Lattice Reduction-Aided Regularized Block Diagonalization for Multiuser MIMO Systems

Keke Za†, Rodrigo C. de Lamare‡ and Martin Haardt†

† Communications Research Group, Department of Electronics, University of York
York Y010 5DD, United Kingdom
Emails: kz511@york.ac.uk, rcdl500@ohm.york.ac.uk
‡ Communications Research Laboratory, Ilmenau University of Technology
PO Box 100565, D-98684 Ilmenau, Germany
Email: martin.haardt@tu-ilmenau.de

Abstract—By employing the regularized block diagonalization (RBD) preprocessing technique, the multi-user multi-input multi-output (MU-MIMO) broadcast channel is decomposed into multiple parallel independent single user multi-input multi-output (SU-MIMO) channels and achieves the maximum diversity order at high data rates. The computational complexity of RBD, however, is relatively high due to two singular value decomposition (SVD) operations. In this paper, a low-complexity lattice reduction aided RBD is proposed. The first SVD is replaced by a QR decomposition, and the orthogonalization procedure provided by the second SVD is substituted by a lattice reduction whose complexity is mainly contributed by a QR decomposition. Simulation results show that the proposed algorithm can achieve almost the same sum-rate as RBD while offering a lower complexity and substantial BER gains with perfect as well as imperfect channel state information at the transmit side.

I. INTRODUCTION

Unlike the received signal in single user multi-input multi-output (SU-MIMO) systems, the received signals of different users in multi-user multi-input multi-output (MU-MIMO) systems not only suffer from the noise and intra-antenna interference but are also affected by the multi-user interference (MUI). Channel inversion strategies such as zero forcing (ZF) and minimum mean squared error (MMSE) precoding [1], [5] can be still used to cancel the MUI, but they result in a reduced throughput or require higher power at the transmitter [2]. Block diagonalization (BD) has been proposed in [2] to improve the sum-rate or reduce the transmitted power. However, BD only takes the MUI into account and suffers a performance loss at low signal to noise ratios (SNRs) when the noise is the dominant factor. Therefore, the regularized block diagonalization (RBD) which introduces a regularization to take the noise term into account has been proposed in [3].

Although the MU-MIMO system performance is improved by BD or RBD compared with a channel inversion scheme, the computational complexity of BD or RBD is relatively high due to two SVD operations. In order to reduce the complexity of BD, the first SVD of RBD is replaced with a less complex QR decomposition in [4]. We term here the RBD in [4] as QR/SVD RBD. The second SVD of RBD is used to orthogonalize the equivalent SU-MIMO channels and obtain a power loading matrix. In this paper, we replace the second SVD with a complex valued lattice reduction (CLR) whose complexity is mainly due to a QR decomposition. Then, an RBD CLR-aided precoding algorithm is proposed, which not only offers a lower complexity but also achieves a better BER performance than the conventional RBD and the QR/SVD RBD. In addition, for the RBD or QR/SVD RBD algorithm we still need a unitary matrix for decoding which is obtained by the second SVD to orthogonalize each user’s stream. However, this SVD is not required any more in the proposed algorithm which only needs the channel state information (CSI) and a quantization procedure at the receiver. For convenience, the proposed algorithm is termed as LC-RBD-LR in this paper.

In order to implement the above precoding algorithms, the CSI has to be known at the transmit side. In time division duplexing (TDD) systems, the CSI can be acquired by the transmitter easily since the downlink and the uplink share the same physical wireless channels. In frequency division duplexing (FDD) systems, the transmitter has to rely on the feedback of the CSI provided by the receiver to perform precoding. In fact, the feedback CSI is inevitably distorted due to the estimation errors, transmission delay and feedback errors. In this paper, we firstly assume the channel is perfectly known at the transmit side in order to illustrate the performance of the proposed algorithms. Then, the impact of imperfect CSI is studied.

This paper is organized as follows. The system model is given in Section II. The proposed LC-RBD-LR algorithm is described in detail in Section III and the computational complexity analysis is given in Section IV. The effect of imperfect channel is investigated in Section V. Simulation results and conclusions are presented in Section VI and Section VII.

