Non-stationary Robust Transmission Expansion Planning

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***Abstract*—Recent breakthroughs in Transmission Network Expansion Planning (TNEP) have demonstrated that the use of robust optimization, as opposed to stochastic programming methods, renders the expansion planning problem computationally tractable for real systems. State-of-the-art robust methods treat this problem as stationary throughout the study period, with all the required investments on expansion of lines being made at the beginning of the project horizon. However, it is common knowledge that socioeconomic and environmental constraints might change considerably during the network life, such as planned increases in wind-power generation capacities, variations of environmental conditions due to climate change, or changing trends in consumption patterns, and these changes might also affect optimal capacity expansion planning. In this paper the classical stationarity assumption is dispensed with and the existing adaptive robust transmission expansion planning formulation for non-stationary approaches is expanded. The solution for this problem provides information not only about what additional lines must be installed but the construction timing during the study horizon as well. Moreover, the more evenly spread scheduling of construction, which, with the method put forward in this paper, is distributed throughout the life of the network, enables corrective measures to be taken in case the variables involved change in a manner different from that initially conceived. Numerical results from an illustrative example and the IEEE 118-bus system are presented and discussed, demonstrating the benefits of this non-stationary approach with respect to classical methods.**

Notation

The main notation used throughout this paper is stated below for quick reference. Other symbols are defined as needed throughout the paper.

Constants:

- b_k Susceptance of line k (S).
- c_i^{G} Generation cost for generator i (€/MWh).
- c_j^{U} Load-shedding cost for consumer j (€/MWh).
- c_k Investment cost of building line k (€).
- $\boldsymbol{D}^{(t)}$ Uncertainty set for time period t.
- $e_j^{(t)}$ Percentage of load shed by the j-th demand for year t.
- $f_k^{\max}$ Capacity of line k (MW).
- I Discount rate, i.e. the rate of return that could be earned on an investment in the financial markets with a similar risk.
- n_y Number of study periods (years).
- $o(k)$ Sending-end bus of line k.
- $r(k)$ Receiving-end bus of line k.
- Π Maximum budget for investment in transmission expansion (€).
- σ Weighting factor for obtaining annual operation and load-shedding costs (h).

Primal variables:

- $\boldsymbol{d}^{(t)}$ Vector of continuous variables representing the random or uncertain parameters for year t, i.e. generation capacities and loads (MW).
- $f_k^{(t)}$ Power flow through line k for year t (MW).
- $g_i^{(t)}$ Power produced by the i-th generating unit for year t (MW).
- $p_j^{(t)}$ Power consumed by the j-th demand for year t (MW).
- $r_j^{(t)}$ Load shed by the j-th demand for year t (MW).
- $x_k^{(t)}$ Actual status (*existing* vs *no existing*) of line k at the beginning of year t.
- $y_k^{(t)}$ Binary variable representing the construction of new line k at the beginning of year t.
- $\theta_n^{(t)}$ Voltage angle at bus n for year t (radians).

Dual variables: Note that dual variables are provided after the corresponding equalities or inequalities separated by a colon.

Indices and Sets:

- $\mathcal{D}$ Set of indices of demand.
- $\mathcal{G}$ Set of indices of generating units.
- $\mathcal{L}$ Set of all prospective and existing transmission lines at the beginning of time horizon considered.
- $\mathcal{L}^+$ Set of all prospective transmission lines.
- $\mathcal{N}$ Set of all networks buses.
- $n(i)$ Bus index where the i-th generating unit is located.
- $n(j)$ Bus index where the j-th demand is located.
- $\mathcal{T}$ Set of indices of different uncertainty sets.
- Ψ_n^{D} Set of indices of the demand located at bus n.
- Ψ_n^{G} Set of indices of the generating units located at bus n.

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I. Introduction

A. Motivation

Throughout the life-cycle of power transmission networks different phases must be considered: pre-design, design, construction, service life, expansion planning, substitution and dismantling. Traditionally, each of these phases has been faced by engineers with the assumption of stationarity, based on the concept that historical natural patterns are reliable predictors for the future. However, in the pertinent technical literature [1] it has been acknowledged that this hypothesis is no longer valid, and we need to find ways to characterize and model the non-stationarity of relevant variables. Changes in temperature, solar radiation, atmospheric circulation, precipitation, sea level, and the frequency and severity of extreme events, i.e. climate change, will, in all probability, affect how much energy is produced, delivered, and consumed worldwide [2]. However, climate change is not the only catalyst for influencing any capacity expansion planning analysis. Energy production and use is interconnected with many other aspects of modern life, such as water consumption, use of goods and services, transportation, economic growth, land use, and population growth. Thus, changes in any of these variables might influence generation and/or load demands, and thereby affect capacity expansion planning. Moreover, all these changes which affect capacity expansion planning occur gradually and are subject to a high degree of uncertainty, therefore we have to adapt to those changes progressively, in such a way that capacity expansion planning is less prone to inaccurate future prognosis of demand loads and power production.

According to [3], decision-making for transmission planning usually entails planning a horizon of approximately 10 years with reviews every 2 years, as described, for instance, in [4]. Moreover, planning should take in the entire life of the infrastructure to be built. Up to now, use of a year-by-year representation of investment decisions has been considered to be a highly complex and computationally intractable problem, and in order to ensure tractability while keeping the model accurate enough, one or few target years are usually selected when planning and investment costs are considered annually (see for instance [5], [6], [7], [8]). Nevertheless, in the pertinent technical literature it has been acknowledged that accurate capture of the effect uncertain parameters have on investment outcomes is vital for obtaining effective transmission investment plans.

B. Literature Review

Transmission network expansion planning analyzes the issue of how to expand or reinforce an existing power transmission network to adequately service system loads over a given time horizon. This problem is challenging due to the great amount of uncertainty associated with i) consumption, ii) renewable energy generation, such as with wind and solar power plants, and iii) equipment failure.

TNEP problems have been dealt with using stochastic programming [9], [10], [6], [11] and robust optimization [12], [13]. Consideration of the effects of climate change in the generation of expansion planning problems has been put forward by [14] by using a stochastic method with different future scenarios considered. However, stochastic programming formulations result in computationally intractable problems for real-size networks. In contrast, recent breakthroughs in robust TNEP problems [7], [15], [3], [16] proved that computational tractability for real-size systems is possible by using Adaptive Robust Optimization (ARO) frameworks [17], [18].

ARO materializes in a three-level formulation: i) the first level minimizes the cost of expansion ([15] and also minimizes the maximum regret), the decision variables are those related to construction or expansion of lines, ii) the second level selects the least desirable outcome for the uncertain parameters maximizing the system's operational costs within the uncertainty set, the variables associated with this level are the uncertain generation capacities and demand, and iii) the third level selects the optimal decision variables to minimize operational costs for given values of first and second level variables. The main difference in methods that use ARO make in TNEP is their way of solving problems and how the uncertainty set is defined.

Specifically spreaking, [7] and [16] merge the second and third levels into one single-level maximization problem using the third level dual. To deal with uncertain parameters and considering that they have to be equal to their upper or lower limits in the least desirable situation, binary variables are used. The limitation of this simplification is that the uncertainty budget must be an integer. However, this strategy does not belittle the benefits of robust optimization, but rather simplifies resolution of the problem substantially. Additionally, [16] cuts the binary variables by half because the least desirable operational costs occur for generation capacities and demand loads below and above their nominal values [3], respectively. Conversely, [15] and [3] merge the second and third levels into one single-level maximization problem by using the Karush-Kuhn-Tucker (KKT) third level conditions. Here, the number of constraints, continuous and binary variables of the subproblem increase with respect to the alternative approach. Note that in all approaches, authors deal with cardinality constrained uncertainty sets.

