ADAPTIVE PREDICTION OF TIME-VARYING CHANNELS FOR CODED OFDM SYSTEMS

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ABSTRACT

We propose adaptive channel predictors for orthogonal frequency division multiplexing (OFDM) communications over time-varying channels. Successful application of the normalized least-mean-square (NLMS) and recursive least-squares (RLS) algorithms is demonstrated. We also consider the use of adaptive channel predictors for delay-free equalization, thereby avoiding the need for regular transmission of pilot symbols. Simulation results demonstrate the good performance of the proposed techniques.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an attractive modulation technique for high data-rate wireless communications [1]. A crucial issue in achieving high quality of service in wireless communications is reliable channel estimation [2, 3]. Beyond that, it was recently recognized that channel prediction is useful for various tasks like delay-free equalization, adaptive modulation, and power control [4–6].

Here, we propose two adaptive, decision-directed predictors for time-varying channels within a coded OFDM system. The adaptive channel predictors extend the minimum mean-square error (MMSE) channel predictor presented in [4]. We also propose a receiver structure in which the adaptive channel predictors are used for delay-free channel equalization without the use of pilot symbols.

This paper is organized as follows. The OFDM system is reviewed in Section 2. In Section 3, the MMSE channel predictor from [4] is extended to prediction horizon $p > 1$. The adaptive predictors are presented in Section 4 and applied to equalization in Section 5. Finally, simulation results are provided in Section 6.

2. OFDM SYSTEM MODEL

We consider an OFDM system with $K$ subcarriers. A block of bits $b[n, i], i \in \{0, 1, \ldots, B - 1\}$ is encoded and mapped to transmit symbols denoted as $a[n, k]$. Here, $n \in \mathbb{Z}$ is the OFDM symbol (time) index and $k \in \{0, 1, \ldots, K - 1\}$ is the subcarrier (frequency) index. The $n$th OFDM symbol is obtained by applying an inverse discrete Fourier transform (IDFT) to the $a[n, k]$ and adding a cyclic prefix of length $L_{cp}$:

$$s_{n}[m] = \begin{cases} \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} a[n,k] e^{j2\pi nk/K}, & m = -L_{cp}, \ldots, K-1, \\ 0, & \text{elsewhere}. \end{cases}$$

Thus, each OFDM symbol has length $N = K + L_{cp}$. The overall transmit signal is $s[n] = \sum_{n=-\infty}^{\infty} s_{n}[m-nN]$.

Assuming a time-varying mobile radio channel with impulse response $h[m, l]$ ($l = 0, 1, \ldots, L$, with $L \leq L_{cp}$ the maximum delay) and additive noise $\eta[n, k]$, the received signal is

$$r[m] = \sum_{l=0}^{L} h[m, l] s[m-l] + \eta[n, k].$$

The noise $\eta[n, k]$ is assumed white and Gaussian with variance $\sigma^2_{\eta}$. The receiver discards the cyclic prefix and demodulates the received signal $r[m]$ by means of a DFT, which yields

$$x[n, k] = \frac{1}{\sqrt{K}} \sum_{m=0}^{K-1} r[nN+m] e^{-j2\pi km/K}.$$

It is assumed that the channel impulse response $h[m, l]$ varies negligibly within one symbol period. The input-output relation of the overall OFDM system can then be shown to equal

$$x[n, k] = H[n, k] a[n, k] + z[n, k],$$

with the channel coefficients

$$H[n, k] = \sum_{l=0}^{K-1} h[nN+l] e^{-j2\pi kl/K}$$

and the noise

$$z[n, k] = \frac{1}{\sqrt{K}} \sum_{m=0}^{K-1} \eta[nN+m] e^{-j2\pi km/K}.$$

In what follows, the time-varying channel will be considered random. Under the wide-sense stationary uncorrelated scattering (WSSUS) assumption, the autocorrelation function of $h[n, l]$ is $E\{h[m+l, l] h^{*}[m', l']\} = \delta[l-l']$ [7, 8], and the channel’s Doppler spectra or scattering function is defined as [7, 8]

$$S_{h}(v, l) = \sum_{m=-\infty}^{\infty} r_{h}[m, l] e^{-j2\pi vm}, \quad l = 0, 1, \ldots, L. \quad (2)$$

Here, $v$ is the normalized Doppler frequency.

3. MMSE CHANNEL PREDICTOR

Before considering adaptive predictors, we extend the MMSE channel predictor from [4] to prediction horizon $p > 1$. As was shown in [4], the MMSE predictor has the structure depicted in Fig. 1. It consists of divisions of the observations $x[n, k]$ by the transmit symbols $a[n, k]$, i.e.,

$$\hat{H}[n, k] = \frac{x[n, k]}{a[n, k]} = H[n, k] + \tilde{z}[n, k],$$

where $\tilde{z}[n, k] = z[n, k]/a[n, k]$, followed by an IDFT, $L+1$ predictors, and a DFT. (For practical operation, the $a[n, k]$ are replaced with the symbols $\hat{a}[n, k]$ detected by the receiver.) The number of

