Multi-Objective Design Problem of Tire Wear and Visualization of Its Pareto Solutions


ABSTRACT: Since tires carry out many functions and many of them have tradeoffs, it is important to find the combination of design variables that satisfy well-balanced performance in conceptual design stage. To find a good design of tires is to solve the multi-objective design problems, i.e., inverse problems. However, due to the lack of suitable solution techniques, such problems are converted into a single-objective optimization problem before being solved. Therefore, it is difficult to find the Pareto solutions of multi-objective design problems of tires. Recently, multi-objective evolutionary algorithms have become popular in many fields to find the Pareto solutions. In this paper, we propose a design procedure to solve multi-objective design problems as the comprehensive solver of inverse problems. At first, a multi-objective genetic algorithm (MOGA) is employed to find the Pareto solutions of tire performance, which are in multi-dimensional space of objective functions. Response surface method is also used to evaluate objective functions in the optimization process and can reduce CPU time dramatically. In addition, a self-organizing map (SOM) proposed by Kohonen is used to map Pareto solutions from high-dimensional objective space onto two-dimensional space. Using SOM, design engineers see easily the Pareto solutions of tire performance and can find suitable design plans. The SOM can be considered as an inverse function that defines the relation between Pareto solutions and design variables. To demonstrate the procedure, tire tread design is conducted. The objective of design is to improve uneven wear and wear life for both the front tire and the rear tire of a passenger car. Wear performance is evaluated by finite element analysis (FEA). Response surface is obtained by the design of experiments and FEA. Using both MOGA and SOM, we obtain a map of Pareto solutions. We can find suitable design plans that satisfy well-balanced performance on the map called “multi-performance map.” It helps tire design engineers to make their decision in conceptual design stage.

KEY WORDS: inverse problem, multi-objective optimization, multi-objective genetic algorithm, self-organizing map, Pareto solution, wear, finite element analysis

Introduction

Previously, it has been thought that the tire is one of the most complex structures to simulate numerically. It is true at present too. However, nowadays, complex multidisciplinary simulation, such as fluid structure interaction analysis of tires considered with real tread pattern, has been developed using FVM, FEM, and SPH for fluid modeling [1–3]. This kind of simulation is useful to
predict tire performance in tire design and development process. However, analysis is insufficient for design and development of products. Numerical simulation or analysis is a so-called “forward” problem. Inputs of analysis are design variables, i.e., size, shape, material constant, etc., and outputs are resultant responses of products. On the other hand, design is an inverse problem. In the design process, the aim of engineers is to find the design variables that lead to desired responses. Processes of design and analysis are opposite as shown in Fig. 1. One possible solution is to use an optimization technique that refines the design variable until a forward computation gives responses close to the desired responses. Optimization is the key issue to solving the inverse problem in the design process.

For the past decade, design optimization has been applied to the various areas in which the nonlinear finite element analysis (FEA) is indispensable due to progress of optimization techniques [4–10]. Response surface methodology (RSM) has triggered the increasing usage of design optimization for highly nonlinear problems [4]. Most highly nonlinear problems spent a lot of CPU time. In addition, it is difficult to calculate the gradient of objective functions with respect to design variables using explicit FEA codes, such as LS-DYNA. The RSM has been recognized as a practical optimization tool in the design process of production models; for example, the crash safety design of vehicle [5]. In addition, new optimization methods have been proposed to find the global optimal using evolutionary algorithms. Genetic algorithm (GA) is getting popular, and it has been applied to the tire design [11]. On the other hand, inverse shape determination of tires has been proposed by Koishi et al. [12] as a direct solution strategy for inverse problems. However, it is not a comprehensive methodology to solve inverse problems.