Once the third-level formulation is merged into a bi-level problem, [7] puts forward the Benders approach where the dual information from subproblems is used to construct additional Benders cuts. The main drawback with this method is the slow convergence typical with this type of decomposition algorithms [19], which made the author include additional linear constraints in order to improve convergence. Conversely, [15], [3], [16] apply a column-and-constraint generation method [20] solely based on primal cuts. This is computationally advantageous with respect to Benders decomposition and converges in a small number of iterations.

C. Contribution

The purpose of this paper is fourfold:

1) To demonstrate how the non-stationary model enables more optimal use of existing financial resources, rendering the solution more robust with respect to the initial selection of uncertainty sets and less prone to wrongful future prognosis of demand loads and generation capacities.
2) To introduce a way of adapting state-of-the-art cardinality constrained uncertainty sets to non-stationary stochastic processes.

3) To show that computational tractability for a year-by-year representation of investment decisions is possible by extending the stationary formulation given by [16].
4) To transform the stationary formulation given by [16] for the non-stationary approach. We have selected this method because it is the most computationally efficient method for robust TNEP to date, according to the extensive numerical simulations given in [16] where comparisons with the methods put forward by [7], [15], [3] were made.

D. Paper Structure

The remainder of the paper is structured as follows. Section II describes the non-stationary adaptive robust formulation of the TNEP problem. In Section III the uncertainty set is defined and how it changes over the time horizon is shown. In section IV further details are provided, there is a discussion on the extension of the approach for finding a solution put forward by [16] for multiple uncertainty sets. Numerical results for an illustrative example and a realistic case study are given in Section V. Finally, the paper is concluded in Section VI.

II. Non-stationary Robust Transmission Network Expansion Planning Formulation

A detailed formulation of the non-stationary adaptive robust TNEP problem can be written as the following three-level mathematical programming problem. Note that the dual variables are provided after the corresponding constraint separated by a colon.

$$\underset{x_k^{(t)},\, y_k^{(t)}}{\text{Minimize}} \quad \sum_{t\in\mathcal{T}} \frac{1}{(1+I)^{t-1}} \left(\sum_{k\in\mathcal{L}} c_k y_k^{(t)} + c_{\text{op}}^{(t)} \right) \tag{1}$$

subject to

$$\Pi \geq \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{L}} \frac{1}{(1+I)^{t-1}} c_k y_k^{(t)} \tag{2}$$

$$x_k^{(t)} = 1;\ \forall k \in \mathcal{L}\backslash\mathcal{L}^{+}, \forall t \in \mathcal{T} \tag{3}$$

$$x_k^{(t)} = \sum_{p=1}^{p=t} y_k^{(p)};\ \forall k \in \mathcal{L}^{+}, \forall t \in \mathcal{T} \tag{4}$$

$$\sum_{t\in\mathcal{T}} y_k^{(t)} \leq 1;\ \forall k \in \mathcal{L}^{+} \tag{5}$$

$$y_k^{(t)} \in \{0,1\};\ \forall k \in \mathcal{L}, \forall t \in \mathcal{T}, \tag{6}$$

where operational costs $c_{\text{op}}^{(t)}$ in (1) for each period $t;\ \forall t \in \mathcal{T}$ and for given values of the first-stage decision variables $x_k^{(t)}, y_k^{(t)}$ are obtained by solving the following inner optimization problem:

$$c_{\text{op}}^{(t)} = \underset{\boldsymbol{d}^{(t)} \in \boldsymbol{D}^{(t)}}{\text{Maximum}} \quad \underset{g_i^{(t)}, p_j^{(t)}, r_j^{(t)}, \theta_n^{(t)}, f_k^{(t)}}{\text{Minimum}} \left(\sigma \sum_{i\in\mathcal{G}} c_i^{\text{G}} g_i^{(t)} + \right.$$
$$\left. + \sigma \sum_{j\in\mathcal{D}} c_j^{\text{U}} r_j^{(t)} \right), \tag{7}$$

subject to

$$\sum_{i\in\Psi_n^{\text{G}}} g_i^{(t)} - \sum_{k|o(k)=n} f_k^{(t)} + \sum_{k|r(k)=n} f_k^{(t)} + \sum_{j\in\Psi_n^{\text{D}}} r_j^{(t)}$$
$$= \sum_{j\in\Psi_n^{\text{D}}} p_j^{(t)} : \lambda_n^{(t)};\ \forall n \in \mathcal{N} \tag{8}$$

$$f_k^{(t)} = b_k x_k^{(t)} (\theta_{o(k)}^{(t)} - \theta_{r(k)}^{(t)}) : \phi_k^{(t)};\ \forall k \in \mathcal{L} \tag{9}$$

$$\theta_n^{(t)} = 0 : \chi_n^{(t)};\ n : \text{slack} \tag{10}$$

$$f_k^{(t)} \leq f_k^{\max} : \hat{\phi}_k^{(t)};\ \forall k \in \mathcal{L} \tag{11}$$

$$f_k^{(t)} \geq -f_k^{\max} : \check{\phi}_k^{(t)};\ \forall k \in \mathcal{L} \tag{12}$$

$$\theta_n^{(t)} \leq \pi : \hat{\xi}_n^{(t)};\ \forall n \in \mathcal{N}\backslash n : \text{slack} \tag{13}$$

$$\theta_n^{(t)} \geq -\pi : \check{\xi}_n^{(t)};\ \forall n \in \mathcal{N}\backslash n : \text{slack} \tag{14}$$

$$g_i^{(t)} \geq 0;\ \forall i \in \mathcal{G} \tag{15}$$

$$r_j^{(t)} \geq 0;\ \forall j \in \mathcal{D} \tag{16}$$

$$p_j^{(t)} = d_j^{\text{D}(t)} : \alpha_j^{\text{D}(t)};\ \forall j \in \mathcal{D} \tag{17}$$

$$g_i^{(t)} \leq d_i^{\text{G}(t)} : \varphi_i^{\text{G}(t)};\ \forall i \in \mathcal{G} \tag{18}$$

$$r_j^{(t)} \leq e_j^{(t)} d_j^{\text{D}(t)} : \varphi_j^{\text{D}(t)};\ \forall j \in \mathcal{D}. \tag{19}$$

The objective function (1) represents the net present cost (NPC) associated with expansion investment and operational costs, defined as the sum of the present values of costs over the time horizon. Constraint (2) keeps the maximum amount of expansion investment throughout the time horizon to within the available budget. Constraints (3) and (4) make the line status equal to 1 for all existing transmission lines at the beginning of the time horizon considered, and once the line has been constructed, respectively, while constraint (5) ensures that no line is constructed more than once throughout the time horizon considered. Constraint (6) establishes the binary nature of investment decisions. Note that variables $\boldsymbol{x}^{(t)};\ t \in \mathcal{T}$ are also binary, but, these integrality constraints can be relaxed and variables such as these can be defined as continuous because their binary nature is ensured by means of the set of equations (3)-(6). Equation (7) represents the least desirable scenario for operational costs made up of maximum generation and load-shedding costs. Constraint (8) sets the power balance at every bus. Constraint (9) shows the power flow through each line. Note that the power flow depends on the actual status of the line $x_k^{(t)}$, thus, if the corresponding line is not physically connected to the network, the power flow through it is zero. Constraint (10) fixes the voltage angle of the reference bus to zero. Constraints (11)-(12) set the upper and lower line flow limits. Constraints (13)-(14) set limits on the voltage angles at every bus, and (15)-(16) ensure the power generation and load-shedding variables are both positive. Finally, constraint (17) makes the level of demand match the uncertain demand variable, (18) sets the power generation to be lower than the uncertain generation capacity variable, and (19) limits load-shedding to a percentage of the uncertain demand variable. The uncertain demand and generation capacity variables are defined in Section III. For purposes of clarity, the number of different uncertainty sets is initially considered equal to the number of years of the time horizon, although different time periods could be considered instead.