Notation: Matrices and vectors are denoted by upper and lowercase boldface letters, and the transpose, Hermitian transposition, inverse, pseudo-inverse of a matrix \( B \) by \( B^T \), \( B^H \), \( B^{-1} \), \( B^+ \), respectively. The trace, rank, determinant, 2-norm are denoted as \( Tr(\cdot) \), \( r(\cdot) \), \( det(\cdot) \), \( \| \cdot \| \). \( I \) and \( 0 \) are identity matrix and zero matrix, respectively.

II. SYSTEM MODEL

We consider an uncoded MU-MIMO broadcast channel, with \( N_T \) transmit antennas at the base station (BS) and \( N_r \) receive antennas at the ith user equipment (UE). With \( K \) users in the system, the total number of receive antennas is
\[ N_R = \sum_{i=1}^{K} N_i. \]
We assume a flat fading MIMO channel and the received signal at the \( i \)th user is given by
\[ y_i = \beta^{-1}(H_i P_i s_i + H_i \sum_{j=1,j \neq i}^{K} P_j s_j + n_i), \] (1)
where \( H_i \in \mathbb{C}^{N_i \times N_T}, P_i \in \mathbb{C}^{N_T \times N_i}, s_i \in \mathbb{C}^{N_i} \) are the \( i \)th user’s channel matrix, precoding matrix and the transmit signal, respectively. The quantity \( \beta \) is a scalar chosen to make sure the energy of the precoded signal still the same as the average transmit power \( E_s \). And \( n_i \in \mathbb{C}^{N_i} \) is the \( i \)th user’s Gaussian noise with independent and identically distributed (i.i.d.) entries of zero mean and variance \( \sigma_n^2 \).

III. PROPOSED LC-RBD-LR ALGORITHM [6]

From the system model, the combined channel matrix is given by \( H = [H_1^T \ H_2^T \ \ldots \ H_K^T]^T \). We exclude the \( i \)th user’s channel matrix and define \( \overline{H}_i = [H_1^T \ H_2^T \ H_{i+1}^T \ \ldots \ H_K^T]^T \), so that \( \overline{H}_i \in \mathbb{C}^{N_i \times N_T} \). The proposed precoder design is performed in two steps. Correspondingly, the precoding matrix for the \( i \)th user can be rewritten as \( P_i = \beta P_i^b P_i^a \).

Step 1: Obtaining the first precoding matrix \( P_i^a \) by a QR decomposition of an extension of the matrix \( \overline{H}_i \).

For user \( i \), the channel extension of \( \overline{H}_i \) is defined as
\[ \overline{H}_i = [\rho I_{N_i}, \overline{H}_i], \] (2)
where \( \rho = \sqrt{\frac{N_R \sigma_n^2}{E_s}} \) and \( I_{N_i} \) is a \( N_i \times N_i \) identity matrix.

The QR decomposition of \( \overline{H}_i^H \) is given by
\[ \overline{H}_i^H = Q_i R_i, \] (3)
where \( Q_i \) is an \((N_i + N_T) \times (N_i + N_T)\) unitary matrix and \( R_i \) is an \((N_i + N_T) \times N_i \) upper triangular matrix. The first precoding matrix \( P_i^a \) for the \( i \)th user is obtained as
\[ P_i^a = Q_i (N_i + 1 : N_i + N_T), \] (4)
the columns of \( P_i^a \) lie in the null space of \( H_j (\forall j \neq i) \), and the first precoding matrix \( P_i^a \) is equivalent to the one obtained by the first SVD in the conventional RBD [4]. Then, the first \( N_T \times N_T \) combined precoding matrix for all users is
\[ P^a = [P_1^a, P_2^a, \ldots, P_K^a]. \] (5)

Step 2: Employing the CLR algorithm instead of the second SVD to implement the size-reduction, and obtaining the second precoding matrix \( P_i^b \) by channel inversion.