Tires carry out many functions. Some aspects of tire performance have tradeoffs, therefore an optimal solution is not unique. In such problems, each design has the best combination of objective values and improving one objective is impossible without sacrificing one or more of the other objectives. The solutions are called the Pareto solutions. Figure 2 shows the schematic figure of

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**FIG. 1 — Analysis and design.**
Pareto solutions of a two-objective maximization problem. Therefore, multi-objective optimization should be considered in the design process of tire. Due to the lack of suitable solution techniques, such problems are converted into a single-objective optimization problem. This is inadequate to obtain the Pareto solutions of tires. A multi-objective genetic algorithm (MOGA) has been studied to find the Pareto solutions directly [13–16].

So far, the objective of optimization is to find the optimal solution. In academia, to find the best solution is the motivation for their studies, and optimization is used to look for a better design than the present one in industry. Although optimal solutions are obtained, the reason why it is the optimal is not explained by any optimization techniques. Obayashi et al. [18] have proposed data mining from Pareto solutions using a self-organizing map (SOM) proposed by Kohonen [19]. This idea has shifted the paradigm of optimization. Namely, optimization is used as the tool for understanding the know-how of design. It will become a practical tool for solving inverse problems.

In this paper, we propose a procedure to solve multi-objective design problems as the comprehensive methodology to solve inverse problems. At first, MOGA is employed to find the Pareto solutions of tire performances, which are in multi-dimensional space of objective functions. RSM is also used to evaluate objective functions in the optimization process to reduce CPU time dramatically. In addition, SOM is used to map Pareto solutions from high-dimensional objective space onto two-dimensional space. Using SOM, design engineers see easily the Pareto solutions of tire performance and can find suitable design plans. The SOM can be considered as an inverse function that defines the relation between Pareto solutions and design variables. To demonstrate the pro-
procedure, multi-objective tire tread design is conducted. The objective of this example is to improve uneven wear and wear life for both the front tire and the rear tire of a passenger car. Wear performance is evaluated by FEA. Response surface is obtained by the design of experiments and FEA. Using both MOGA and SOM, we obtain a map of Pareto solutions. Here we call the map a “multi-performance map.” We can find suitable design plans that satisfy well-balanced performance on the multi-performance map. It helps tire design engineers make their decision in the design stage.

Multi-Objective Design Problem

Design of products is an inverse problem. The difficulty of an inverse problem is caused by the ill conditioning problem. One possible solution is to use an optimization technique that refines the design variable until a forward computation gives responses close to the desired responses. Many multi-objective optimization problems are considered as a single-objective optimization problem because of the lack of suitable solution techniques. However, an inverse function, which maps the desired response onto the design variables, cannot be generated by the converted single-objective optimization. The idea in this paper is to use SOM of Pareto solutions as an inverse function. We call the inverse function a multi-performance map. We propose a comprehensive methodology to solve inverse problems using MOGA and SOM. By means of MOGA and SOM, we can obtain a map of Pareto solutions called the multi-performance map. We can find easily some combinations of design variables that satisfy well-balanced performance by the multi-performance map. This helps tire design engineers make their decision in the design process.

Tires carry out many functions, i.e., handling and stability, noise and vibration, riding comfort, and so on. At the first period of the tire development process, the target performance of each function is determined. Then, the design process can be divided into two stages, i.e., conceptual design stage and detailed design stage. FEA had been used for troubleshooting and verification of detailed design in various manufactures for years. Recently, design optimization combined with FEA has been applied to the design process. Since a good conceptual design is important for the reduction of development time and costs, the optimum design should be applied to the conceptual design. As many of the tire functions have tradeoffs, understanding the relationship of tradeoffs helps to plan the best design that satisfies the target performance.

Due to the lack of suitable solution techniques, multi-objective design problems are converted into a single-objective problem. The converted single-objective function can be written as the following linear combination of original objective functions:
where $N_{\text{OBJ}}, w_i$, and $SF_i$ are number of objective functions, weight, and scale factor, respectively. Using ordinal optimization techniques, design variables can be found to minimize or maximize the converted new function $F_c$. However, this approach can find only one of the Pareto solutions corresponding to an arbitrary set of weights. Therefore, it is difficult to obtain whole information in multi-objective solution space and design variable space. To obtain the information of Pareto solutions is the key in the multi-objective design problem.