It must be stressed that the main difference with respect to previous ARO formulations for TNEP problems is the consideration of different costs for each year throughout the time horizon, which made us include the additional constraints (3)-(5). This is because there are different uncertainty sets for each year $\boldsymbol{D}^{(t)}$, which represent possible changes in loads, generation capacities, etc. The advantage this formulation has over traditional stationary approaches is that investment decisions can be made at any time throughout the study horizon. On the downside, computational complexity increases.

III. Uncertainty Modelling

Uncertainties that are pertinent to the transmission expansion planning problem in a market setting include:

1) demand growth,
2) spatial distribution of demand growth,
3) generation capacities
4) availability of transmission facilities and
5) availability of generation facilities.

In this paper we only consider uncertainties associated with demand and generation capacities. To be specific, we use the same definition of an uncertainty set as that given by [16] as a starting point, i.e.:

$$\begin{aligned} d_i^{\mathrm{G}} &= \bar{d}_i^{\mathrm{G}} - \hat{d}_i^{\mathrm{G}} z_i^{\mathrm{G}};\ \forall i \in \mathcal{G} && (20)\\ d_j^{\mathrm{D}} &= \bar{d}_j^{\mathrm{D}} + \hat{d}_j^{\mathrm{D}} z_j^{\mathrm{D}};\ \forall j \in \mathcal{D} && (21)\\ \sum_{i\in\mathcal{G}} z_i^{\mathrm{G}} &\leq \Gamma^{\mathrm{G}} && (22)\\ \sum_{j\in\mathcal{D}} z_j^{\mathrm{D}} &\leq \Gamma^{\mathrm{D}} && (23)\\ z_i^{\mathrm{G}} &\in \{0,1\}; \forall i \in \mathcal{G} && (24)\\ z_j^{\mathrm{D}} &\in \{0,1\}; \forall j \in \mathcal{D}, && (25) \end{aligned}$$

where d_i^{G} is the uncertain generation limit for the generating unit i, and is related to the ith variable within vector $\boldsymbol{d}$, $\bar{d}_i^{\mathrm{G}}$ is the corresponding nominal value, $\hat{d}_i^{\mathrm{G}}$ is the maximum positive distance from the nominal value that can take the random parameter, and z_i^{G} is an auxiliary variable. Likewise, d_j^{D}, $\bar{d}_j^{\mathrm{D}}$, $\hat{d}_j^{\mathrm{D}}$ and z_j^{D} correspond to the same values but for demand. Note that according to [3], in the least desirable outcome arising from "nature" with a fixed network configuration, would be one in which there is maximum load shedding and, consequentially, maximum operational costs. Therefore, generation capacity would be as low as possible with respect to nominal values and the demand load as high as possible with respect to nominal values. This explains the signs in (20)-(21).

Since the aim of this paper is to use non-stationary stochastic processes for TNEP, the uncertainty set (20)-(25) must be associated with each year in the study horizon and allow for changes in successive years. Thus, (20)-(25) transforms into the set $\boldsymbol{D}^{(t)}$ for each year t as follows:

$$\begin{aligned} d_i^{\mathrm{G}(t)} &= \bar{d}_i^{\mathrm{G}} r_{\mu,i}^{(t)} - \hat{d}_i^{\mathrm{G}} r_{\sigma,i}^{(t)} z_i^{\mathrm{G}(t)};\ \forall i \in \mathcal{G}; \forall t \in \mathcal{T} && (26)\\ d_j^{\mathrm{D}(t)} &= \bar{d}_j^{\mathrm{D}} r_{\mu,j}^{(t)} + \hat{d}_j^{\mathrm{D}} r_{\sigma,j}^{(t)} z_j^{\mathrm{D}(t)};\ \forall j \in \mathcal{D}; \forall t \in \mathcal{T} && (27)\\ \sum_{i\in\mathcal{G}} z_i^{\mathrm{G}(t)} &\leq \Gamma^{\mathrm{G}};\ \forall t \in \mathcal{T} && (28)\\ \sum_{j\in\mathcal{D}} z_j^{\mathrm{D}(t)} &\leq \Gamma^{\mathrm{D}};\ \forall t \in \mathcal{T} && (29)\\ z_i^{\mathrm{G}(t)} &\in \{0,1\}; \forall i \in \mathcal{G}; \forall t \in \mathcal{T} && (30)\\ z_j^{\mathrm{D}(t)} &\in \{0,1\}; \forall j \in \mathcal{D}; \forall t \in \mathcal{T}. && (31) \end{aligned}$$

In these uncertainty sets, the nominal values $\bar{d}_i^{\mathrm{G}}, \bar{d}_j^{\mathrm{D}}$ and maximum positive distances from the nominal values $\hat{d}_i^{\mathrm{G}}, \hat{d}_j^{\mathrm{D}}$ given in (20)-(25) are taken as being the initial reference values, while the final values are obtained by multiplying them by the positive constants $r_{\mu,i}^{(t)}$ or $r_{\mu,j}^{(t)}$ and $r_{\sigma,i}^{(t)}$ or $r_{\sigma,j}^{(t)}$. Note that these values affect the nominal or expected value and the dispersion, respectively. Thus, this methodology is highly flexible as it takes different circumstances into account, such as increases and/or decreases in nominal values, increases and/or decreases in dispersion or both.

Figure 1 exemplifies how the uncertainty sets can be built based on historical time series. When applied here, it corresponds to the change in aggregate demand in Spain, although we could make a similar analysis for demand and production associated with every node in our study network if data were available. Note that we can fit a heteroscedastic regression model onto it [21] in such a way that there are two different linear trends fitted: one for the expected value and another one for the uncertainty or standard deviation. Once the regression parameters are obtained, it is possible to plot the change in expected value for the future (the black line in Figure 1), and confidence bands associated with a pre-established confidence level α (0.9, 0.95, 0.99). These confidence bands set the lower and upper limits of the uncertainty sets. Reference values $\bar{d}_j^{\mathrm{D}}$ and $\hat{d}_j^{\mathrm{D}}$ correspond, to the expected value and half confidence band to the present, respectively as shown in Figure 1, while for the remaining years within the study horizon these values increase or decrease according to the distributions fitted.

A simple and user-friendly method for defining the uncertainty sets is to select the annual rate of change associated with the expected value β_μ and standard deviation β_σ, which corresponds to the slopes of the heteroscedastic model put forward in this paper. Given these rates of change, the factors from equations (26) can be obtained as follows:

$$\begin{aligned} r_{\mu,i}^{(t)} &= (1+\beta_\mu)^{(t-1)};\ \forall i \in \mathcal{G}; \forall t \in \mathcal{T} && (32)\\ r_{\sigma,i}^{(t)} &= (1+\beta_\sigma)^{(t-1)};\ \forall i \in \mathcal{G}; \forall t \in \mathcal{T}. && (33) \end{aligned}$$

Likewise, factors $r_{\mu,j}^{(t)}$ and $r_{\sigma,j}^{(t)}$ related to the demand from the equation in (27) can be written.

