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Time Truncated Chain Sampling Plans for Log - Logistic Distribution

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Abstract:- Chain sampling plan for Log - Logistic distribution when the life-test is truncated at a pre-specified time is discussed. The design parameters such as the minimum sample size and the acceptance number necessary to assure a specified mean life time are obtained by satisfying the producer's and consumer's risks at the specified quality levels, under the assumption that the termination time and the number of items are pre-fixed. The operating characteristic functions of the sampling plan according to various quality levels are obtained.

Keywords:- Truncated life test, Log - Logistic distribution, Operating characteristics, Consumer's risk, Producer's risk.

I. INTRODUCTION

Acceptance sampling plans are the practical tools for quality assurance applications involving product quality control. Acceptance sampling systems are advocated when small sample size are necessary or desirable towards costlier testing for product quality. Whenever a sampling inspection is considered, the lot is either accepted or rejected along with associated producer and consumer's risk. In a time truncated sampling plan a random sample is selected from a lot of products and put on the test where the number of failures is recorded until the pre specified time. If the number of failures observed is not greater than the specified acceptance number, then the lot will be accepted. Sampling inspection in which the criteria for acceptance and nonacceptance of the lot depend in part on the results of the inspection of immediately preceding lots is adopted in Chain Sampling Plan. More recently, Aslam and Jun (2009) proposed the group acceptance sampling plan based on the truncated life test when the lifetime of an item follows the inverse Rayleigh and Log-logistic distribution. These truncated lifetests were discussed by many authors, Kantam R.R. L., Rosaiah K. and Srinivasa Rao G. (2001) discussed acceptance sampling based on life tests with Log-logistic models. Rosaiah K. and Kantam R.R.L. (2005) discussed acceptance sampling based on the inverse Rayleigh distribution. Gupta and Groll(1961), Baklizi and EI Masri(2004), and Tsai, Tzong and Shou(2006), Balakrishnan, Victor Leiva & Lopez (2007).All these authors developed the sampling plans for life tests using single acceptance sampling.
This paper presents the probability of Acceptance for chain sampling plan based on the truncated life tests when the lifetime of the product follows a Log - Logistic distribution. Procedures and necessary tables are provided involving the producer and consumer quality levels.

II. GLOSSARY OF SYMBOLS

n - Sample size
γ - Shape parameter
σ - Scale parameter
T - Prefixed time
β - Consumer's risk
p* - Minimum probability
d - Number of defectives
p - Failure probability
L(p) - Probability of acceptance
i - Acceptance criteria

III. LOG-LOGISTIC DISTRIBUTION

The Log - Logistic distribution has been studied by Shah and dav (1963) and Tadikamalla and Johnson (1982). The cumulative distribution function of the Log - Logistic distribution is given by

$$F(t,\sigma)=\frac{\left(\frac{t}{\sigma}\right)^{\gamma}}{1+\left(\frac{t}{\sigma}\right)^{\gamma}} \qquad \text{-----------} \qquad \textbf{(1)}$$

let us assume the shape parameter $\gamma = 2$, σ is the scale parameter. If some other parameters are involved, then they are assumed to be known, for example, if shape parameter of a distribution is unknown it is very difficult to design the acceptance sampling plan. In quality control analysis, the scale parameter is often called the quality parameter or characteristics parameter. Therefore it is assumed that the distribution function depends on time only through the ratio of t/σ.

IV. DESIGN OF THE PROPOSED SAMPLING PLAN

Chain Sampling Plan (ChSP-1) proposed by Dodge (1955) making use of cumulative results of several samples help to overcome the shortcomings of the Single Sampling Plan. It avoids rejection of a lot on the basis of a single nonconfirming unit and improves the poor discrimination between good and bad quality that occurs with the c = 0 plan. The conditions for application and operating procedure of chsp-1 are as follows

4.1 Conditions for application of ChSP -1:

The cost of destructiveness of testing is such that a relatively small sample size is necessary, although other factors make a large sample desirable.

1) The product to be inspected comprises a series of successive lots produced by a continuing process.
2) Normally lots are expected to be of essentially the same quality.
3) The consumer has faith in the integrity of the producer.

4.2 Operating Procedure

The plan is implemented in the following way:

1) For each lot, select a sample of n units and test each unit for conformance to the specified requirements.
2) Accept the lot if d (the observed number of defectives) is zero in the sample of n units, and reject if d > 1.
3) Accept the lot if d is equal to 1 and if no defectives are found in the immediately preceeding i samples of size n.