**Multi-Objective Genetic Algorithm (MOGA)**

GA simulates biological evolutionary theories on computer to solve optimization problems \cite{13}. According to the evolutionary theories, only the most fitting elements in a population are likely to survive and transmit their biological heredity to the next generation. This leads to the evolution of species through operators such as competition among individuals, natural selection, and mutation of the DNA. GA is likely to obtain a global optimum instead of a local one. One of the shortcomings of GA is the need for a large number of evaluations in order to find the optimum solution. Moreover, it is difficult in order to find the Pareto solutions by GA as well as by other optimization techniques.

MOGA has been developed to find the Pareto solutions directly. It compensates for those shortcomings of GA. Originally, the MOGA that adopted the idea of Pareto optimality shown in Fig. 2 was proposed by Fonseca \textit{et al.} \cite{14}. Since then, many improved techniques of MOGA have been proposed \cite{15–17}. In this paper, we employ the neighborhood cultivation genetic algorithm (NCGA) \cite{17}. With this technique, each objective function is treated separately. Standard genetic operation of mutation and crossover is performed on the designs. The crossover process is based on the neighborhood cultivation mechanism, where the crossover is performed mostly between individuals with values close to one of the objectives. In the following, NCGA in iSIGHT \cite{20} is used to find the Pareto solutions. Here, a simple numerical test is conducted to demonstrate how to capture Pareto solutions by MOGA. An optimization problem with two objective functions is considered as follows:

Maximize $f_1(r, \theta) = r \cdot \cos \theta$ and $f_2(r, \theta) = r \cdot \sin \theta$,

subject to $0.0 \leq r \leq 1.0, \quad 0.0 \leq \theta \leq \pi/2$.

The analytical solutions of this problem are points on a circle with the radius of 1.0. MOGA is applied in which the population size is 30, the number of generations is 20, and the crossover rate is 1.0. The conventional procedure of
multi-objective optimization can be converted into a single-objective problem as follows:

$$\text{Minimize } F(r, \theta) = w_1 \cdot r \cdot \cos \theta + w_2 \cdot r \cdot \sin \theta,$$

Subject to $0.0 \leq r \leq 1.0, \quad 0.0 \leq \theta \leq \pi/2,$

where $w_1$ and $w_2$ are weight; for example, $w_1 = w_2 = 0.5$. Figure 3 shows optimum solutions obtained by MOGA and the converted single-objective problem with a weight of $w_1 = w_2 = 0.5$. The solid line in the figure shows the analytical Pareto solutions, i.e., Pareto front. MOGA can find a total of 135 solutions which are uniformly dispersed. The solution of the converted single-objective problem is the intersection between Pareto front and line of $w_1 \cdot f_1 + w_2 \cdot f_2 = \alpha$, where $\alpha$ is arbitrary constant. Figure 4 shows the evolution of elements. As elements evolve from first generation, they approach Pareto front. The figure shows the global optimum searching process of MOGA. In the 20th generation, most elements reach Pareto front. However, two elements are far from Pareto front to search the global optimum.

Next, a three-objective optimization problem is considered to demonstrate the ability of MOGA to capture Pareto solutions,
FIG. 4 — History of generation in MOGA.
maximize $X(r,s,t) = r \cdot \sin(s) \cdot \cos(t)$,

$Y(r,s,t) = r \cdot \sin(s) \cdot \sin(t)$,

$Z(r,s,t) = r \cdot \cos(s)$,

subject to $0.0 \leq r \leq 1.0$, $0.0 \leq s \leq \pi/2$, $0.0 \leq t \leq \pi/2$.

Figure 5 shows analytical Pareto solutions and Pareto solutions obtained by MOGA. The figure shows that resulting Pareto solutions are uniformly dispersed and can be visualized in a three-dimensional objective function space.