Use of these annual rates of change is highly intuitive, and appropriate values for those parameters can be obtained using regression models, as put forward in this paper, or any other method, such as expert criterion, or alternatively, the forecasting tools put forward by [3], such as ARIMA, GARCH, dynamic factors or transfer function models. These tools enable the behaviour of different demand levels and production capacities to be forecast, after which this data can be used to derive appropriate upper and lower bounds for the uncertainty sets. The difference with respect to the stationary approach is that those limits are defined for each year considered within the study horizon. Note that according to (32)-(33), the nominal values and maximum positive distances from the nominal values are equal to the reference values, and increase or decrease according to the selected rates of change for subsequent years.

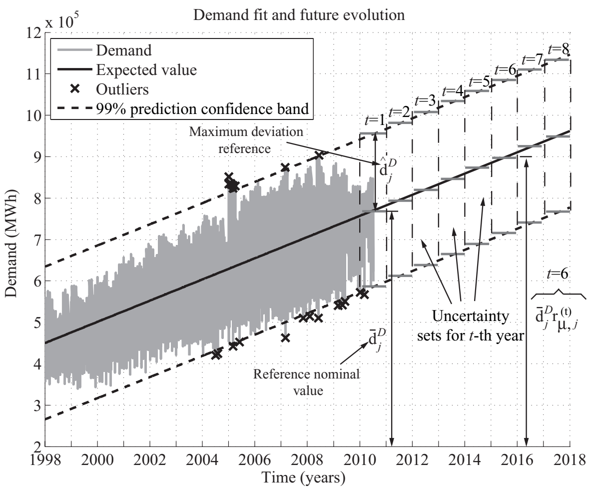

Fig. 1. Graphical illustration of how uncertainty sets can be defined.

Another example corresponds to the same data sets considered by [3] and is taken from the Spanish electricity market (OMIE, http://www.omie.es/), where wind power production and demand is considered. In both approaches a least squares (LS) linear regression has been plotted (continuous and dashed black lines respectively) to check possible long term trends associated with nominal or expected values. The change in the installed capacity of wind power results in a positive trend, which has been prolonged until 2020 as shown in Figure 2 (a). Moreover, the linear lower production envelope has been traced and prolonged up to 2020. Note that only values below nominal values are considered. Our proposal consists of defining constant uncertainty sets for each year, which are represented by light gray boxes $\boldsymbol{D}^{(1)}, \ldots, \boldsymbol{D}^{(5)}$, that change according to foreseen nominal and dispersion values of wind power production. Regarding demand, the least squares trend has decreased as a result of the European economic downturn. However, we have used an annual increase rate of 6%, similar to the rate before the downturn. For demand, only values above nominal values are considered. Note that the aim of this figure is to illustrate the concept of uncertainty set evolution and to provide an example of possible selection of uncertainty sets, not to state what the appropriate statistical technique is to make future prognosis (which is beyond the scope of this paper).

Finally, it must be stressed that as with the treatment of uncertain variables within each uncertainty set for stationary approaches, where the temporal autocorrelation among variables is disregarded because we are only interested in

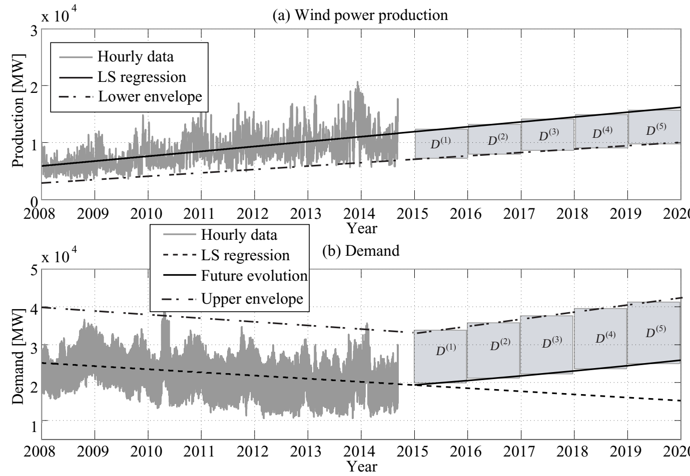

Fig. 2. Graphical illustration of the evolution of uncertainty sets associated with Spanish electricity market (OMIE) historical data (2008-2013).

the situation within the uncertainty set in which the highest operational costs occur, no correlation among uncertainty sets associated with each time period (year) is required. In the non-stationary approach, our aim is to find the highest operation cost for each uncertainty set independently.

IV. Proposed Decomposition Method

In this section, the extension of the approach to obtaining a solution put forward by [16] for multiple uncertainty sets is described in detail.

The decomposition method put forward has a bi-level structure, so, the first step is to merge the initial three-level formulation (1)-(19) into a two-level problem. For given values for the first-stage variables $x_k^{(t)}$ and $y_k^{(t)}$ for each line at any year, the problem set out by (7), (8)-(19) might be decomposed by the time period. Thus, the operational cost definition transforms into the following single-level maximization problem for each time period t; $\forall t \in \mathcal{T}$:

$$\underset{\boldsymbol{d}, \lambda_n^{(t)}, \phi_k^{(t)}, \chi_n^{(t)}, \hat{\phi}_k^{(t)}, \check{\phi}_k^{(t)}, \hat{\xi}_n^{(t)}, \check{\xi}_n^{(t)}, \alpha_j^{\mathrm{D}(t)}, \varphi_i^{\mathrm{G}(t)}, \varphi_j^{\mathrm{D}(t)}}{\text{Maximize}} \left\{ \begin{array}{l} \sum_{k \in \mathcal{L}} \left(\hat{\phi}_k^{(t)} - \check{\phi}_k^{(t)} \right) f_k^{\max} + \sum_{n \in \mathcal{N} \setminus n:\text{slack}} \pi \left(\hat{\xi}_n^{(t)} - \check{\xi}_n^{(t)} \right) \\ + \sum_{i \in \mathcal{G}} \left(d_i^{\mathrm{G}(t)} \varphi_i^{\mathrm{G}(t)} \right) + \sum_{j \in \mathcal{D}} \left(d_j^{\mathrm{D}(t)} \alpha_j^{\mathrm{D}(t)} + e_j^{(t)} d_j^{\mathrm{D}(t)} \varphi_j^{\mathrm{D}(t)} \right) \end{array} \right\} \tag{34}$$

subject to:

$$\lambda_{n(i)}^{(t)} + \varphi_i^{\mathrm{G}(t)} \le \sigma c_i^{\mathrm{G}}; \ \forall i \in \mathcal{G} \tag{35}$$

$$-\lambda_{n(j)}^{(t)} + \alpha_j^{\mathrm{D}(t)} \le 0; \ \forall j \in \mathcal{D} \tag{36}$$

$$\lambda_{n(j)}^{(t)} + \varphi_j^{\mathrm{D}(t)} \le \sigma c_j^{\mathrm{U}}; \ \forall j \in \mathcal{D} \tag{37}$$

$$-\lambda_{o(k)}^{(t)} + \lambda_{r(k)}^{(t)} + \phi_k^{(t)} + \hat{\phi}_k^{(t)} + \check{\phi}_k^{(t)} = 0; \ \forall k \in \mathcal{L} \tag{38}$$