The aim of the CLR transformation is to find a new basis \( \tilde{H} \) which is nearly orthogonal compared to the original matrix \( H \) for a given lattice \( L(H) \). After the first precoding, the effective channel matrix for the \( i \)th user is
\[ \tilde{H}_{eff,i} = H_i P_i^a. \] (6)

We perform the CLR transformation on \( \tilde{H}_{eff,i}^T \) in the precoding scenario [9], that is
\[ \tilde{H}_{eff,i} = U_i H_{eff,i}\], (7)
where \( U_i \) is an unimodular matrix which satisfies \(|\det(U_i)| = 1\) and \( u_{l,k} \in \mathbb{Z} + j\mathbb{Z} \). The physical meaning of the constraints is that the transmit power is unchanged after the CLR transformation.

By using the ZF precoding, the second precoding matrix for user \( i \) is given as
\[ P_{ZF,i}^b = \tilde{H}_{eff,i}^H (\tilde{H}_{eff,i}^H \tilde{H}_{eff,i})^{-1}. \] (8)

As shown in [10], [11], the MMSE precoding is equivalent to ZF precoding with respect to an extended channel matrix \( H \) which is defined below for the precoding scenario
\[ H = [H, \sigma_n I_{N_R}]. \] (9)
The MMSE precoding filter can be rewritten as \( P_{MMSE} = A \tilde{H}_{eff,i}^H (\tilde{H}_{eff,i}^H \tilde{H}_{eff,i})^{-1} \), where \( A = [I_{N_T}, 0_{N_R \times N_R}] \). Actually, it is the rows of \( H \) determine the effective transmit power amplification. Thus, the CLR transformation should be applied to the transpose of the extended channel matrix \( \tilde{H}_{eff,i}^H = [H_{eff,i}, \sigma_n I_{N_R}]^T \) to obtain the CLR transformed channel matrix \( \tilde{H}_{eff,i}^H \). Then, the CLR-aided MMSE precoding filter is given by
\[ \tilde{P}_{MMSE,i} = \tilde{A} \tilde{H}_{eff,i}^H (\tilde{H}_{eff,i}^H \tilde{H}_{eff,i})^{-1}. \] (10)

Finally, the second precoding matrix \( P_i^b \) for all users is
\[ P^b = \begin{bmatrix} P_1^b & 0 & \ldots & 0 \\ 0 & P_2^b & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & P_K^b \end{bmatrix}. \] (11)
The resulting precoding matrix is \( P = \beta P^a P^b \), where the gain factor \( \beta = \sqrt{E_s/||P^a P^b||^2} \). The received signal is finally obtained as
\[ y = \beta^{-1}(H P s + n). \] (12)
The mainly processing work left for the receiver is to quantize the received signal \( y \) to the nearest transmitted symbols.

IV. COMPUTATIONAL COMPLEXITY ANALYSIS

In this section we use the total number of FLOPs to measure the computational complexity of the proposed and existing algorithms. According to [8], the average complexity of the CLR algorithm is almost 1.6 times of the QR decomposition. The FLOPs for the real QR, SVD and complex QR decomposition are given in [7]. In real arithmetic, a multiplication followed by an addition needs 2 FLOPs; in a complex scenario, a multiplication followed by an addition needs 8 FLOPs. Thus, the complexity of a complex matrix multiplication is nearly 4 times that of its real counterpart. For a complex \( m \times n \) matrix \( B \), its SVD is given by \( B = U \Sigma V^T \), where \( U \) and \( V \) are unitary matrices and \( \Sigma \) is a diagonal matrix containing the singular values of matrix \( B \). The equivalent real-valued SVD can be obtained by rewriting the formulation as
\[ \begin{bmatrix} B_r & B_i \\ -B_i & B_r \end{bmatrix} = \begin{bmatrix} U_r & U_i \\ U_i & -U_r \end{bmatrix} \begin{bmatrix} \Sigma & 0 \\ 0 & \Sigma \end{bmatrix} \begin{bmatrix} V_r^T & V_i^T \\ V_i^T & -V_r^T \end{bmatrix}. \] (13)
From (13), the number of FLOPs required by a $m \times n$ complex SVD is equivalent to the complexity required by its extended $2m \times 2n$ real matrix. We summarize the total FLOPs needed for the matrix operations below:

