SPATIALLY-VARIANT LUCY-RICHARDSON DECONVOLUTION FOR MULTIVIEW FUSION OF MICROSCOPICAL 3D IMAGES

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SPIM =  
Single Plane Illumination Microscopy

[Huisken 2004]
Goal: Joint Fusion and Deconvolution

Missing parts → Fusion

Variant PSF → Deconvolution
Estimation of the PSF at Bead Positions

Variation of the PSF along y-axis
Related Work

➢ Blending [Preibisch 2010]
  ➢ Combines Gray values without a prior model
    ➢ Fast Computation
    ➢ Smearing of the points + blur

➢ Average PSF for Multiview Deconvolution [Krzic 2009]
  ➢ Assumes constant PSF
    ➢ Good in the center
    ➢ Bad at the corners of the image
Contributions

- Location **variant PSF estimation** for joint deconvolution and fusion

**Approach:**

- PSF Estimation
- Overlap-Save Deconvolution
- Lucy-Richardson Algorithm
- Multiview deconvolution
- TV Regularization

**LRMOS-TV**
Problem Formulation: Multiview Fusion

➢ Given:
  ➢ Recorded images $Y_1, \ldots, Y_N$
  ➢ PSF at bead positions $H_1, \ldots, H_N$

➢ Goal:
  ➢ Find true image $X$
    that maximizes

$$p(X|Y_1, \ldots, Y_N, H_1, \ldots, H_N) = \prod_{i=1}^{N} p(X|Y_i)$$
Solution: Regionwise Multiview Fusion

\[ H_{1,4,1}^1 * X \]
PSF Estimation

\[ H(x, y) = \frac{d_B d_C H_A + d_A d_C H_B + d_A d_B H_C}{d_B d_C + d_A d_C + d_A d_B} \]

\[ d_A = \sqrt{(x - x_A)^2 + (y - y_A)^2} \]
Overlap-Save Deconvolution

- Model spatially-variant PSF by blockwise constant PSFs
- Consider large overlapping regions to overcome boundary artifacts

\[
\begin{bmatrix}
Y_{11} & Y_{12} & \ldots & Y_{1n} \\
Y_{21} & Y_{22} & \ldots & Y_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{n1} & Y_{n2} & \ldots & Y_{nn}
\end{bmatrix}
\]

Size of the blocks: \( s \times s \)

\[
Y_{ij}^{(r+s)} = \begin{bmatrix}
\times & \times & \times \\
\times & Y_{ij} & \times \\
\times & \times & \times
\end{bmatrix}
\]

Size of the padded blocks: \((s+r) \times (s+r)\)
Image Formation Model

- Convolution with the PSF of the system:

\[ Y = P \left( H \ast X \right) \]

- Recorded Image
- PSF
- "True" Image
- Noise Model
Deconvolution: MLE Estimation

- Image Statistics Modeled by Poisson Process [Herbert 1989]:

\[
p(X|Y) = \prod_v \frac{[(H \ast X)(v)]^{Y(v)}}{Y(v)!} \exp\left(-[(H \ast X)(v)]\right)
\]

Likelihood Probability

\[
J(X) = \int_Y Y(v) \log \left( [(H \ast X)(v)] - (H \ast X)(v) \right) d\nu
\]

log likelihood
Lucy-Richardson Algorithm

\[ \hat{X}^{p+1}(v) = \hat{X}^p(v) \cdot C^p(v) \]

Correction Factor: \[ C^p(v) = (H^s \ast \frac{Y}{S^p})(v) \]

Simulated Image

\[ H^s(v) = H(-v) \]

\[ S^p = (H \ast \hat{X}^p)(v) \]
Multiview Deconvolution

➢ Total Correction Factor (CF) as average of the individual correction factors [Krzic 2009]:

\[
C^p = \frac{1}{N} \sum_{i=1}^{N} C^p_i
\]