$$- \sum_{k|o(k)=n} b_k x_k^{(t)} \phi_k^{(t)} + \sum_{k|r(k)=n} b_k x_k^{(t)} \phi_k^{(t)}$$
$$+ \hat{\xi}_n^{(t)} + \check{\xi}_n^{(t)} = 0;\ \forall n \in \mathcal{N} \backslash n : \text{slack} \quad (39)$$
$$- \sum_{k|o(k)=n} b_k x_k^{(t)} \phi_k^{(t)} + \sum_{k|r(k)=n} b_k x_k^{(t)} \phi_k^{(t)}$$
$$+ \chi_n^{(t)} = 0;\ n : \text{slack} \quad (40)$$
$$-\infty \leq \lambda_n^{(t)} \leq \infty;\ \forall n \in \mathcal{N} \quad (41)$$
$$-\infty \leq \phi_k^{(t)} \leq \infty;\ \forall k \in \mathcal{L} \quad (42)$$
$$-\infty \leq \chi_n^{(t)} \leq \infty;\ n : \text{slack} \quad (43)$$
$$\hat{\phi}_k^{(t)} \leq 0;\ \forall k \in \mathcal{L} \quad (44)$$
$$\check{\phi}_k^{(t)} \geq 0;\ \forall k \in \mathcal{L} \quad (45)$$
$$\hat{\xi}_n^{(t)} \leq 0;\ \forall n \in \mathcal{N} \backslash n : \text{slack} \quad (46)$$
$$\check{\xi}_n^{(t)} \geq 0;\ \forall n \in \mathcal{N} \backslash n : \text{slack} \quad (47)$$
$$-\infty \leq \alpha_j^{\mathrm{D}(t)} \leq \infty;\ \forall j \in \mathcal{D} \quad (48)$$
$$\varphi_i^{\mathrm{G}(t)} \leq 0;\ \forall i \in \mathcal{G} \quad (49)$$
$$\varphi_j^{\mathrm{D}(t)} \leq 0;\ \forall i \in \mathcal{D} \quad (50)$$
$$d_i^{\mathrm{G}(t)} = \bar{d}_i^{\mathrm{G}} r_{\mu,i}^{(t)} - \hat{d}_i^{\mathrm{G}} r_{\sigma,i}^{(t)} z_i^{\mathrm{G}(t)};\ \forall i \in \mathcal{G} \quad (51)$$
$$d_j^{\mathrm{D}(t)} = \bar{d}_j^{\mathrm{D}} r_{\mu,j}^{(t)} + \hat{d}_j^{\mathrm{D}} r_{\sigma,j}^{(t)} z_j^{\mathrm{D}(t)};\ \forall j \in \mathcal{D} \quad (52)$$
$$\sum_{i \in \mathcal{G}} z_i^{\mathrm{G}(t)} \leq \Gamma^{\mathrm{G}} \quad (53)$$
$$\sum_{j \in \mathcal{D}} z_j^{\mathrm{D}(t)} \leq \Gamma^{\mathrm{D}} \quad (54)$$
$$z_i^{\mathrm{G}(t)} \in \{0,1\}; \forall i \in \mathcal{G} \quad (55)$$
$$z_j^{\mathrm{D}(t)} \in \{0,1\}; \forall j \in \mathcal{D}. \quad (56)$$

Subproblems (34)-(56) result from substituting the third-level problem by its dual and incorporation of definition equations for the uncertainty set (51)-(56), and there are the same number of subproblems as there are years under consideration n_y within the time horizon. These subproblems provide the uncertain parameter values $\boldsymbol{d}^{(t)}$ within the uncertainty sets to give the least desirable operational costs for each year.

The only additional detail required in order to define the subproblems properly is linealization of the bilinear terms included in (34), i.e. $\sum_{i \in \mathcal{G}} \left(d_i^{\mathrm{G}(t)} \varphi_i^{\mathrm{G}(t)} \right) + \sum_{j \in \mathcal{D}} \left(d_j^{\mathrm{D}(t)} \alpha_j^{\mathrm{D}(t)} + e_j^{(t)} d_j^{\mathrm{D}(t)} \varphi_j^{\mathrm{D}(t)} \right)$. Taking into account equations (51)-(56), this bilinear term becomes:

$$\sum_{i \in \mathcal{G}} \left(d_i^{\mathrm{G}(t)} \varphi_i^{\mathrm{G}(t)} \right) + \sum_{j \in \mathcal{D}} \left(d_j^{\mathrm{D}(t)} \alpha_j^{\mathrm{D}(t)} + e_j^{(t)} d_j^{\mathrm{D}(t)} \varphi_j^{\mathrm{D}(t)} \right)$$
$$= \sum_{i \in \mathcal{G}} \left(\bar{d}_i^{\mathrm{G}} r_{\mu,i}^{(t)} \varphi_i^{\mathrm{G}(t)} - \hat{d}_i^{\mathrm{G}} r_{\sigma,i}^{(t)} z_i^{\mathrm{G}(t)} \varphi_i^{\mathrm{G}(t)} \right)$$
$$+ \sum_{j \in \mathcal{D}} \left(\bar{d}_j^{\mathrm{D}(t)} r_{\mu,j}^{(t)} \alpha_j^{\mathrm{D}(t)} + \hat{d}_j^{\mathrm{D}(t)} r_{\sigma,j}^{(t)} z_j^{\mathrm{D}(t)} \alpha_j^{\mathrm{D}(t)} \right)$$
$$+ \sum_{j \in \mathcal{D}} e_j^{(t)} \left(\bar{d}_j^{\mathrm{D}(t)} r_{\mu,j}^{(t)} \alpha_j^{\mathrm{D}(t)} + \hat{d}_j^{\mathrm{D}(t)} r_{\sigma,j}^{(t)} z_j^{\mathrm{D}(t)} \alpha_j^{\mathrm{D}(t)} \right). \quad (57)$$

The terms to be linearized correspond to products of binary and dual variables, $z_i^{\mathrm{G}(t)} \varphi_i^{\mathrm{G}(t)}$ and $z_j^{\mathrm{D}(t)} \alpha_j^{\mathrm{D}(t)}$. Details about the technique used for linearization are divulged in [16]. The resulting formulation associated with subproblems is a mixed-integer linear programming problem, which can be solved efficiently by using state-of-the-art mixed-integer mathematical programming solvers such as CPLEX or Gurobi.

The optimal solutions for subproblems (34)-(56) provide the uncertain parameter values $\boldsymbol{d}^{(t)}$ for each year in order to construct primal cuts for the master problem, which corresponds to the following optimization problem at iteration ν:

$$\underset{\substack{x_k^{(t)}, y_k^{(t)}, g_{i,l}^{(t)}, p_{j,l}^{(t)}, \\ r_{j,l}^{(t)}, \theta_{n,l}^{(t)}, f_{k,l}^{(t)} \\ \forall t \in \mathcal{T} \\ l = 1, \ldots, \nu - 1}}{\text{Minimize}} \quad \sum_{t \in \mathcal{T}} \frac{1}{(1+I)^{t-1}} \left(\sum_{k \in \mathcal{L}} c_k y_k^{(t)} + \gamma^{(t)} \right) \quad (58)$$

subject to

$$\gamma^{(t)} \geq \sigma \sum_{i \in \mathcal{G}} c_i^{\mathrm{G}} g_{i,l}^{(t)} + \sigma \sum_{j \in \mathcal{D}} c_j^{\mathrm{U}} r_{j,l}^{(t)};$$
$$\forall t \in \mathcal{T}, l = 1, \ldots, \nu - 1 \quad (59)$$
$$\gamma^{(t)} \geq 0;\ \forall t \in \mathcal{T} \quad (60)$$
$$\Pi \geq \sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{L}} \frac{1}{(1+I)^{t-1}} c_k y_k^{(t)} \quad (61)$$
$$x_k^{(t)} = 1;\ \forall k \in \mathcal{L} \backslash \mathcal{L}^+, \forall t \in \mathcal{T} \quad (62)$$
$$x_k^{(t)} = \sum_{p=1}^{p=t} y_k^{(p)};\ \forall k \in \mathcal{L}^+, \forall t \in \mathcal{T} \quad (63)$$
$$\sum_{t \in \mathcal{T}} y_k^{(t)} \leq 1;\ \forall k \in \mathcal{L}^+ \quad (64)$$
$$y_k^{(t)} \in \{0,1\};\ \forall k \in \mathcal{L}, \forall t \in \mathcal{T} \quad (65)$$
$$\text{Equations } (8)-(19);\ l = 1, \ldots, \nu - 1. \quad (66)$$

