Online social networks are multi-dimensional and dynamic. By combining these two perspectives, we can differentiate between various temporal patterns, better understand the mechanisms behind network formation, and test theories that are potentially associated with the behaviors of individuals online. In this study, we develop a temporal network analysis model and a multi-theoretical framework for examining various temporal network patterns in multi-dimensional networks. The proposed framework suggests a list of temporal patterns that may be observed in online social networks, including temporal reciprocity, co-occurrence, triangle, and k-star. We also provide theoretical explanations for why these patterns could be observed. This study provides a generalized framework to explore, analyze, and explain various temporal patterns in online social networks. An empirical test of our framework in the context of online social communities is outlined. The extended multi-theoretical framework can be easily applied to any social network that shows multi-dimensionality.

**Keywords:** Exponential Random Graph Models, social media, social network,
Introduction

Recent years have witnessed a rapid proliferation of Web 2.0 technologies including social media, where people can establish online communities to communicate, share information, and collaborate. As Kane et al. (2014) pointed out, one of the core features of these online social media sites is the existence of relational ties among community members, which provides fruitful opportunities for using social network analysis (SNA) to understand how and why people build connections in online social networks. As a result, a growing number of studies have applied SNA in an online context (Oestreicher-Singer and Sundararajan 2012; Shi et al. 2014; Singh et al. 2011; Stieglitz and Dang-Xuan 2013; Susarla et al. 2012).

One of the important characteristics of online social networks is multi-dimensionality, which indicates that multiple types of network nodes and ties co-exist in the same network (Contractor 2009). For example, some users in online platforms may be well known in the virtual community, while others may be anonymous temporary visitors. Also, these users often develop different types of relational ties, such as communication ties which represent message exchanges, and friendship ties which are enabled by social platforms to help develop long-term relationships. A consequence of network multi-dimensionality is that the seemingly same network patterns can be resulting from different underlying processes of network formation, depending on the order in which network ties develop. For example, Figure 1 shows two different processes of forming a two-star pattern. Assume that the black nodes represent well-known users in the community, and the numbers next to ties indicate the order in which relationships develop. The pattern 1 illustrates a process in which a well known user is prioritized over others in developing relationships, while the pattern 2 illustrates a process where this user is de-prioritized. Differentiating between various temporal sequences of tie formation is important for understanding the mechanisms behind network formation, because different theories may be associated with each scenario of the temporal sequences, which however leads to the seemingly same network patterns.

In this study, we propose an extended model for temporal network analysis and a multi-theoretical framework for using the model to examine various temporal patterns in multi-dimensional networks. The framework is an extension of Contractor et al. (2006)'s multi-level multi-theoretical framework for exploring organizational networks. The proposed framework suggests a list of temporal patterns that may be observed in online social networks, and provide potential theoretical explanations for why these patterns are observed. We will empirically test our framework in the context of online social media, and theoretically explain how and why online social ties in these communities develop. The objective of this study is to provide a generalized framework to explore, analyze, and explain online social networks.

The remainder of the paper is organized as follows. We first review relevant social network literature to provide the background for network multi-dimensionality and address the need for analyzing temporal patterns in online social network analysis. Then we introduce a statistical model, Exponential Random Graph Model (ERGM), which is particularly useful for analyzing multi-dimensional networks and is the basis of our extended model. Based on ERGM, we then describe the extended model and the proposed multi-theoretical framework for exploring temporal network patterns, and discuss relevant theories that can be tested under the framework. An initial plan of testing the framework is discussed subsequently. Finally, we summarize expected contribution of this research.
Background