Finally, to show the difficulty of seeing Pareto solutions in more than three-dimensional objective functional space, a simple four-objective optimization problem is conducted. Objective functions to maximize are

$X_1(r,s,t) = r \cdot \sin(s) \cdot \cos(t)$,

$X_2(r,s,t) = r \cdot \sin(s) \cdot \sin(t)$,

$X_3(r,s,t) = r \cdot \cos(s) \cdot \sin(t)$,

$X_4(r,s,t) = r \cdot \cos(s) \cdot \cos(t)$.

The range of design variables is the same as in the above three-objective optimization problem. Figure 6 shows analytical solutions of each objective func-
tion in design space in which $r$ is fixed to 1.0. Figure 7 shows four-dimensional Pareto solutions obtained by MOGA in which the population size is 60 and number of generations is 50. This figure shows that all of the objective functions have tradeoffs. However, it is difficult to see the relation among objective functions in Pareto solutions. It is also difficult to see the relation between Pareto solutions and design variables.

Self-Organizing Map (SOM)

In two-objective optimization problems, Pareto solutions form curves. In three-objective problems, those form surfaces in three-dimensional solution space. In more than three-dimensional objective function space, Pareto solutions cannot be visualized in the usual way.

To visualize higher dimensions, a self-organizing map proposed by Kohonen is employed in this paper. SOM is one of the neural network models. The SOM algorithm is based on unsupervised and competitive learning. It provides a mapping with preserving topology from the high-dimensional space to map
units. Map units, or neurons, usually form a two-dimensional lattice of hexagonal cells. SOM is a mapping from high dimensions onto two-dimensions. The mapping with preserving topology means that nearby points in the input space

FIG. 7 — Four-dimensional Pareto solutions obtained by MOGA.
are mapped to the nearby units in SOM. Roughly speaking, a relation between high-dimensional data and SOM is similar to the relation between the earth and world map. Nearby countries on the earth are mapped to nearby positions on the world map. SOM can be used as a cluster analysis tool for high-dimensional data. SOM is useful not only to visualize high-dimensional Pareto solutions but also for the cluster analysis. Here, Viscovery SOMine 4.0 Plus [21] is used to generate SOM. SOM-Word distance is used for clustering [21].

Figure 8 shows the SOM of a four-objective optimization problem shown

![Self-organizing map of four-dimensional Pareto solutions.](image)
in the previous section. The borderlines in each map are divided into four clusters. Four maps in Fig. 8 are the same maps, but each map is colored by the value of each objective function. The four-dimensional space of objective functions is mapped onto two-dimensional space. Namely, hexagonal units in SOM contain four-dimensional data of Pareto solutions. For objective function $x_1$, the right upper corner is the desired area. Desirable areas of objective functions, $x_2$, $x_3$, and $x_4$ are the left upper corner, left lower corner, and right lower corner, respectively. Those maps show that four of the objective functions have tradeoffs with each other. Figure 9 shows the map of design variables. Design variables are mapped onto SOM constructed by the objective functions shown in Fig. 8. In the above, it is mentioned that a hexagonal cell in SOM has four-dimensional data; however, it has seven-dimensional data including three design variables. Figure 9 shows a design variable $r$ is 1.0 at anywhere. Design variable $s$ is almost constant in the horizontal direction and varies linearly in the vertical direction. On the other hand, design variable $t$ is constant in the vertical direction and varies linearly in the horizontal direction. Therefore, it is recognized that the horizontal axis of maps shown in Fig. 8 represents design variable $s$, and the vertical axis of maps represents design variable $t$. In this example, axes have physical meanings because the example is simple and has two design variables. However, axes of SOM usually have no physical meaning. SOM is recognized as a database that visualizes the relation between design variables and Pareto solutions. In this paper, the database, i.e., the inverse function, is called the multi-performance map. The multi-performance map enables us to find the design plan that satisfies desired functions. Consequently, it is recognized as the comprehensive methodology to solve inverse problems.