Note that the master problem, besides variables $\gamma^{(t)}$ relates to year on year operational costs, includes one variable $g_{i,l}^{(t)}$, $p_{j,l}^{(t)}$, $r_{j,l}^{(t)}$, $\theta_{n,l}^{(t)}$ and $f_{k,l}^{(t)}$ for each year and for each realization of the uncertain parameters obtained from the subproblem (34)-(56) at every iteration. As pointed out by [3] in the stationary approach, the master problem does not pose any computational challenge since it only incorporates a small number of primal cuts (small number of iterations ν). However, in the non-stationary method, the number of primal cuts and first-level binary variables is multiplied by the number of time periods (years) under consideration, thereby increasing exponentially the computational time required to solve the master problem. Alternatively, instead of working with time periods in years, longer periods could be used. The optimal time period must be long enough to reduce the number of subproblems and, therefore, the computational complexity and short enough for the uncertainty set to accurately display the non-stationary nature of the uncertain variables. In other words, there must be a suitable tradeoff between complexity and display of the non-stationary characteristics. This fundamentally depends on how fast those parameters might change throughout the time horizon. Nevertheless, appropriate selection of this period is beyond the scope of this paper.

Once the subproblems and master problem formulations are given, the solution method consists of iteratively solving the following problems:

Subproblems, one for each year: For given values for the first-stage variables $x_k^{(t)}$ and $y_k^{(t)}$, the subproblems in (34)-(56) obtain the values for the uncertain parameters within the uncertainty set to obtain the least desirable operational costs (7).

Master problem: Given the least desirable realizations of the uncertain parameters in terms of operational costs, new values for the first-stage variables $x_k^{(t)}$ and $y_k^{(t)}$ are calculated by means of (58)-(66).

The proposed iterative scheme put forward is described step by step in the following algorithm:

Algorithm 1: Non-stationary robust transmission network expansion planning

Input: Selection of uncertainty budgets Γ^{G} and Γ^{D}, time periods to divide the time horizon, interest rate I, definition of the uncertainty sets for each time period, and tolerance of the process ε. These data are selected by the decision maker.

Step 1: Initialization. Initialize the iteration counter to $\nu = 1$, and upper and lower bounds of the objective function $z^{(\mathrm{up})} = \infty$ and $z^{(\mathrm{lo})} = -\infty$.

Step 2: Solving the master problem at iteration ν. Solve the master problem (58)-(66). The result provides values of the decision variables $x_{k,\nu}^{(t)}$, $y_{k,\nu}^{(t)}$ and $\gamma^{(t)}$. Update the optimal objective function lower bound $z^{(\mathrm{lo})} = \sum_{t\in\mathcal{T}} \frac{1}{(1+I)^{t-1}} \left(\sum_{k\in\mathcal{L}} c_k y_{k,\nu}^{(t)} + \gamma^{(t)}\right)$. Note that at the first iteration the optimal solution matches the no investment case. Alternatively, we could start with any other vector for decision variables.

Step 3: Solving subproblems at iteration ν for each year t. For given values of the decision variables $x_{k,\nu}^{(t)}$, $y_{k,\nu}^{(t)}$, we calculate the least desirable operational costs within the uncertainty set $c_{\mathrm{op},\nu}^{(t)}$, whereby we also obtain the corresponding uncertain parameters $\boldsymbol{d}_\nu^{(t)}$. This is achieved by solving subproblems (34)-(56). Update the optimal objective function upper bound $z^{(\mathrm{up})} = \sum_{t\in\mathcal{T}} \frac{1}{(1+I)^{t-1}} \left(\sum_{k\in\mathcal{L}} c_k y_{k,\nu}^{(t)} + c_{\mathrm{op},\nu}^{(t)}\right)$.

Step 4: Convergence checking. If $(z^{(\mathrm{up})} - z^{(\mathrm{lo})})/z^{(\mathrm{up})} \le \varepsilon$ go to *Step 5*, else update the iteration counter $\nu \to \nu + 1$ and continue from *Step 2*.

Step 5: Output. The solution for a given tolerance corresponds to $y_k^{(t)*} = y_{k,\nu}^{(t)}$.

V. Examples

It is worth stressing that the examples presented in this section have been strategically designed to enable both stationary and non-stationary approaches to be comparable. We have assumed that the network must withstand ever higher traffic loads which always increase by the same proportion. Therefore, the least desirable situation from a capacity design perspective, i.e. one in which there are the highest operational costs, is that which corresponds to the last year n_y in the study horizon. The uncertainty set associated with this last year is the one required to carry out a stationary analysis. Unlike stationary analysis, with the non-stationary approach it is assumed that the same final design situation will be reached at the end of the study horizon but the variation of uncertain parameters is distributed throughout the study period. This is important because since expansion planning is conditioned by the uncertainty set at the end of the study horizon, i.e. assuming that is the one in which the highest operational costs among all the uncertainty sets occurs, then the solution in terms of expanding lines at the end of the study horizon for both approaches must be the same.

A. Illustrative Case Study. Garver System

The model put forward is illustrated with the Garver 6-bus system, depicted in Figure 3. This system is made up of 6 buses, 3 generators, 5 levels of inelastic demand and 6 lines. Nominal values for generation capacities and demand and their supply and bidding prices can be found in [16]. The load-shedding cost is equal to the bidding price for each level of demand. It has been thought that up to three lines

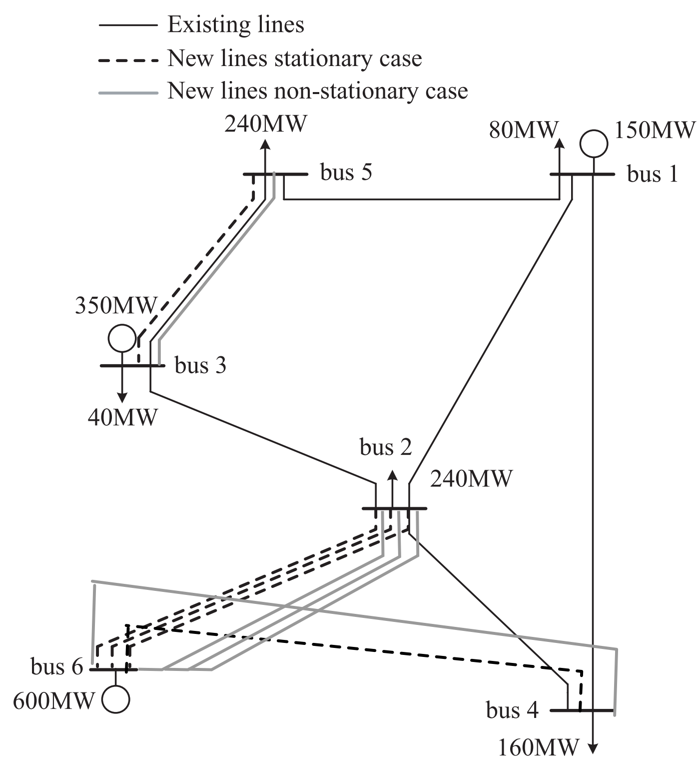

Fig. 3. Garver's 6-bus test system.

could be installed between each pair of buses. Line data are obtained from Table I of [6], including construction costs, and the maximum available investment budget is 40 million euros.