Multi-dimensionality of Online Social Networks and Temporal Network Patterns

Multi-dimensional networks are networks that consist of multiple types of nodes and multiple types of ties inter-connecting the nodes (Contractor 2009; Jiang et al. 2013; Su and Contractor 2011). The nodes can be of distinct categories. Some nodes may be individual persons while others may be non-human agents such as machines, documents, and tools (Contractor 2009). These distinct categories of nodes are connected by ties that represent various relationships. For instance, Su and Contractor (2011) used relational ties to link consultants and digital knowledge depositories in consultancy firms, representing knowledge seeking relationships between human and machines. For the same category of nodes, nodal attributes can also differ, which further enriches the variety of nodes in multi-dimensional networks. For example, Jiang et al. (2013) examined a knowledge diffusion network where both funded and unfunded inventors were present; the different types of inventors showed different knowledge exchange behaviors. Network ties between nodes can also have multiple types, which is the other aspect of the network multi-dimensionality. In prior studies, various types of relational ties between online users have been identified. Based on the classification in Kane et al. (2014), the four major types are proximities, interactions, social relations, and flows. Proximity ties represent that two individuals belong to the same online sub-communities. Interaction ties represent information exchanges between two parties (Stewart and Abidi 2011). The information is often textual, such as message replies in online web forums. Social relation ties include virtual friendship and subscription relationships in micro-blogging sites (Kwak et al. 2010; Shriver et al. 2013). They are usually established based on a mutual agreement between users that they are interested in each other and like to keep in touch, thus forming “strong ties” (Nelson 1989), which are stable and long-lasting compared to other types of ties. Flow ties represent the movement of goods between network nodes, such as retweets and social bookmarking. Most prior studies modeled these types of relationships in separate networks. However, different types of network ties can co-exist in a multi-dimensional online network, and interact with each other (Kane et al. 2014).

Note that multi-dimensional networks are different from two (or higher order)-mode networks in several ways (Borgatti and Everett 1997; Carrington et al. 2005). First, in a two-mode network, network ties only exist between a pair of nodes belonging to different sets, while any two nodes can be connected in a multi-dimensional network. Second, two-mode networks can be (and are often) transformed to a one-mode network by projection methods (Newman 2001), whereas multi-dimensional networks are well-integrated and difficult to decompose. The complex interactions between multiple types of nodes and ties can be examined using statistical network modeling methods, such as ERGMs (Robins et al. 2007a).

In conventional network studies on offline social networks, the networks are often static. In other words, the networks are often “given” in the sense that all nodes and ties appear all at once; or at least the order in which the ties develop is ignored. Online social networks are inherently multi-dimensional, and the ties are usually associated with temporal information about when they develop (Susarla et al. 2012; Choudhary et al. 2010), thereby providing rich opportunities for dynamic network analysis. For instance, assume that some users are associated with attribute A, while others are not. Both types of users may develop relationships with another user, and hence result in a star structure in the network. By differentiating which type of users develop the ties first, the star structure can branch into two distinct temporal patterns—a star led by A nodes and a star followed up by A. The differentiation would help reveal the impact of attribute A on the formation of the network. For another example, two users can develop multiple types of relational ties. By considering which types of the ties develop first, we may be able to infer causal relationships between different types of ties, instead of merely observing the co-occurrence of ties. Taking into account the temporal patterns of multi-dimensional network formation is critical in understanding how and why online users develop relationships.

Exponential Random Graph Models

ERGMs are statistical models that can be used to test whether the observed networks exhibit theoretically hypothesized structural tendencies (Robins and Pattison 2005; Wasserman and Pattison 1996). These structural tendencies, or network patterns, are called configurations. Technically, a configuration is a subset of nodes and ties in the network, reflecting a certain type of network sub-structure. Examples of
typical configurations including “triangle” and “k-star” have been described in prior social network literature (Robins et al. 2007a; Robins et al. 2007b). In addition, nodal attributes can be incorporated in a configuration, thereby making ERGMs particularly fit for analyzing multi-dimensional networks. Formula (1) specifies the expression of ERGMs. In the formula, 1) Y is the random variable for the network status, where y is its realization; 2) \( \eta_A \) is a parameter corresponding to the configuration A, positively related to its tendency to occur; 3) \( g_A(y) \) is network statistics corresponding to A, present in the observed network y; 4) \( \kappa \) is a normalizing constant ensuring that \( \Pr(Y) \) is a probabilistic distribution.

\[
\Pr(Y = y) = \left(\frac{1}{\kappa}\right) \exp \left\{ \sum_A \eta_A g_A(y) \right\}
\]  

(1)

Given an observed network, the primary task of ERGMs is to examine which configurations appeared statistically more than by chance. If a parameter \( \eta_A \) is estimated to be significant, it will lead to support of the underlying hypotheses and theories that explain the formation of corresponding configurations.

