LETTER

A Stopping Criterion for Low-Density Parity-Check Codes*

Donghyuk SHIN†, Jeongseok HA†, Kyoungwoo HEO‡, Nonmembers, and Hyuckjae LEE†, Member

SUMMARY We propose a new stopping criterion for decoding LDPC codes which consists of a measure of decoder behaviors and a decision rule to predict decoding failure. We will show that the proposed measure, the number of satisfied check nodes, does not need (or minimizes) additional complexity, and the decision rule is efficient and more importantly channel independent, which was not possible in the previous work.

key words: LDPC codes, belief propagation, stopping criterion

1. Introduction

Recently, rate-compatible LDPC codes [1] have been introduced and there have been many attempts to incorporate the rate-compatible LDPC codes into the type-II Automatic Repeat reQuest (ARQ) protocol [2]. In such attempts, the decoding latency of iterative decoders becomes, in some cases, prohibitively longer since the decoder iterates for a maximum number of iterations. Such a practical issue motivates us to propose a stopping criterion for LDPC codes which predicts decoding failure instead of detecting successful decoding. If the decoder for LDPC codes predicts the decoding failure before reaching the maximum number of iterations, the receiver can more rapidly respond to the transmitted request for additional parity bits with the type-II ARQ protocol. Together with the better efficiency of the ARQ protocol, the receiver can save power consumption by avoiding the unnecessary iterations.

We find that there is a previous work [3] on the prediction of decoding failure which consists of two key elements: 1) a measure of decoding behaviors and 2) a decision rule. The measure of decoding behaviors is the changes of log-likelihood ratio (LLR) values during iterations. However, the previous work requires additional complexity since the LLR values are in a form of floating point or multi-bit resolution. Thus, in some cases, the power consumption due to the additional complexity for the prediction may offset the power saving from the early termination. The decision rule also needs a channel dependent parameter.

In this letter, we propose a simple stopping criterion to predict the decoding failure without additional complexity or at a minimized computational cost. Instead of LLR values, we propose to use the number of satisfied check nodes as a measure of decoder behaviors, which does not need additional complexity since the parity-check constraints are always tested in the BP decoding and the result of each parity-check test is a single bit, i.e., “success” or “failure.” In addition to the computational advantage, the proposed decision rule is channel independent and provides a better prediction performance than that of the previous work.

2. Stopping Criteria for LDPC Codes

2.1 The Behaviors of the BP Decoders at Finite Lengths

The behaviors of the BP decoding can be categorized into three different types in terms of the evolutions of LLR values at the variable nodes during iterations: 1) convergence, 2) stuck, and 3) oscillation. In the case of the convergence, the average magnitude of LLR (AM-LLR) values of coded bits is ever increasing at each iteration, and the decoder eventually has a valid codeword which is not necessarily to be the transmitted codeword. However, in the stuck case, the AM-LLR values are stuck to a fixed value after a certain number of iterations, and the decoder does not have a valid codeword. In the last case, the AM-LLR values commute between an increasing and decreasing phases. Thus, the decoder cannot have a valid codeword at any iteration. In some cases, the decoding behavior changes from the oscillation to the convergence. We call such a case slow convergence to tell it from the convergence. However, the slow convergence is categorized into the oscillation if the oscillation phase lasts longer than a preset maximum number of iterations and the decoding ends before the phase changes to the convergence. Thus, the decision of the slow convergences also depends on the maximum number of iterations. Our idea is about predicting the stuck and the oscillation in as a small number of iterations as possible. However, the slow convergence behavior makes it very hard to decide the decoding behavior in the early stage of the decoding.

We demonstrate the evolutions of the AM-LLR values at the variable nodes during decoding an LDPC code in the lower part of Fig. 1 where the four different types of the behaviors are clearly depicted. Especially, the slow convergence would have been categorized into the oscillation if the maximum iteration were set to less than 40.

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†The authors are with the School of Engineering, Information and Communications University, 119 Munjiro Yuseong-gu Daejeon 305-732, Korea.