- Multiplication of $m \times n$ and $n \times p$ complex matrices: $8mnp$.
- QR decomposition of an $m \times n$ ($m \leq n$) complex matrix: $16(n^2m - mn^2 + \frac{3}{4}n^3)$.
- SVD of an $m \times n$ ($m \leq n$) complex matrix where only $U$ and $V$ are obtained: $32(mn^2 + 2n^3)$.
- SVD of an $m \times n$ ($m \leq n$) complex matrix where only $U$, $V$, and the singular values are obtained: $8(4m^2n + 8mn^2 + 9n^3)$.
- Inversion of an $m \times m$ real matrix: $2m^3 - 2m^2 + m$.

For the case shown in Table I, Table II and Table III, the complexity of the proposed LC-RBD-LR-ZF is about 46.1% of RBD and 70.3% of the QR/SVD RBD, while the complexity of the proposed LC-RBD-LR-MMSE is about 55.8% of RBD and 85.1% of the QR/SVD RBD. Clearly, the proposed algorithm requires the lowest complexity.

### Table I: Computational complexity of proposed LC-RBD-LR algorithm

<table>
<thead>
<tr>
<th>Steps</th>
<th>Operations</th>
<th>FLOPs</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 QR($\mathbf{H}^H$)</td>
<td>$16K(N_c^2-N_t^2+\frac{1}{4}N_i^2)$</td>
<td>12544</td>
<td></td>
</tr>
<tr>
<td>2 ZF</td>
<td>$8\sqrt{N_c}N_r^2$</td>
<td>1728</td>
<td></td>
</tr>
<tr>
<td>3 CLR($\mathbf{H}_e^H$)$^T$</td>
<td>$25.6K(N_c^2-N_t^2+\frac{1}{4}N_i^2)$</td>
<td>3891</td>
<td></td>
</tr>
<tr>
<td>4 ZF</td>
<td>$K(2N_t^2-2N_c^2)+N_r+16N_cN_r^2$</td>
<td>Total 19345</td>
<td></td>
</tr>
<tr>
<td>5 CLR($\mathbf{H}_e^H$)$^H$</td>
<td>$25.6K(N_c^2-N_t^2+\frac{1}{4}N_i^2)$</td>
<td>7578</td>
<td></td>
</tr>
<tr>
<td>6 ZF</td>
<td>$K(2N_t^2-2N_c^2)+N_r+16N_cN_r^2$</td>
<td>Total 23416</td>
<td></td>
</tr>
</tbody>
</table>

### Table II: Computational complexity of Conventional RBD

<table>
<thead>
<tr>
<th>Steps</th>
<th>Operations</th>
<th>FLOPs</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $U_h^H\sum_i V_{e_i}^H$</td>
<td>$32K(N_rN_c^2+\frac{1}{4}N_i^2)$</td>
<td>21504</td>
<td></td>
</tr>
<tr>
<td>2 $\sum_i\sum_j V_{e_i}^H D_i^j (D_i^j ← 2)$</td>
<td>$K(18N_c^2+N_r)$</td>
<td>336</td>
<td></td>
</tr>
<tr>
<td>3 $V_{e_i}^H P_{e_i}$</td>
<td>$8KN_c^2$</td>
<td>5184</td>
<td></td>
</tr>
<tr>
<td>4 $H_e^H P_h$</td>
<td>$8N_cN_r^2$</td>
<td>1728</td>
<td></td>
</tr>
<tr>
<td>5 $U_h^H \sum_i V_{e_i}^H$</td>
<td>$64K(2N_t^2+\frac{1}{4}N_i^2)$</td>
<td>Total 42000</td>
<td></td>
</tr>
</tbody>
</table>