ERGMs have been widely used in management and social science studies (Contractor et al. 2006; Ellwardt et al. 2012; Gondal 2011), and recently have been receiving attention in IS research as well (Faraj and Johnson 2011; Jiang et al. 2013; Su and Contractor 2011). To use ERGMs for exploratory analysis on a social network, the multi-level, multi-theoretical framework proposed by Contractor et al. (2006) is helpful. The framework identifies a set of possible network configurations that can be tested for organizational networks, including actor-level configurations, dyadic-level configurations, triadic-level configurations, and global-level configurations. The framework also suggests a set of theories that can explain the formation of these organizational network patterns. The framework has been used to suggest theoretical models or methodological approaches in later social network research under various contexts (Eberly et al. 2011; Fehr and Gelfand 2012; Moliterno and Mahony 2011). However, the framework did not consider temporal patterns of network configuration, as described in the previous subsection.

Recently, improved ERGM techniques have been devised to extend ERGM’s application beyond “static” network models (Guo et al. 2007; Krivitsky and Handcock 2014; Snijders et al. 2010). For example, Snijders et al. (2010) have formulated estimation techniques for ERGM parameters when the network data are discrete observations in continuous time. Separable Temporal ERGMs (STERGMs) address the problems of network dissolution in addition to tie formation (Krivitsky and Handcock 2014). However, none of these extensions address address network dynamics by differentiating between different processes that lead to the same network patterns, and develop frameworks for testing multiple underlying theories that potentially explain these temporal patterns.

### Extended ERGM model and Multi-theoretical Framework for Testing Temporal Network Patterns

Formula (2) specifies the expression of our extended model. The major difference with the existing model is that for each configuration type A, the model accounts for an additional layer of summation over all possible temporal patterns related to A, represented by the space T (A). This allows for differentiation between various temporal patterns associated with each type of configuration. The various temporal patterns constructing T(A) are suggested by our framework next.

\[
\Pr(Y = y) = \left(\frac{1}{\kappa}\right) \exp \left\{ \sum_A \sum_{t \in T(A)} \eta_{tA} g_t(y) \right\}
\]  

(2)

Since the temporal patterns of networks mainly result from the sequence of how different types of nodes form different types of ties, our proposed testing framework focuses on extending the dyadic, triadic, and global level variables in Contractor et al. (2006)’s work. In particular, we show that reciprocity, tie co-occurrence, triangle, and k-star configurations can have extended theory-testing capabilities when the sequence of the tie formation is modeled. Table 1 summarizes the extended multi-theoretical framework for testing temporal network patterns based on the configuration types, their corresponding hypotheses, and related theories. Markov dependence assumption is used, which indicates that a possible tie from i to
j is contingent on all other ties that involve i or j. In the following subsections, we explain each configuration in detail and provide examples of what theories can be tested by these configurations.

