Substantial simplification is possible. First premultiply (9) and (10) by \((s_1 I - F)^{-1}\) and postmultiply by \((s_2 I - F^T)^{-1}\), noting the conditions in Lemma 3.1, then substitute to obtain

\[
\Phi_{zz}(s_1, s_2) = (s_1 I - F)^{-1} M_0 (s_2 I - F^T)^{-1} - \frac{1}{s_1 + s_2} \left\{ (s_2 I + F)^{-1} \Gamma \right. \\
+ (s_1 I + F^T)^{-1} \Gamma (s_2 I - F^T)^{-1} \left. \right\} \times (s_2 I - F^T)^{-1} + (s_1 I - F)^{-1} \Gamma (s_1 I + F^T)^{-1}.
\]

But we can also show that the first half of the third term is \((s_1 + s_2)^{-1} (s_1 I - F)^{-1} \Gamma (s_2 I - F^T)^{-1}\), with a similar result holding for the second half of the third term. The second term therefore cancels and there results (6).

ACKNOWLEDGMENT

The author wishes to thank Dr. C. R. Offer and the anonymous reviewers for helpful remarks.

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2This observation is due to Dr. C. R. Offer.

An Exact Method for the Stability Analysis of Linear Consensus Protocols With Time Delay

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Abstract—This technical note presents a methodology for the stability analysis of linear consensus protocols with time-delayed communications. Second order agent dynamics with a fixed and undirected communication topology and uniform delays are considered. This class of group dynamics is very complex and is not fully explored to date. The proposed technique takes advantage of the general structure of the control protocols in performing a state transformation that allows a decomposition of the characteristic equation into a set of factors. These factors distribute the imprint of the delay in the characteristic equation in a much simpler form to achieve the stability analysis in parts. The procedure also prepares the characteristic equation for the deployment of the Cluster Treatment of Characteristic Roots paradigm, a recent method which declares the stability features of the system for various compositions of the time delay and other control parameters. In order to show the effectiveness of this approach, it is applied to different consensus protocols under the assumptions of fixed and undirected communication topologies and uniform communication time delays.

Index Terms—Cluster treatment of characteristic roots (CTCR), Consensus, multiagent systems.

I. INTRODUCTION

Among many aspects of the swarming dynamics, the consensus generation is one of the most widely studied topics. The main objective is to drive all the agents of the group in a way such that they will reach a common value in some variable of interest that may or may not be related to the motion of the agents. In the past few years researchers have introduced different protocols for consensus over group behaviors of agents with both first and second order dynamics in continuous and discrete time, with and without time delays, and using directed and undirected information flow. The work of Olfati–Saber and Murray [1] is one of the earlier studies published. They focus on continuous time agents with first order dynamics, considering fixed and switching communication topologies. Under the simplifying features of the first order dynamics, they also handle the communication delays, with fixed topology. Several other researchers [2]–[11] have performed further extensions on this earlier work, proposing different protocols for agents that are driven by second order dynamics in continuous time, also including variations on the topologies and communication delays. Some others have studied the consensus problem for groups of agents with discrete time first order dynamics, including communication and input delays [11] and references therein. These results, however, are out of the scope of this work, because the analysis of discrete time delayed systems is different (and simpler) from the continuous time case.

For the analysis of the stability in the delay space, some of the earlier works [2]–[4] use approximate methods, based on LMs. The process yields only conservative bounds in the time delay for stable operations. In this technical note, we propose a new methodology which can be used both for agents with first or second order dynamics. The main