Figure 1: Channel predictor structure.
The adaptive channel predictors presented in this section perform a continual update of the predictor coefficients that replaces the exponential forgetting factor $\beta$ with a forgetting factor $\lambda$. Furthermore, $\mathbf{r}^{(p)}[\ell] \triangleq \mathbf{h}[\ell][n] \mathbf{h}^H[\ell][n] + \sigma_0^2 \mathbf{I}$ denotes the Hermitian Toeplitz matrix of the vector $\mathbf{h}[\ell][n] = [\mathbf{h}[\ell][n], \mathbf{h}[\ell][n-1], \ldots, \mathbf{h}[\ell][n-M+1]]^T$. The MMSE channel coefficients can be shown to equal

$$
epsilon_{\min} = \mathcal{E}^{\text{opt}} |_{\mathbf{w}_n = \mathbf{w}^{\text{opt}}} = \sum_{\ell=0}^{\ell_{-1}} \bigl[ \mathbf{r}^{(0)}[\ell] - \mathbf{w}^{\text{opt}} H^{(p)}[\ell] \bigr].$$

Calculation of the MMSE channel predictor in (7) requires knowledge of the correlation function of the time-varying channel and the variance of the noise. In practice, these quantities would have to be estimated [10]. Even worse, since the statistics of real-world channels are stationary only over a certain time, they would have to be reestimated, and the MMSE channel predictor would have to be recalculated once in a while. These problems are avoided by the adaptive channel predictors discussed next.

4. ADAPTIVE CHANNEL PREDICTORS

The adaptive channel predictors presented in this section perform a continual update of the predictor coefficients that replaces the explicit design (5). They do not assume knowledge of the channel and noise statistics and are capable of tracking nonstationary statistics. Assuming without loss of generality that the adaptation starts at $n = 0$, the predicted channel taps are (cf. (5))

$$\hat{h}_1[n+p] = \mathbf{w}[n-1] + \frac{\mu}{\|\mathbf{h}[n-1]\|^2} e^H[n] \mathbf{h}[n-1], \quad n \geq 0, \quad p = 0, 1, \ldots, L.$$
5. OFDM RECEIVER WITH PREDICTIVE EQUALIZER

As mentioned in Section 1, channel prediction is useful for several tasks. An important example, delay-free equalization without the use of regular pilot symbols, will be considered in this section. Fig. 2 shows a block diagram of the proposed receiver [4]. The upper branch is a conventional OFDM receiver with equalizer. Based on the approximate input-output relation (1), the observed vector $x_n = [x(n, 0), x(n, 1), \ldots, x(n, K - 1)]^T$ is equalized according to

$$y[n,k] = \frac{x[n,k]}{H[n,k]}, \quad k = 0, 1, \ldots, K-1.$$ 

Here, the $H[n,k]$ are estimates of the current channel coefficients that are calculated by the lower branch. The equalized sequence $y_n = [y(n, 0), y(n, 1), \ldots, y(n, K - 1)]^T$ is then passed through a slicer and a decoder to obtain the (error-corrected) bits $\hat{b}_n = [\hat{b}(n, 0), \hat{b}(n, 1), \ldots, \hat{b}(n, B - 1)]^T$.

The lower branch of the receiver in Fig. 2 produces estimates $\hat{H}_{n+p} = [\hat{H}[n+p, 0], \hat{H}[n+p, 1], \ldots, \hat{H}[n+p, K - 1]]^T$ of the channel coefficients $H[n+p, k]$, to be used for equalization in a subsequent symbol interval (hence the delay by $\hat{p}$ in Fig. 2). The central part of the lower branch is an adaptive channel predictor as described in Section 4. The inputs of the channel predictor are the observed vector $x_n$ and the estimated symbols $\hat{a}_n = [\hat{a}(n, 0), \hat{a}(n, 1), \ldots, \hat{a}(n, K - 1)]^T$ that are obtained by re-encoding the error-corrected detected bits $b_n$ (see Fig. 2). The prediction horizon $p$ is chosen to equal the decoding/re-encoding delay. Note that $\hat{a}_n = a_n$ only if all bit errors were corrected; otherwise, error propagation will result.

6. SIMULATION RESULTS

We simulated a coded OFDM system with $K = 120$ subcarriers, cyclic prefix length $L_p = 20$, and QPSK modulation with $|a[n, k]| = 1$. We used a (15, 7) Reed-Solomon (RS) code with each code symbol consisting of two QPSK symbols grouped in frequency. The RS code symbols were interleaved in frequency. The channel was simulated using the technique described in [11]. The scattering function (see (2)) of the simulated channel had a Jakes Doppler profile $[7]$ and an exponentially decaying delay profile, i.e., the observed channel was approximated by a Rayleigh fading channel with a constant Doppler spectrum and a slowly decaying delay $\tau$. The scattering function was approximated by a Rayleigh fading channel with a constant Doppler spectrum and a slowly decaying delay $\tau$. The prediction MSE of the MMSE predictor from (3) is then passed through a slicer and a decoder to obtain the (error-corrected) bits $\hat{b}_n = [\hat{b}(n, 0), \hat{b}(n, 1), \ldots, \hat{b}(n, B - 1)]^T$.