### Table 1. An Extended Multi-theoretical Framework for Testing Temporal Network Patterns

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Illustration*</th>
<th>Hypotheses†</th>
<th>Selected Relevant Theories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reciprocity-standard</td>
<td></td>
<td>Reciprocity has a high tendency to occur.</td>
<td>Equity theory</td>
</tr>
<tr>
<td>Reciprocity-temporal</td>
<td></td>
<td><strong>Left</strong>: The focal node is likely to receive feedbacks when it initiates a relationship. <strong>Right</strong>: The focal node is likely to respond to incoming ties.</td>
<td>Social Capital theory; Norm of Reciprocity</td>
</tr>
<tr>
<td>Co-occurrence-standard</td>
<td></td>
<td>Two types of ties are likely to co-occur.</td>
<td>Reinforcement theory;</td>
</tr>
<tr>
<td>Co-occurrence-temporal</td>
<td></td>
<td>One type of tie is likely to lead to the other type of tie.</td>
<td>Social Penetration Theory; Social Capital Theory</td>
</tr>
<tr>
<td>Triangle-standard</td>
<td></td>
<td>Triangle relationships have a high tendency to occur.</td>
<td>Theory of Cognitive Balance</td>
</tr>
<tr>
<td>Triangle-temporal</td>
<td></td>
<td><strong>Left</strong>: A node with some attributes is likely to bridge a relationship between other nodes. <strong>Middle</strong>: The focal node is likely to follow the behaviors of a linked friend. <strong>Right</strong>: The focal node is likely to build ties with two nodes that are already inter-connected.</td>
<td>Theory of Homophily; Theory of Imitation; Observational Learning; Reinforcement-affect Theory</td>
</tr>
<tr>
<td>K-star-standard</td>
<td></td>
<td>Star structures have a high tendency to occur.</td>
<td>Theory of Collective Action</td>
</tr>
<tr>
<td>K-star-temporal</td>
<td></td>
<td><strong>Left</strong>: The focal node is likely to be prioritized in a k-star relationship. <strong>Right</strong>: The focal node is likely to be deprioritized in a k-star relationship.</td>
<td>Rational Choice Theory; Social Exchange Theory</td>
</tr>
</tbody>
</table>

* The numbers next to the ties indicate the sequence of occurrence.
† In the hypotheses, the focal node refers to the color node, representing some nodal attributes.
**Reciprocity**

Reciprocity is a relationship of mutuality between two actors where outgoing relations are always associated with incoming ones. Reciprocity is a commonly observed phenomenon in social science and communication research (Ellwardt et al. 2012; Olk and Gibbons 2010). Reciprocity patterns in social networks are most relevant to equity theory which states that individuals are most satisfied in a relationship when the give and take are about equal (Adams 1963; Hatfield et al. 1978). Individuals are going to be unhappy if their outgoing relations are not acknowledged, and the individuals who are not responding to the incoming ties may also feel guilty about the imbalance. The theory would be supported if reciprocity patterns were statistically more likely to be observed than by chance in a social network.

For multi-dimensional networks, it is important to differentiate between the individuals sending out the outgoing tie first and the individuals responding to an incoming tie in a reciprocity relationship, as shown in row 2 of the Table 1. For example, assume that the individual’s attribute is related to reputation, which is a common individual attribute in online virtual communities (Ba and Pavlou 2002; Wasko and Faraj 2005). In social capital theory, one of the important facets of the relational dimension of social capital is trust (Coleman 1994; Nahapiet and Ghoshal 1998), which is developed through a history of direct and indirect interactions in the community. An individual with a good reputation is trusted by others, and can expect positive future interactions (Wasko and Faraj 2005). Thus, the social capital theory would be supported if we observed that online community members with a high reputation have a higher chance of getting responses when they send out outgoing ties, such as messages or friendship requests. By contrast, if we observed that individuals with a high reputation are likely to respond to incoming ties in reciprocity relationships, this would support that high reputation members are associated with the norm of reciprocity, which maintains that people feel obliged to return the favors of others (Cialdini 2001; Kunz and Woolcott 1976). In sum, by incorporating these temporal patterns of reciprocity in social network analysis, we are able to explain whether reciprocity is due to the relational social capital in online communities, or the norm of reciprocity of individuals, or both.

**Co-occurrence**

The co-occurrence of ties indicates that two network actors are connected with at least two different types of ties. Without considering which type of tie occurred first, co-occurrence in an online social network can be used to support reinforcement theory (Surlin and Gordon 1976) and confirmation bias (Clifford et al. 1977; Nickerson 1998), which suggest that people tend to establish multiple types of relationships with others in order to reinforce their initial belief, such as “connecting to a certain individual is beneficial.”