FIG. 9 — Design variables mapped onto four-dimensional Pareto solution space.
Response Surface Methodology (RSM)

RSM is important for multi-objective design problems in which the nonlinear FEA is indispensable. Although nonlinear dynamic simulation using explicit FEA is powerful for the tire analysis, some simulations, such as wear simulation, take more than 1 CPU day on a supercomputer. It is not suitable for optimization, especially for multi-objective optimization. The RSM reduces the computational time required for objective function evaluation in the optimization process. The RSM maps the design space onto an approximated design space. Optimized solution is found in an approximated design space defined by the RSM. The most widely used response surface model is the polynomial-based model, due to its simplicity and ease of use. Recently, the Kriging model [22] has gained popularity. Kriging has its roots in the field of geostatistics and is useful for predicting temporally and spatially correlated data. Kriging is named after D.G. Krige, a South African mining engineer who developed empirical methods for determining true ore grade distributions from distributions based on sample ore grades. One more important issue of RSM is the design of experiments (DOE). DOE is the methodology to obtain a set of sampling points for RSM. Latin-hypercube, orthogonal array, and D-optimal are used widely. In the following application, a polynomial-based response surface model is constructed based on an orthogonal array for simplicity.

Multi-Objective Design Problem of Tire Wear

Here, the tire tread design to improve the wear performance is considered as an example of a multi-objective design problem.

Problem Setup

Many functionalities of a tire have tradeoffs as mentioned before. Uneven wear and wear life are important wear performance. Here, tire tread design is studied to improve uneven wear and wear life for both the front tire and the rear tire mounted on a passenger car. Generally, the form of uneven wear of a front tire and a rear tire are different. In the case of the front tire, the shoulder area is easy to wear. On the other hand, the center area of the rear tire is easy to wear. Namely, shoulder wear is observed at the front tire and center wear is observed at the rear tire. Uneven wear needs to be improved for both the shoulder wear at the front position and the center wear at rear position. In this example, tires mounted on a vehicle with rear-wheel drive are studied. The tire size is 215/60R16 as shown in Fig. 10. Three design variables are chosen from the geometric parameters shown in Fig. 10. To simplify the problem, tread pattern is not considered.

Two objective functions are selected to indicate the uneven wear and wear
life for each, the front tire and the rear tire. The objective function for indicator of uneven wear is the ratio of frictional energy of shoulder area to that of center area. The objective function to measure wear life is the mean frictional energy. Four objective functions are considered.

The procedure of the study is follows:

1. Wear simulation by FEA to generate response surface models.
2. Multi-objective optimization using MOGA to find Pareto solutions.
3. Visualization of Pareto solutions by SOM.

**Wear Simulation**

Tire wear is one of the most important properties of tires. The evaluation of tire wear takes a long time with a large cost, and it is difficult to maintain a constant test condition. Tire wear can be evaluated by the frictional energy, because tire wear is in proportion to the frictional energy and the wear index of tread compounds. A tread wear simulation based on FEM steady-state rolling simulation has been proposed to predict the tire cross-section wear profile [23]. Also, uneven wear of passenger car tire with real tread pattern can be predicted accurately using explicit finite element analysis [24]. In this paper, tire wear simulation is carried out using LS-DYNA [25]. To predict tread wear of a front tire, four driving conditions are considered: free rolling, braking, left cornering, and right cornering as shown in Fig. 11. On the other hand, driving, braking, left cornering, and right cornering conditions are considered to accumulate wear of
a rear tire shown in Fig. 12. Specified loading condition and vehicle acceleration history of the front tire are different from that of the rear tire. At the front tire cornering condition affects shoulder wear. On the other hand, driving condition influences center wear of the rear tire. Although, Figs. 11 and 12 are the

FIG. 11 — Frictional energy distribution (above) at left cornering mode and mean frictional energy in circumferential direction (below). FES and FEC stand for frictional energy at shoulder area and that at center area, respectively.
results of a tire FE model with real tread pattern, to simplify the problem a tire FE model with only straight grooves is used in the following study.