The prediction MSE shown was estimated from 100 realizations of the adaptive channel predictors for decision-directed startup. Fig. 4(a) shows the temporal evolution of the squared prediction error of the various predictors for one realization. Fig. 4(b) shows the cumulative distribution function (estimated from 200 realizations) of the convergence time for SNR = 15 dB. (The convergence time is the time the prediction MSE requires to fall below −13 dB.) It can be seen that for decision-directed startup, on average, the NLMS algorithm converges more rapidly than the RLS algorithm. Note that the MMSE filter is fixed; its prediction error behavior is due to decoding errors and error propagation.

Convergence with decision-directed startup. Next, we study the behavior of the channel predictors for decision-directed startup. (At initial time $n = 0$ a randomly selected OFDM symbol was used in (3).) Fig. 4(a) shows the temporal evolution of the squared prediction error of the various predictors for one realization. Fig. 4(b) shows the cumulative distribution function (estimated from 200 realizations) of the convergence time for SNR = 15 dB. (The convergence time is the time the prediction MSE requires to fall below −13 dB.) It can be seen that for decision-directed startup, on average, the NLMS algorithm converges more rapidly than the RLS algorithm. Note that the MMSE algorithm converges more rapidly than the RLS algorithm. Note that the MMSE algorithm is more reliable than the RLS algorithm.

Convergence with ideal symbol feedback. Fig. 3 shows the convergence of the adaptive channel predictors assuming ideal symbol feedback, i.e., without decoding errors. Both a “slow” channel ($v_{\text{max}} K = 0.01$) and a “fast” channel ($v_{\text{max}} K = 0.05$) are considered.

The prediction MSE of the MMSE predictor from Section 3 and the theoretical MMSE in (8) are also plotted for comparison. The SNR is $\text{SNR} = \sum \int S(v,l) dv / \sigma_n^2$ where $S(v,l)$ was 15 dB. The adaptive predictors were initialized as explained in Section 4.

It is seen that the RLS algorithm converges more slowly with an excess MSE of about 3 dB. For the slower channel, channel prediction is more accurate but convergence is slower (since for small $v_{\text{max}}$ the channel taps are highly correlated).

Convergence with nonstationary channel statistics. The ability of the adaptive channel predictors to track nonstationary channel statistics is studied in Fig. 6. Two different channels were generated: channel 1 has a constant scattering function (uniform Doppler and delay profile) for $|v| < v_{\text{max}}$ and $l = 0, \ldots, L$ with $v_{\text{max}} K = 0.01$ and $L = 15$; channel 2 has the Jakes exponential scattering function used previously, with $v_{\text{max}} K = 0.03$ and $L = 20$. We used channel 1 for the first 1000 OFDM symbols and channel 2 for the last 1000 OFDM symbols. For the intermediate transition period (corresponding to $n = 1001, \ldots, 2000$), the channel was constructed by linearly interpolating between channel 1 and channel 2.
operation (the initial convergence to channel 1 is not shown). It is seen that during each stationary period, the RLS algorithm performs as well as the respective MMSE predictor and nearly achieves the theoretical MMSE. The RLS algorithm is also good at tracking the nonstationary channel statistics during the transition period. The tracking results of the NLMS algorithm are much poorer, with a relatively high MSE during and even after the transition period.

Receiver performance. Finally, we study the performance of the OFDM receiver proposed in Section 5 (see Fig. 2) using the various channel predictors in decision-directed mode. The channel has the Jakes-exponential scattering function used previously, with $v_{\text{max}}K = 0.01$. The theoretical MMSE (8) and the MSE using pilot symbol assisted (PSA) channel estimation [12] are also shown for comparison. For PSA channel estimation, approximately 10% of the transmitted symbols were used as pilots. Due to the decision-directed operation, a threshold SNR is necessary for good performance of the predictors. For sufficiently high SNR, the predictors outperform PSA channel estimation and both the MMSE and the RLS predictors achieve the theoretical optimum. Again, the NLMS algorithm performs slightly worse and has about 3 dB excess MSE.

Fig. 7(e) shows the bit error rates (BERs) after the channel decoder for the “slow” channel when the result of channel prediction is used for equalization. The BERs of a “genius” receiver knowing the true channel and of a receiver using PSA channel estimation are also shown for comparison. Above the threshold SNR, the performance of all receivers is very similar; this is due to the relatively robust QPSK modulation. Fig. 7(b) and Fig. 7(d) show the prediction MSEs and BERs for a “fast” channel ($v_{\text{max}}K = 0.05$). Generally speaking, the performance is worse than for the “slow” channel; in particular, the threshold SNR is about 2 dB higher. Note that with PSA channel estimation the effective data rate is significantly reduced.

7. CONCLUSIONS

We developed adaptive channel predictors for OFDM transmission over time-varying mobile radio channels. The adaptive predictors do not require knowledge of the channel and noise statistics. With the RLS algorithm, almost optimum performance can be achieved. The NLMS algorithm has poorer performance but lower computational complexity.

REFERENCES