Co-occurrence can show different temporal patterns by considering the sequence of tie formation. Assume that in rows 3 and 4 in the Table 1, the bold ties in “co-occurrence” illustration represent friendship relationship in online social media, and the thinner ties represent communication links such as exchanging messages. On one hand, if we observed that friendship ties have a tendency to precede communication ties (as shown in the left part of Table 1), it would support one of the perspectives of social capital theory in that the structural dimension of social capital plays an important role in the multi-dimensional network formation (Nahapiet and Ghoshal 1998; Putnam 1995; Wasko and Faraj 2005). On the other hand, if we observed that communication ties are likely to form before friendship ties, it would support social penetration theory (Altman and Taylor 1973), which basically states that interpersonal relationships develop from relatively shallow and non-intimate levels to deeper and more intimate ones. Therefore, by incorporating these temporal patterns of co-occurrences in social network analysis, we are able to test whether social capital theory or social penetration theory is the driving factor for forming co-occurrence relationships in an online social network.

**Triangle**

A triangle relationship is a triadic relationship between three network actors where each pair of two is connected. Theories of cognitive balance (Heider 1958; Holland and Leinhardt 1974) suggest that individuals have a tendency to maintain consistency in personal relationships. In other words, people tend to think “a friend’s friend is also my friend.” We can test whether such tendencies are present in an online social media context by examining the triangle configurations in online social networks.
In multi-dimensional networks, the triangle configuration can be extended to a number of different temporal patterns by taking into account the timing when a node with attributes develops ties with the other two nodes, as illustrated in row 6 in the Table 1. Assume that the attribute is the activity level of an online user, which is one of the important characteristics of individual members in virtual communities often measured by the number of messages, the frequency of postings, and the likelihood of replies (Abbasi and Chen 2005; Choudhury et al. 2010; Jiang and Chen 2013). First, theory of homophily (McPherson et al. 2001; Monge and Contractor 2003) suggests that individuals who share common features and similar characteristics are likely to interact with each other. In online social networks, if we observed that many triangles are formed by an active user initiating friendships with two other users, who subsequently develop a third friendship tie between them (the left illustration in row 6), then the observation might support the theories of homophily because sharing a common friend might have led to equivalent topological positions as well as similar information exposure in the network. Moreover, we may infer that active users in online communities play a bridging role for friendships by creating structural homophily in the network. In another possible scenario, an active user establishes a friendship tie with another user first. Later, after noticing that the friend developed a new friendship tie with a third person, the active user may follow up a friendship tie with this third person (the middle illustration in row 6). If we observed many triangle patterns in a social network resulting from this scenario, it would support theories of imitation and observational learning (Apesteguia et al. 2007; Davis and Luthans 1980; Yi and Davis 2003), which suggest that exposure to a model for learning is an important stimuli to individuals’ behaviors. Furthermore, we may infer that active users in virtual communities tend to follow the activities of their close friends. Finally, the reinforcement-affect theory (Clore and Byrne 1974) argues that an individual will have a positive attitude towards another person, say, B, if B is always present when the individual is feeling good, even if B is not the driving factor for being happy. The theory would be supported if we observed that active users tend to develop friendship ties with a new friend’s existing friends (the right illustration in row 6), because this would indicate that the active users were susceptible to the influence described by the theory. In any case, by examining the temporal patterns for a triangle configuration in social network analysis, we are able to explain which theories led to the formation of the triangle relationships in online social networks.

The number of temporal patterns can be further increased if we model directed ties in the triangle configuration and differentiate transitivity and cyclicity (Contractor et al. 2006). However, in favor of model parsimony, we only focus on undirected ties in this research.

