly at a straight and level configuration. By adding twist and camber to the wing, it is possible place the aircraft at an angle-of-attack which eliminates the aerodynamic pitching moment and provides a constant, positive lift force.

4. A wing with an asymmetric configuration is able to effectively produce rolling and yawing moments which can be used to counter disturbances in rolling and yawing motion.

VII. Acknowledgment

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Figure 13. Leading Edge Moment Curve for Example #2

References


Table 1. Initial Conditions

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Figure 14. Time History of Morphing Aircraft Shape for Example 1


Figure 15. Morphing Aircraft States for Example 1

Figure 16. Time History of $\alpha$ for Example 1
Figure 17. Time History of Morphing Aircraft Shape for Example 2

Figure 18. Time History of Morphing Aircraft States for Example 2