### Table III: Computational complexity of QR/SVD RBD [4]

<table>
<thead>
<tr>
<th>Steps</th>
<th>Operations</th>
<th>FLOPs</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $\mathbf{H}_e^H = \mathbf{Q}_r \mathbf{R}_r$</td>
<td>$16K(N_c^2-N_t^2+\frac{1}{4}N_i^2)$</td>
<td>12544</td>
<td></td>
</tr>
<tr>
<td>2 $\mathbf{H}_{\text{eff}} = \mathbf{H}_e \mathbf{P}_h^a$</td>
<td>$8\sqrt{N_c}N_r^2$</td>
<td>1728</td>
<td></td>
</tr>
<tr>
<td>3 $\mathbf{H}<em>{\text{eff}} = U_h^H \sum_i V</em>{e_i}^H$</td>
<td>$64K(2N_t^2+\frac{1}{4}N_i^2)$</td>
<td>Total 27520</td>
<td></td>
</tr>
</tbody>
</table>

Coordinately, the precoding matrix $\mathbf{P}$ has to be designed based on the feedback channel $\mathbf{H}_e$ while the physical channel is $\mathbf{H}$ during each transmission, therefore, the BER performance will be degraded by the distortion term $\mathbf{E}$. Assuming that the precoding matrix $\mathbf{P}$ is designed according to the RBD-ZF-LR algorithm, the received signal is given by:

$$y = (\mathbf{H}_e - \mathbf{E})\mathbf{P}_s + \beta^{-1}\mathbf{n} = \mathbf{s} - \mathbf{E}\mathbf{P}_s + \beta^{-1}\mathbf{n},$$

(15)

where $\mathbf{E}\mathbf{P}_s$ is the interference term caused by the imperfect CSI. The error covariance matrix is obtained as

$$\Phi_{\text{err}} = \mathbb{E}[(y - s)(y - s)^H] = \sigma_e^2 \mathbb{E}[\mathbf{P}\mathbf{P}^H] + \beta^{-2}\sigma_n^2,$$

(16)

With perfect CSI, $\sigma_e^2$ is zero in $\Phi_{\text{err}}$ and the total error is only determined by the noise term $\mathbf{n}$; if there exists estimation errors or feedback errors, however, the total error $\Phi_{\text{err}}$ is not only affected by the noise $\mathbf{n}$ but also influenced by the distortion term $\mathbf{E}$. And the BER performance would become worse with the increase of the distortion power $\sigma_e^2$.

Another factor that we should take into account is the spatial correlation caused by sparse scattering and insufficient spacing between adjacent antennas. The Kronecker model of a correlated channel matrix can be written as [13]

$$\mathbf{H}_e = \mathbf{R}_h^H \mathbf{R}_T^{\frac{1}{2}},$$

(17)

where $\mathbf{R}_R$ and $\mathbf{R}_T$ are receive and transmit covariance matrix with $\mathbb{Tr}(\mathbf{R}_R) = N_R$ and $\mathbb{Tr}(\mathbf{R}_T) = N_T$. Both $\mathbf{R}_R$ and $\mathbf{R}_T$ are positive semi-definite Hermitian matrices. In the presence of receive or transmit correlation, the rank of $\mathbf{H}_e$ is constrained by $\min(r(\mathbf{R}_R), r(\mathbf{R}_T))$, therefore, the system will suffer both BER and sum-rate performance loss because of the rank deficiency. For the case of an urban wireless environment, the UE is always surrounded by rich scattering objects and the channel is most likely independent Rayleigh fading at the receive side; from the transmitter’s point of view, however, the spatial structure of the channel is governed by remote scattering objects and will most likely result in a highly spatially correlated scenario [14]. Hence, we assume $\mathbf{R}_R = I_R$, and thus we have

$$\mathbf{H}_e = \mathbf{H} \mathbf{R}_T^{\frac{1}{2}},$$

(18)

To study the effect of antenna correlations, random realizations of correlated channels are generated according to the exponential correlation model [15] such that the element of $\mathbf{R}_T$ is given by
where \( r \) is the correlation coefficient between any two neighboring antennas. This correlation model is suitable for our study since, in practice, the correlation between neighboring channels is higher than that between distant channels. In the following, we examine the performance of the above algorithms with \( |r| = 0.2, 0.5 \) and \( 0.7 \).