**K-star**

K-star configurations model the tendency of nodes to develop relationships with multiple (usually in a relatively large number of) nodes. The theory of collective action (Coleman 1986; Marwell and Oliver 1993; Olson 2009) suggests that actors in a network are more likely to obtain public goods if the network exhibits a centralized structure. In an online social network, the public goods can be the collective knowledge depository (Majchrzak et al. 2013), or the social capital embedded in the network itself (Singh et al. 2011; Wasko and Faraj 2005). Therefore, the k-star configuration can be used to test the aforementioned theory by examining whether individual members of online communities try to place themselves in a central position of the network (Contractor et al. 2006).

In a multi-dimensional network, considering that the central actor can develop ties with different types of other nodes, examining the order in which these ties are developed can provide new perspectives for k-star patterns. For example, the rational choice theory (Coleman and Fararo 1993; Goode 1997) states that individuals make choices to maximize their own benefits, and always prefer more than less of a good. In a similar fashion, the social exchange theory (Blau 1964; Emerson 1976) posits that individuals engage in social relations in an expectation of receiving social rewards. In row 8 in Table 1, if the node with attributes represents some preferred characteristics in the online collective, such as possessing more information than others, then observing that this type of individuals is likely to be prioritized in k-star relationships would support the abovementioned theories, because developing a tie with individual of this attribute would bring high marginal benefits to the central actor. These theories would also be supported if the attribute of interest is expected to have a negative impact on obtaining the public good of the collectivity, and meanwhile we observed that individuals with such attributes tend to be neglected in k-star relationships.
Extended Framework and Theory-testing

When applying the proposed extended framework for exploring multi-dimensional networks, the following implications need to be noted. First, it is not necessary to examine all temporal patterns for a given network and include all the temporal configurations listed in Table 1 into the extended ERGM. Instead, selection of temporal configurations should be guided by theories and a-priori knowledge about the network. For example, with traditional ERGMs we may find that the reciprocity pattern has a high tendency to occur in a network. We can then further include temporal reciprocity patterns using the extended model to examine whether the reciprocity pattern is more likely to be initiated by a special type of node. Also note that inclusion of a temporal pattern necessitates the inclusion of its corresponding static configuration due to their dependency. For instance, if the “reciprocity-temporal” is included in the model, “reciprocity-standard” should also be included. Second, if one of the temporal patterns is significant in a network, it does not necessarily refute the theories associated with the corresponding network pattern without the temporal information. Actually, more than one mechanism may be at work during network formation. For example, if we observed that high reputation individuals tend to receive responding ties in reciprocity relationships, it may also support equity theory in addition to confirming the effect of the relational social capital. It needs to be pointed out that which set of theories receives support depends on the context of the problem. Finally, the listed theories in Table 1 are suggestive rather than comprehensive. They are used as examples to illustrate the enhanced theory-testing capability of ERGM when temporal network patterns are considered. Theories outside the list may also explain some temporal patterns of multi-dimensional network formation, depending on the context of problems.

Data and Research Method

Research Test-bed

To provide empirical examples of applying the proposed framework, we focus on online social networks because they allow us to observe and model various temporal patterns. We have collected data from multiple platforms to test different temporal patterns. Our preliminary data collection includes an online knowledge seeking community, WikiAnswer.com, and an online healthcare community, Patientslikeme.com. WikiAnswer.com is a community where people can post open questions and wait for others’ answers, thereby forming knowledge networks. Patientslikeme.com provides a platform where patients can follow other patients who have similar healthcare concerns and share their experiences, resulting in subscription networks.