The Chain sampling Plan is characterized by the parameters n and i. We are interested in determining the sample size required for in the case of Log - Logistic distribution and various values of acceptance number i.

The probability (α) of rejecting a good lot is called the producer's risk, whereas the probability(β) of accepting a bad lot is known as the consumer's risk. Often the consumer risk is expressed by the consumer's confidence level. If the confidence level is p* then the consumer's risk will be β = 1- p*.We will determine the sample size so that the consumer's risk does not exceed a given value β.

The probability of acceptance can be regarded as a function of the deviation of specified average from the true average. This function is called operating characteristic (oc) function sampling plan. Once the minimum sample size is obtained one may be interested to find the probability of acceptance of a lot when the quality of the product is good enough. The probability of acceptance for the chain sampling plan is calculated using the following equations,

$$L(p)=(1-p)^{n}+np(1-p)^{n-1}(1-p)^{ni}$$

where $$p=\frac{\left(\frac{t}{\sigma}\right)^{\gamma}}{1+\left(\frac{t}{\sigma}\right)^{\gamma}}$$

It is clear that p depends only on the ratio t/σ

The minimum values of n, satisfying equation are found for p* = 0.75, 0.90, 0.95, 0.99 and for t/σ = 0.628,0.942,1.257,1.571,2.356,3.141,4.712. These choices are consistent with Gupta and Groll (1961), Gupta (1962),Kantam et al (2001), Baklizi and EI Masri (2004),Balakrishnan et al (2007).

V. DESCRIPTION OF TABLES AND AN EXAMPLE

In Table 1, we provide the minimum sample size required for the proposed sampling plan assuming that the life time distribution is an Log - Logistic distribution with $\gamma = 2$. For example assume that the experimenter is interested in knowing that the true unknown average life is atleast 1000 hours with confidence 0.99. It is assumed that the maximum affordable time is 767 hours and $t/\sigma = 0.942$, From the table 1, we obtain n = 10. Therefore, out of 10 items if not more than 1 item fail and if no defectives are found in the immediately preceeding i samples before T = 767 units of time, the lot can be accepted with the assurance that the true mean life is atleast 1000 with probability 0.99.

For the sampling plan(n = 10, i = 2, $t/\sigma = 0.942$) and confidence level $p^* = 0.99$ under Log - Logistic distribution with $\gamma = 2$ the values of the operating characteristic function from Table 2 as follows

σ/σ_0	2	4	6	8	10	12
L(p)	0.140291	0.692714	0.902605	0.963093	0.983522	0.991666

Table 1:
Minimum sample size for the specified ratio t/σ,confidence level p*,and the corresponding acceptance number i when the shape parameter $\gamma = 2$

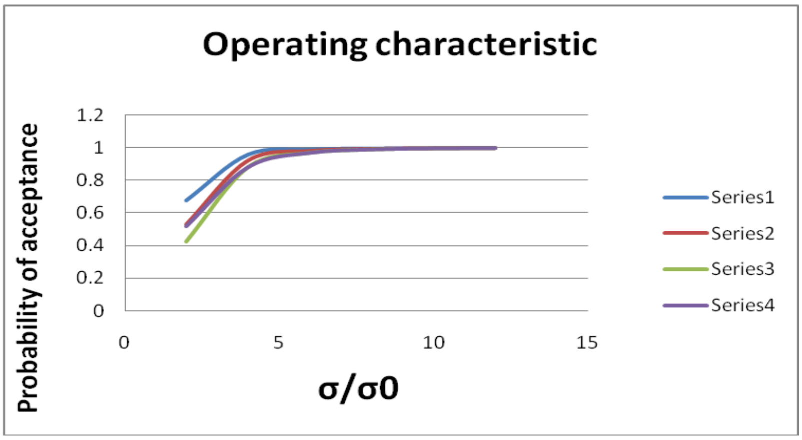
p*	i	t/σ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	1	7	4	3	2	2	1	1	1
	2	6	4	3	2	1	1	1	1
	3	6	4	2	2	1	1	1	1
	4	6	3	2	2	1	1	1	1
	5	6	3	2	2	1	1	1	1
	6	6	3	2	2	1	1	1	1
0.90	1	11	6	4	3	2	2	1	1
	2	10	6	4	3	2	1	1	1
	3	10	5	4	3	2	1	1	1
	4	10	5	4	3	2	1	1	1
	5	10	5	4	3	2	1	1	1
	6	10	5	4	3	2	1	1	1
0.95	1	13	7	5	4	2	2	2	1
	2	13	7	5	4	2	2	1	1
	3	13	7	5	4	2	2	1	1
	4	13	7	5	4	2	2	1	1
	5	13	7	5	4	2	2	1	1
	6	13	7	5	4	2	2	1	1
0.99	1	19	11	7	5	3	2	2	2
	2	19	10	7	5	3	2	2	2