➢ Computation of the individual CF:

\[
C^p_i(v) = (H_i^s * \frac{Y_i}{S^p_i})(v)
\]

\[
S^p_i = (H_i * \hat{X}^p)(v)
\]
TV Regularization

- Regularization of the initial energy by Total Variation [Dey 2004]:

\[ J_{TV}(X) = J(X) + \lambda \int \nabla X(v) \, dv \]

- Resulting iteration using Green's one-step-late (OSL) algorithm:

\[ \hat{X}^{p+1}(v) = \frac{\hat{X}^p(v)}{1 - \lambda \text{div} \left( \frac{\nabla (\hat{X})^p(v)}{\left| \nabla (\hat{X})^p(v) \right|} \right)} \cdot C^p(v) \]
Results: Visual Comparison to Blending

Parameters:
\[ r = 11, \quad p = 4, \]
\[ s + r = 64 \]
Computation Time: 40 min

[Preibisch 2010]
Computation Time: 20 min
Results: Visual Comparison to Blending

LRMOS-TV fusion

Blending fusion
Quantitative Evaluation

Original bead shape (single view)

Part I: PSF Estimation
Part II: Multiview Fusion
Results: Comparison to Average PSF

Original bead shape (single view)

Deconvolved with variant PSF (fused image, upper)

Deconvolved with average PSF (fused image, upper)
Conclusions

➢ A new framework for the fusion of the SPIM images was presented.

➢ Spatially-variant Deconvolution better models the optical properties of the system than existing methods.

➢ The structure borders are well preserved due to the TV regularization.

➢ The algorithm is fast and can be easily parallelized.
Thank you for your attention!
Proposed Algorithm ("LRMOS-TV")

\[
\text{for } m = 1 \text{ to } T_1 \text{ do} \\
\quad \text{for } n = 1 \text{ to } T_2 \text{ do} \\
\quad \quad \text{for } k = 1 \text{ to } T_3 \text{ do} \\
\quad \quad \quad 1. \text{ Extract extended region } R_i = Y_{mnk}^{(r+s)} \\
\quad \quad \quad \text{from } Y_i \text{ for each view } i. \\
\quad \quad \quad 2. \text{ Obtain } H_i = H_{m,n,k}^{(r+s)} \\
\quad \quad \quad \text{by padding with zeros for each view } i. \\
\quad \quad \quad 3. \text{ Compute the initial estimate:} \\
\quad \quad \quad \hat{X}^0 = \frac{1}{N} \sum_{i=1}^{N} R_i^p \\
\quad \quad \quad 4. \text{ Iterate:} \\
\quad \quad \quad \hat{X}_{m,n,k}^{p+1}(v) = \frac{\hat{X}_{m,n,k}^p(v)}{1 - \lambda \text{div} \left( \frac{\nabla \hat{X}_{m,n,k}^p(v)}{|\nabla \hat{X}_{m,n,k}^p(v)|} \right)} \cdot C^p(v) \\
\quad \quad \quad 5. \text{ Extract } \hat{X}_{mnk} \text{ from } \hat{X}_{mnk}^{(r+s)} \text{ and save into } \hat{X}. \\
\quad \quad \text{end for} \\
\quad \text{end for} \\
\text{end for}
\]
Outlook

➢ Algorithm:
   ➢ Additional regularization strategies
   ➢ Optimal number of iteration steps
   ➢ A parametric model of the PSF along the lightsheet

➢ Microscopy:
   ➢ Insert and record beads inside the sample for better PSF modeling inside the tissue
   ➢ Automatic centering of the sample
Coverage of the Beads in xz
Average PSF vs Variant PSF

<table>
<thead>
<tr>
<th></th>
<th>deconvolved with average PSF</th>
<th>deconvolved with variant PSF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_x$</td>
<td>1.5375 (max 1.6409)</td>
<td>1.4835 (max 1.6179)</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>1.3598 (max 1.7707)</td>
<td>1.2937 (max 1.7233)</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>2.0252 (max 2.4188)</td>
<td>2.0354 (max 2.038)</td>
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