These platforms serve as promising research test-beds to test the proposed framework. In WikiAnswer.com, the questions and answers are organized into Q&A entries based on which various types of users can be identified, including registered users and unregistered anonymous visitors. These users develop at least two types of relational ties including knowledge transfer ties representing question answering relationships between two individuals, and collaboration ties which are established when two contributors work together to create a Q&A entry. Information about user types and tie types was collected, allowing us to test whether temporal co-occurrence and temporal k-star patterns exist. For example, we plan to examine if knowledge transfer ties tend to precede collaboration ties (the “co-occurrence-temporal” in Table 1). We can also test whether registered members are prioritized over unregistered visitors in k-star relationships that represent knowledge transfer from a central node (the “k-star-temporal” in Table 1). Moreover, temporal reciprocity and temporal triangle patterns can be potentially observed in Patientslikeme.com. For example, the community awards “3-stars” to patients who share complete health information. We can test whether patients with 3-stars are more likely to receive reciprocated subscription ties when they send follow requests (the “reciprocity-temporal” in Table 1). We can also test if patients with 3-stars tend to bridge subscription relationships between others when forming triangles (the “triangle-temporal” in Table 1). To be able to conduct such analyses, we have extracted subscription ties between patients and identified patients with 3-stars. Note that given a network, it is not necessary to observe all temporal patterns. Rather, we should first make hypotheses about which patterns are likely to be observed, and test these hypotheses using selected configurations in the proposed framework.
In this research, we focus on diabetes-related Q&A entries and patients with Type-2 Diabetes in these platforms, considering the topic’s broad societal impact. Table 2 shows the statistics of our data collection.

<table>
<thead>
<tr>
<th></th>
<th>Number of Users</th>
<th>Number of Ties</th>
<th>Time Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>WikiAnswer.com</td>
<td>70,164 (5,038 registered)</td>
<td>Knowledge transfer: 11,444 Collaboration: 32,180</td>
<td>2008-2013</td>
</tr>
<tr>
<td>Patientslikeme.com</td>
<td>14,701 (179 3-star)</td>
<td>Follow ties: 80,425</td>
<td>2005-2013</td>
</tr>
</tbody>
</table>

To model the temporal network configurations and test them with ERGMs, an open source package in R, statnet, is used. Ergm.userterms package is modified and used to reflect the temporal patterns.

**Validation**

To evaluate the ability of the proposed approach in capturing dynamic network patterns, the following validation framework will be used. First, the data are divided into time spells with monthly intervals. The proposed ERGM model \((2)\) is trained in each month by estimating the parameters for temporal configurations \((\eta_t)\). The estimated model is used to generate an ensemble of simulated networks and key network statistics including density, reciprocity, triangle, and k-star are compared against that of the actual network in next month, based on which average absolute error (AAE) is calculated. The process repeats for each month and produces a vector of AAEs. The same procedure is performed on a baseline model where only static network configurations are included, also resulting in a vector of AAEs. Finally, two vectors are compared using t-test. If the proposed model leads to statistically lower AAEs, it will indicate that the temporal model can predict future networks better than traditional ERGMs by capturing network dynamics.

**Expected Contribution**

The expected contribution of this study is manifold. First, the framework in this research extends the multi-level multi-theoretical framework proposed by Contractor et al. (2006). We believe that our extended model and framework allows richer insights into understanding the formation mechanisms of online social networks. Note that the proposed framework is able to capture some network dynamics that existing ERGMs cannot. For example, when applying existing ERGMs in different time periods in a longitudinal dataset, we can examine if a pattern, say, reciprocity, disappears or continues to manifest over time. However, we are unable to understand whether the reciprocity patterns is often initiated or completed by a special type of nodes. The proposed extended framework provides a finer analysis by taking into account the role of nodal attributes in temporal patterns. Methodologically, by incorporating temporal patterns of network configurations, the theory-testing capability of ERGMs can be greatly enhanced. Since multi-dimensionality has become a prevailing feature of online social networks, combining ERGMs with the proposed framework can help us better understand the dynamic formation mechanisms of online social networks. In addition, the extended multi-theoretical framework is not limited to online social networks, but can be easily applied to any social network that shows multi-dimensionality. Therefore, the proposed framework has a high generalizability. Using this framework, various theories in social science, communication, and psychology can be tested in the context of dynamic networks.

Given a network, our current framework focuses on testing a selected set of temporal patterns guided by theories and a-priori knowledge about the network. If we have absolutely no knowledge about a network and include all temporal configurations in the extended model, the model estimation procedure is expected to be slow. To address this limitation, some heuristic algorithms may be needed to expedite the estimation. This work is left for the future.
References