	3	19	10	7	5	3	2	2	2
	4	19	10	7	5	3	2	2	2
	5	19	10	7	5	3	2	2	2
	6	19	10	7	5	3	2	2	2

Table 2:
OC values for the time truncated chain sampling plan (n, i, t/σ) for a given p*, when i = 2 and γ = 2.

p*	n	t/σ_0	σ/σ_0					
			2	4	6	8	10	12
0.75	6	0.628	0.677691	0.959482	0.990743	0.996914	0.998705	0.999367
	4	0.942	0.528839	0.921885	0.980806	0.993423	0.997203	0.998623
	3	1.257	0.427583	0.880808	0.968547	0.988916	0.995222	0.997630
	2	1.571	0.520760	0.881036	0.967907	0.988578	0.995052	0.997539
	1	2.356	0.347690	0.884407	0.966692	0.987770	0.994614	0.997296
	1	3.141	0.239642	0.764522	0.917414	0.966704	0.984596	0.992041
	1	3.927	0.172401	0.636467	0.847092	0.931916	0.966682	0.982191
	1	4.712	0.451481	0.520760	0.764468	0.884407	0.939976	0.966692
0.90	10	0.628	0.449210	0.902605	0.975777	0.991666	0.996450	0.998251
	6	0.942	0.336682	0.849295	0.959482	0.985630	0.993788	0.996914
	4	1.257	0.293140	0.813707	0.946983	0.980751	0.991581	0.995791
	3	1.571	0.261007	0.777522	0.932848	0.974991	0.988922	0.994421
	2	2.356	0.190385	0.667409	0.881110	0.952063	0.977870	0.988587
	1	3.141	0.347690	0.764522	0.917414	0.966704	0.984596	0.992041
	1	3.927	0.239642	0.636467	0.847092	0.931916	0.966682	0.982191
	1	4.712	0.172401	0.520760	0.764468	0.884407	0.939976	0.966692
0.95	13	0.628	0.327257	0.852626	0.961046	0.986284	0.994092	0.997071
	7	0.942	0.269090	0.810310	0.946756	0.980795	0.991629	0.995823
	5	1.257	0.202676	0.744684	0.921942	0.970821	0.987056	0.993477
	4	1.571	0.153991	0.673701	0.890774	0.957502	0.980762	0.990193
	2	2.356	0.190385	0.667409	0.881110	0.952063	0.977870	0.988587
	2	3.141	0.086063	0.451724	0.744220	0.881147	0.940664	0.967943
	1	3.927	0.239642	0.636467	0.847092	0.931916	0.966682	0.982191
	1	4.712	0.172401	0.52076	0.764468	0.884407	0.939976	0.966692
0.99	19	0.628	0.176332	0.746500	0.924862	0.972324	0.987816	0.993887
	10	0.942	0.140291	0.692714	0.902605	0.963093	0.983522	0.991666
	7	1.257	0.099809	0.612912	0.864971	0.946611	0.975646	0.987532
	5	1.571	0.092739	0.577764	0.844596	0.936936	0.970838	0.984953
	3	2.356	0.075114	0.480566	0.777644	0.902257	0.952791	0.97501
	2	3.141	0.086063	0.451724	0.744220	0.881147	0.940664	0.967943
	2	3.927	0.043008	0.292898	0.591018	0.781316	0.881081	0.932219
	2	4.712	0.023444	0.190385	0.451643	0.667409	0.802970	0.881110

FIGURE 1:



probability of acceptance against mean ratio

VI. CONCLUSIONS

In this paper, chain sampling plan for the truncated life test was proposed in the case of Log - Logistic distribution. It is assumed that the shape parameter is known , the minimum sample size required and the acceptance number were calculated. From the figure 1 we can see that the operating characteristics values increases more rapidly as the quality improves and reaches the maximum value 1 when σ / σ_0 is greater than

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