VI. SIMULATION RESULTS

A system with \( N_T = 6 \) transmit antennas and \( K = 3 \) users each equipped with \( N_i = 2 \) receive antennas is considered; this scenario is denoted as \((2, 2, 2)\times 6\) case. The transmitted \( i \)th user’s symbols are QPSK points. The \( i \)th user’s channel matrix is assumed a complex Gaussian channel matrix with zero mean and unit variance. We assume a block fading channel, that is, the channel is static during each transmit packet. The perfect CSI is first considered, and then the impact of imperfect CSI is evaluated. Moreover, the system performance with spatial correlation channel matrix is also simulated. For simplicity, the power loading between users and streams are not be considered and this strategy is termed as no power loading (NPL). The number of simulation trials is 1000 and the packet length is 100 symbols. The \( E_b/N_0 \) is defined as \( E_b/N_0 = \frac{N_b E_s}{N_C M} \) with \( M \) being the number of transmitted information bits per channel symbol.

Fig. 1 shows the BER performance of the proposed and existing algorithms with perfect CSI. It is clear that the proposed algorithm displays a better performance compared to the BD, RBD and QR/SVD RBD algorithms. At the BER of \( 10^{-2} \), LC-RBD-LR-ZF has more than 6 dB gains compared to the RBD, whereas LC-RBD-LR-MMSE has more than 7 dB gains over RBD. It is worth noting that the BER gains of the proposed algorithm get larger with the increase of \( E_b/N_0 \).

The proposed LC-RBD-LR-MMSE shows the same sum-rate as RBD at low \( E_b/N_0 \)s. At high \( E_b/N_0 \)s, it is slightly inferior to the RBD but requires a lower computational complexity.

Fig. 2 illustrates the sum-rate of the above algorithms with perfect CSI. The information rate is calculated using [16]:

\[
C = \log(\det(I + \sigma_n^{-2} H P P^H H^H)).
\]  

(20)

Fig. 2. Sum-rate performance, \((2, 2, 2) \times 6\) MU-MIMO, QPSK

Fig. 3. BER with \( \sigma_n^2 \) for a fixed \( E_b/N_0=10dB \), QPSK

Fig. 3. gives the BER performance of the above algorithms with imperfect CSI of fixed \( E_b/N_0 = 10dB \). It is clear that by increasing the distortion noise power \( \sigma_n^2 \), the BER gets worse for all the above algorithms. The proposed LC-RBD-LR-MMSE outperforms RBD when \( \sigma_n^2 \) is below \( 10^{-1} \), however, for severe distortions, RBD is more robust and reliable than the other algorithms.

Fig. 4. and Fig. 5. display the BER and sum-rate performance of the above algorithms with spatial correlation. It is obvious that both BER and sum-rate performance deteriorate with the increase of the correlation coefficient \( r \). The BER performances of the proposed LC-RBD-LR-ZF and LC-RBD-LR-MMSE outperform the BD and RBD algorithms from slight to the severe correlations. Due to the second step being based on the channel inversion strategy, the proposed algorithms still suffer a little sum-rate loss at high SNRs. At low SNRs, however, the sum-rate of the proposed LC-RBD-LR-MMSE gradually becomes better compared to the RBD.
For example, with the highly correlated scenario $|r| = 0.7$, the sum-rate of LC-RBD-LR-MMSE is better than RBD from 0 dB to 10 dB, which illustrates the robustness when the proposed algorithms encounter spatial correlation.

The BER performance of RBD is actually dependent on the power loading algorithm being used, an improved diversity (impD) power loading algorithm is proposed in [3] to achieve a better BER performance for RBD. For a fair comparison, the RBD with impD power loading is simulated and the comparison with the proposed algorithm is shown in Fig. 6. As we can see, RBD-impD shows a 5 dB gains over RBD-NPL at BER around $2.8 \times 10^{-5}$; however, it still gets 5 dB loss compared to the proposed LC-RBD-LR-MMSE algorithm.

**VII. CONCLUSION**

In this paper, a low-complexity precoding algorithm for MU-MIMO systems has been proposed. The complexity of the precoding process is reduced and a considerable BER gain is achieved at a cost of a slightly sum-rate loss at high SNRs. The proposed algorithm shows a robust performance in the presence of imperfect CSI and spatial correlation. It is worth noting that, the receiver is simplified by employing the proposed algorithm at transmit side.

**REFERENCES**