The time horizon is thought to be 25 years, and the discount rate is 10%. Since the pseudo-stationary periods are deemed to be one year, the weighted factor σ associated with load-shedding and power generation costs is equal to the number of hours in a year, i.e. 8760.

In order to compare results yielded from this method with respect to the stationary approach, we can consider the same example given by [16], where power generation capacities can increase or decrease by up to 50% of their nominal values, while demand levels may change by up to 20%. We assume that the reference uncertainty set, including nominal values and deviations from nominal values, is the same as that given in [16] but reduced by 25%. For the remaining years, we assume that the growth rates for nominal values and deviations are equal to 1.2% so that the uncertainty set variations are equal to:

$$\begin{aligned} r_{\mu,i}^{(t)} &= (1+0.012)^{(t-1)};\ \forall i \in \mathcal{G}; \forall t \in \mathcal{T} \\ r_{\mu,j}^{(t)} &= (1+0.012)^{(t-1)};\ \forall j \in \mathcal{D}; \forall t \in \mathcal{T} \\ r_{\sigma,i}^{(t)} &= (1+0.012)^{(t-1)};\ \forall i \in \mathcal{G}; \forall t \in \mathcal{T} \\ r_{\sigma,j}^{(t)} &= (1+0.012)^{(t-1)};\ \forall j \in \mathcal{D}; \forall t \in \mathcal{T}. \end{aligned} \tag{67}$$

Using these rates of change the uncertainty set defined for year 25 is equal to the uncertainty set given in [16]. Note that this last uncertainty set corresponds to the least desirable possible outcome from a capacity expansion perspective, i.e. the one in which the highest operational costs occur.

Using the uncertainty budgets $\Gamma^G = 2$ and $\Gamma^D = 2$, in the solution given by the stationary approach [16] a total of 27.031 million euros is invested for constructing the lines shown in Figure 3 (dashed black lines), these lines are constructed at the beginning of the study period. Conversely, in the non-stationary solution 22.775 million euros is invested for constructing the lines shown in Figure 3 (gray lines). Both constructed lines are exactly the same in terms of their optimal solutions,although with the non-stationary approach the rate of investment is lower because three of the lines have been constructed at the beginning of the time horizon, while one of the lines joining nodes 2 and 6 is constructed for year 6 and the line between nodes 3 and 5 is constructed for year 9.

Note that in the end, the final solution given by the stationary and non-stationary approaches is the same, as it is dominated by the last uncertainty set, which is the one in which the highest operational costs occur and the one used for the stationary approach. This is a particular case strategically selected by the authors in order to compare both approaches in a more meaningful way. To date the advantage the non-stationary approach has is associated with the cost of money over time, which allows the overall costs to be reduced. However, this is not the only advantage. Let us assume that in year 8, an analysis of the data yielded throughout those seven years reveals that the 1.2% trend which was initially assumed would be needed for design was higher than in reality, then it would be possible to re-run the model incorporating this new information so that corrective actions could be taken. Let us consider that growth rates throughout those 7 years were indeed 0.4% instead of 1.2%. If we re-ran the model for the remaining years using said 0.4% rate of growth, the results would confirm that the line between nodes 3 and 5 would not be necessary, by which we could save an additional 1.8015 million euros. However, even though the stationary capacity expansion planning is oversized, no correction actions are possible because those lines were already constructed. The non-stationary alternative enables more rational use of existing financial resources making the model less prone to wrongful design assumptions.

Note that these situations are not unusual, as shown in Figures 1 and 2 (b). Figure 1 depicts aggregate demand in Spain from 1998 to 2010, while Figure 2 (b) also shows demand in Spain from 2008 to 2015. It can be seen that from 2009 to 2015 the growth trend stops and demand starts decreasing due to the economic downturn. Note that a stationary design using data from 1998 to 2010 would have probably resulted in oversized expansion planning. However, the non-stationary approach enables corrective measures to be carried out in order to adapt to the real change in uncertain variables.

Additional observations regarding computational tractability are pertinent:

1) Computational times for the stationary, and non-stationary versions are, respectively, 0.842 and 68.701 seconds, respectively. Note that the non-stationary assumption increases computational times considerably with respect to the stationary approach.
2) Both the stationary and non-stationary approaches require the same number of iterations, i.e. four.
3) The computational complexity associated with each approach is provided in Table I. Note that the possibility of making investments at any time within the project horizon increases the discrete variable number by the number of time periods considered, i.e. $45 \times 25 = 1125$. This explains the increased complexity.

TABLE I
COMPUTATIONAL COMPLEXITY RELATED TO GARVER'S 6-BUS TEST SYSTEM ILLUSTRATIVE EXAMPLE.

	♯ cont. variables	♯ discrete variables	♯ equations
Stationary	266	45	644
Non-stationary	7751	1125	17216

B. *Case study. IEEE 118-bus test system*

We run additional computational tests using the more realistic IEEE 118-bus test system [22]. The system is made up of 118 buses, 186 existing lines, 54 generating units and 91 loads. Additionally, and as with the example given in [16], it is possible to construct up to 61 additional lines to duplicate each one of the following existing lines: 8, 12, 23, 32, 38, 41, 51, 68, 78, 96, 104, 118, 119, 121, 125, 129, 134, 159, 7, 9, 36, 117, 71, 131, 133, 147, 103, 65, 144, 168, 4, 13, 132, 69, 66, 67, 5, 89, 29, 167, 145, 70, 42, 90, 16, 174, 98, 99, 185, 93, 94, 128, 164, 97, 153, 146, 116, 163, 31, 92, 130. Data for lines in existing corridors are taken from [22]. The investment budget is 100 million euros, the time horizon period considered is 10 years, and the discount rate is 10%. Data for generation capacities and demand loads are given in [16]. The load-shedding cost equals the bidding price of each level of demand multiplied by 1.2

As with the illustrative example, and in order to compare results from this method with respect to the stationary method, the same example given by [16] is considered, where power generation capacities and levels of demand can increase or decrease by up to 50% of their nominal values. We assume the same uncertainty set considered in [16] as the reference set but reduce its expected value and interval range by two thirds. For the remaining years, we assume that the growth rates of nominal values and deviations are equal to 3.25% so that the uncertainty set variations are equal to:

$$\begin{aligned} r_{\mu,i}^{(t)} &= (1+0.0325)^{(t-1)};\ \forall i \in \mathcal{G}; \forall t \in \mathcal{T} \\ r_{\mu,j}^{(t)} &= (1+0.0325)^{(t-1)};\ \forall j \in \mathcal{D}; \forall t \in \mathcal{T} \\ r_{\sigma,i}^{(t)} &= (1+0.0325)^{(t-1)};\ \forall i \in \mathcal{G}; \forall t \in \mathcal{T} \\ r_{\sigma,j}^{(t)} &= (1+0.0325)^{(t-1)};\ \forall j \in \mathcal{D}; \forall t \in \mathcal{T}. \end{aligned} \quad (68)$$

Use of these ratios make the uncertainty set defined for year 10 equal to the uncertainty set given in [16] but with the nominal values and interval range reduced by one third. Generation and demand nominal values and their possible deviations grow linearly reaching their maximum values at the end of the study period. Note that the uncertainty set for year $n_y = 10$ corresponds to the least desirable outcome possible from a capacity expansion perspective, i.e. the uncertainty set in which the maximum operational costs occur among all uncertainty sets.