**Multi-Objective Optimization**

Figure 13 shows the frictional energy distribution and the mean frictional energy in the circumferential direction of a front tire. The chart of the mean frictional energy also shows a wear profile in the tire cross section. Two indicators are defined to characterize the wear performance. The frictional energy at shoulder area (FES) and the frictional energy at center area (FEC) are shown in Fig. 13. FES and FEC are the mean frictional energy at each area. Two objective functions are selected to indicate both uneven wear and wear life for each of the front tire and the rear tire. Four objective functions used here are

1. ratio of FES to FEC of a front tire, \( f_1 \),
2. ratio of FES to FEC of a rear tire, \( f_2 \),
3. mean frictional energy density of a front tire, \( f_3 \),
4. mean frictional energy density of a rear tire, \( f_4 \).
In the present optimization, all of the objective functions are to be minimized. The $f_1$ and $f_2$ indicate the uneven wear. The $f_3$ and $f_4$ are indicators of wear life. Design variables are three parameters that define tread geometry as shown in Fig. 10.

Design of experiments L9 is employed to obtain the response surface models of four objective functions. Since computation time of a wear response by FEA for both the front tire and the rear tire is more than 150 hours on an NEC SX-6 supercomputer, it is very efficient for a MOGA run. Response surface evaluates the fitness function of MOGA. NCGA is used to obtain the Pareto solutions. The population size is 40 and the number of generations is 100. Figure 14 shows the four-dimensional Pareto solutions obtained by NCGA. About 2500 of the Pareto solutions are found in this example. Figure 14 shows that all of the objective functions have tradeoffs. Although $f_3$ and $f_4$ seem not to have tradeoffs, they have tradeoffs in a small area. These figures in Fig. 14 are useful to understand the tradeoffs among objective functions. However, it is difficult to visualize Pareto solutions in a space more than three dimensions. To visualize a four-dimensional Pareto solution space, SOM is used here.

FIG. 13 — Frictional energy at each driving mode of a rear tire.
FIG. 14 — Computed four-dimensional Pareto solutions.
Visualization of Pareto Solutions

Figure 15 shows the clusters computed by SOM of Pareto solutions. In this example, we consider ten clusters. Figure 16 shows the maps of objective functions. The maps are colored by the values of each objective function. The values of objective functions are normalized as follows:

FIG. 15 — Cluster of Pareto solutions.

FIG. 16 — Self-organizing map of Pareto solutions.
All of the objective functions are to be minimized. The blue area on maps leads to even wear and long wear life. The figure shows that the desirable areas of objective functions $f_3$ and $f_4$ are the same areas on the map. However, desirable areas of $f_1$ and $f_2$ are slightly different.

Figure 17 shows the mean value of the objective function in each cluster. This chart characterizes each cluster. Cluster #2 excels in uneven wear performance. Cluster #9 excels in wear life. There are several clusters between #2 and #9. This means the compatibility of uneven wear performance with wear life is difficult to achieve because they have tradeoffs. Uneven wear and wear life have tradeoffs in the tread geometry design. However, different design variables such as material may have the possibility to cancel the tradeoffs. If one can use the cap compound, which leads to a long wear life, one can choose the design plan in cluster #2. However, if one cannot use the compound, one has to select another cluster. The clusters whose sum of mean values of objective functions is low, are #5, #7, #8, and #10. If one makes a point of uneven wear, the choice is #5. Cluster #10 corresponds to wear life. A well-balanced cluster is #8.