Using the uncertainty budgets $\Gamma^{\mathrm{G}} = 15$ and $\Gamma^{\mathrm{D}} = 20$, in the solution given by the stationary approach [16] (reducing also the nominal values and interval ranges by one third) a total of 79.384 million euros are invested for constructing the lines 7, 8, 9, 38, 41, 133, 134, 153, 159. These lines are constructed at the beginning of the study period. Conversely, with the non-stationary solution 73.626 million euros are invested for constructing exactly the same lines as in the stationary solution. However,with the non-stationary approach investment is lower because lines 38 and 133 are planned to be constructed for year 3 and line 41 is constructed for year 8.

Note that as in the previous example, the advantage the non-stationary approach has so far is related to the cost of money over time, because both solutions are conditioned by the last uncertainty set. Due to this it can be confirmed that the non-stationary model provides consistent solutions. However, let us assume that in year 7, an analysis of data throughout those seven years reveals that the 3.25% trend initially assumed for design was higher than the actual one, it would then be possible to re-run the model with this new information added so that corrective actions can be taken. Let us consider that growth rates throughout those 7 years were indeed below 2.52% instead of 3.25%. If we re-ran the model for the remaining years using, for instance, 2.4% growth rates, the results would confirm line 41 to be unnecessary, with which we could save an additional 2.0285 million euros.

Additional observations regarding computational tractability are pertinent:

1) Computational times for the stationary and non-stationary versions are 18.563 and 366.245 seconds respectively using a Microsoft Windows Server 2012 with four processors clocking at 2.00GHz. Note that the non-stationary assumption increases computational times considerably with respect to the stationary approach.
2) In the stationary approach six iterations are required, while in the non-stationary case eight iterations are needed to reach the optimal solution.
3) The computational complexity associated with each model is provided in Table II. Note that the possibility of making investments at any time within the project horizon increases the discrete variable number by the number of time periods considered, i.e. $247 \times 10 = 2470$ which can explain the increase in computational time.

TABLE II
COMPUTATIONAL COMPLEXITY RELATED TO IEEE 118-BUS TEST SYSTEM EXAMPLE.

	♯ cont. variables	♯ discrete variables	♯ equations
Stationary	$3,984$	247	$8,083$
Non-stationary	$57,241$	2470	$114,923$

4) The change in the algorithm during the solution process associated with the stationary approach is given in Table III. Note how the change in the lower and upper bounds tend to converge to the same value, and how investment costs $c_{\mathrm{in},\nu}$ change among iterations.

TABLE III
CHANGE IN THE STATIONARY SCENARIO ALGORITHM FOR THE IEEE 118-BUS TEST SYSTEM EXAMPLE.

ν	1	2	3	4	5	6
$c_{\mathrm{op},\nu}$	18718.57	15263.49	15499.74	15369.24	15317.92	15263.49
$c_{\mathrm{in},\nu}$	0	97.817	19.416	57.294	68.154	79.384
$z^{(\mathrm{up})}$	18718.57	15273.27	15273.27	15273.27	15273.27	15271.43
$z^{(\mathrm{lo})}$	$-\infty$	13807.49	15265.43	15269.29	15270.31	15271.43
error$^{(\S)}$	1	0.09597	0.000513	0.000260	0.000194	0.0

Units in million €
(§): Adimensional value

5) The change in the algorithm during the solution process associated with the non-stationary scenario is given in Table IV. The change in lower and upper bounds tend to converge on the same value, due to the variations in investment costs $c_{\mathrm{in},\nu}$ among iterations. It is also important to verify in the Table that the operational costs for the last year $n_y = 10$ at each iteration are the highest, because this is the most critical uncertainty set. Additionally, it should be noted how the operation costs for the last year $n_y = 10$ at the beginning of the iterative process (18718.57 million euros) and at the end (15263.49 million euros) match the operational costs found with the stationary solution.

Finally, it is worth stressing that the non-stationary approach is more flexible in so far as it can accommodate different rates of growth associated with uncertain parameters.

VI. CONCLUSIONS

In this paper the use of robust optimization for solving the transmission expansion planning problem for non-stationary situations has been extended, which is more realistic as regards the variability of energy resources and requirements. The model put forward herein provides the initial design and the expansion plan as regards forthcoming years, assuming that the

TABLE IV
EVOLUTION OF THE NON-STATIONARY CASE ALGORITHM FOR THE IEEE 118-BUS TEST SYSTEM EXAMPLE.

ν	1	2	3	...	6	7	8
$c_{\mathrm{op},\nu}^{(1)}$	14038.928	11405.560	11558.403	...	11453.623	11407.980	11405.560
$c_{\mathrm{op},\nu}^{(2)}$	14558.888	11830.358	11948.694	...	11830.358	11830.358	11830.358
$c_{\mathrm{op},\nu}^{(3)}$	15078.848	12255.157	12392.574	...	12264.433	12257.387	12255.157
$c_{\mathrm{op},\nu}^{(4)}$	15598.809	12679.956	12836.455	...	12710.111	12689.654	12679.956
$c_{\mathrm{op},\nu}^{(5)}$	16118.769	13104.755	13284.671	...	13104.755	13104.755	13104.755
$c_{\mathrm{op},\nu}^{(6)}$	16638.729	13529.554	13735.844	...	13529.554	13529.554	13529.554
$c_{\mathrm{op},\nu}^{(7)}$	17158.690	13961.002	14168.095	...	13961.002	13961.002	13961.002
$c_{\mathrm{op},\nu}^{(8)}$	17678.650	14395.165	14611.976	...	14395.165	14395.165	14395.165
$c_{\mathrm{op},\nu}^{(9)}$	18198.610	14829.328	15055.856	...	14829.328	14829.328	14829.328
$c_{\mathrm{op},\nu}^{(10)}$	18718.570	15263.492	15504.033	...	15263.492	15263.492	15263.492
$c_{\mathrm{in},\nu}$	0.0	99.178	9.796	...	69.359	71.081	73.625
$z^{(\mathrm{up})}$	107982.298	87924.811	87924.811	...	87924.811	87908.264	87899.258
$z^{(\mathrm{lo})}$	$-\infty$	78666.752	87835.871	...	87894.992	87896.714	87899.258
error$^{(\S)}$	1	0.1053	0.0010	...	0.0003	0.0001	0.0000

Units in million €
(§): Adimensional value

probability distributions for the random variables (uncertainty sets) change between consecutive years. These changes might be due to climate change or economic development of the area. With such a strategy, investment is made little by little on request rather than entirely at the beginning of the project.

The additional requirement for implementing the non-stationary model consists in considering different time periods in which the uncertainty sets change. This means that the subproblems related to obtaining the least desirable operational costs are replicated for each uncertainty set.

Results from the non-stationary model mean corrective actions can be carried out throughout the study horizon, especially in outcomes in which the future variable prognosis differs from true evolution. This alternative enables more rational use of existing financial resources and the model is less prone to wrongful design assumptions.

Finally, in this paper it has been demonstrated that the non-stationary approach remains computationally tractable even for realistic cases.

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