Figure 18 shows design variables mapped onto SOM generated by objective functions. Figure 19 shows mean values of design variables at each cluster. One can see a relation between Pareto solutions and design variables by Figs. 16 and 18. Cluster #2, which is good at uneven wear performance, consists of higher $x_2$ and lower $x_3$, and does not depend on $x_1$. Cluster #9, which leads to a long wear life, consists of higher $x_1$ and lower $x_2$ and does not depend on $x_2$. 

$$f_{\text{normalized}} = \frac{f - f_{\text{min}}}{f_{\text{max}} - f_{\text{min}}}.$$
Clusters #7 and #8 seem to be the same in Fig. 17. However, one can see the difference between #7 and #8 in Fig. 19. Responses of #7 and #8 are the same, but their design variables are quite different. In this example we consider only two functions of wear. In the real development process, many functions should be considered and some of them are not predicted by numerical simulation. Moreover, constraints of manufacturing exist. In such a situation, design plans like #7 and #8 are very useful. To find a lot of design plans in the conceptual design stage is to reduce development time and costs. Also, design points in Pareto solutions can be selected from SOM. After design engineers select the unit from the cells in SOM generated by Pareto solutions, design variables corresponding to the unit are obtained. RSM is the mapping between the response and the objective function, however, SOM of Pareto solutions is the mapping between Pareto solutions and design variables. Pareto solutions and design variables are contained in units of SOM. SOM of Pareto solutions is an inverse function to solve inverse problems.

Here, we select one unit in cluster #8 to compare the results of SOM with those of FEA. Figure 20 shows the frictional energy and the wear profile of a
front tire and a rear tire. The figure shows that the uneven wear is improved for both the front tire and the rear tire. Computed wear performance by FEA correlates with that obtained by SOM within 5% error.

Figure 21 shows two optimized solutions obtained by the single-objective optimization problems on the SOM of objective functions. OPT1 is the solution of the problem that minimizes the following single-objective function:

![FIG. 19 — Mean values of design variables of each cluster.](image)

![FIG. 20 — Frictional energy and wear profile of one solution in cluster #8.](image)
OPT1 = \frac{f_1}{SF_1} + \frac{f_2}{SF_2} + \frac{f_3}{SF_3} + \frac{f_4}{SF_4},

where \( SF_i \) are scale factors. On the other hand, OPT2 is the solution of the problem that minimizes the following objective function:

\[ F_{OPT2} = 4\frac{f_1}{SF_1} + \frac{f_2}{SF_2} + \frac{f_3}{SF_3} + \frac{f_4}{SF_4} \]

The difference between OPT1 and OPT2 is the weight factor of objective function \( f_1 \). The figure shows that OPT1 is in cluster #8 and OPT2 is in cluster #5. According to Table 1 objective function \( f_1 \) of OPT1 is smaller than that of OPT2. This supports the effect of the weight in single-objective optimization. However, dependence between the weights in single-objective function and the resulting objective functions are not definite. So, it is difficult to obtain uniformly dispersed Pareto solutions by the converted single-objective optimization.

TABLE 1 — Design variables and objective function of converted single-objective optimization.

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Conclusions

In this paper, we propose a procedure to solve multi-objective design problems as the comprehensive methodology to solve inverse problems using MOGA and SOM. The idea is to use SOM of Pareto solutions as an inverse function. At first, MOGA is employed to find the Pareto solutions, which are in a multi-dimensional space of objective functions. RSM is also used to evaluate objective functions in the optimization process and can reduce CPU time dramatically. In addition, SOM is used to map Pareto solutions from a high-dimensional objective space onto the two-dimensional space. To demonstrate the procedure, a multi-objective tire tread design to improve wear performance is conducted. Using SOM, design engineers see easily the Pareto solutions of tire performance and can find suitable design plans. Therefore, SOM can be considered as an inverse function that defines the relation between Pareto solutions and design variables. Here, we call the inverse function a multi-performance map. It helps tire design engineers make their decision in the conceptual design stage. In this paper, we only demonstrate an application to the tread shape design for wear improvement, however, this methodology is also useful for multi-objective design problems. Consequently, SOM of Pareto solutions called multi-performance map is the comprehensive tool to solve inverse problems.

References