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a) E-mail: hkw06@icu.ac.kr
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2.2 Previous Work

The stopping criterion in [3] is based on the observation of the variable node reliability (VNR) which is defined as

\[ VNR^{(\ell)} = \sum_{k=1}^{n} |q_{k}^{(\ell)}| \]

(1)

where \( 1 \leq k \leq n, 1 \leq \ell \leq N_{\text{max}}, N_{\text{max}} \) is a maximum number of iterations, and \( |q_{k}^{(\ell)}| \) is the magnitude of the LLR values of the \( k \)-th variable node at the \( \ell \)-th iteration obtained in Vertical Step [5] of the BP decoding.

At each iteration, the decoder decides the decoding failure based on the following decision rule:

**S1** If the VNR\(^{(\ell)}\) does not change or decreases within two successive decoding iterations, then stop the decoding process. That is, \( VNR^{(\ell-1)} \geq VNR^{(\ell)} \) for \( \ell > 1 \).

**S2** But, if VNR\(^{(\ell)}\) exceeds a threshold (VNR\(_{\text{off}}\)), S1 has to be switched off and a decoder proceeds to the next iteration. VNR\(_{\text{off}}\) is defined as \( MB \times n \) where \( n \) is the number of variable nodes, \( MB \approx 4 \times \left( \frac{E_{b}}{N_{0}} \right)_{\text{WR}} \) and \( \left( \frac{E_{b}}{N_{0}} \right)_{\text{WR}} \) is a SNR point around the waterfall region in the BER curve.

This decision rule is based on a fact that \( VNR^{(\ell-1)} \geq VNR^{(\ell)} \) indicates the stuck and oscillation. Even if the inequality holds, in S2, it assumes the slow convergence when \( VNR^{(\ell)} \geq VNR_{\text{off}} \).

2.3 Proposed Stopping Criterion

The measure of the decoding behavior VNR\(^{(\ell)}\) in (1) is acquired by summing the LLR values in a form of floating point or multi-bit resolution, which needs additional complexity. In this letter, we use a simpler measure, the number of satisfied parity-check constraints \( N_{\text{spc}} \).

\[ N_{\text{spc}}^{(\ell)} = m - 1^{T} \hat{H} \overline{e}^{(\ell)} = m - \sum_{i=1}^{m} s_{i}, \]

(2)

where \( \mathbf{1} \) is the all-one column vector of length \( m \), \( \mathbf{H} \) is parity-check matrix which consists of \( m \) rows and \( n \) columns, and \( \overline{e}^{(\ell)} \) is the decoded word of the BP decoding at the \( \ell \)-th iteration.

The proposed measure is a simple summation of the parity-check results in binary. Moreover, there is no additional complexity in computing \( N_{\text{spc}} \) since the parity-check test is inherently provided from Stopping Criterion Test [5] in the BP decoding.

To see that \( N_{\text{spc}} \) is an effective measure of the decoding behaviors, we compare the evolutions of \( N_{\text{spc}} \)'s during iterations with those of LLR values in Fig. 1 where the evolutions of \( N_{\text{spc}} \)'s exactly match with those of LLR values. In addition, \( N_{\text{spc}} \) needs a smaller number of additions than VNR\(^{(\ell)}\) does since \( m < n \), which becomes more distinctive at higher rates.

We also realize that the threshold VNR\(_{\text{off}}\) = \( n \times MB \) must be avoided since VNR\(_{\text{off}}\) is channel dependent. That is, for an LDPC code \( MB \) comes from evaluations of required \( E_{b}/N_{0} \) around the waterfall region in the BER curve on a channel. When the LDPC code is working on a different channel, the decision rule needs different \( MB \) which may not be possible in some applications.

The proposed stopping criterion consists of the simple measure (\( N_{\text{spc}} \)) of the decoding behaviors and a channel independent decision rule. The decision rule is as follows:

**S0** If \( \ell = 1 \), initialize \( c_{d} \) to zero \( (c_{d} \leftarrow 0) \).

**S1** Wait for the test of the parity-check constraints \( (\mathbf{H} \hat{e} = \mathbf{S}) \) at the \( \ell \)-th iteration of the BP decoding.

**S2** Compute \( N_{\text{spc}}^{(\ell)} \) with \( \mathbf{S} \).

**S3** If \( \ell > 1 \), compute \( d_{\text{spc}}^{(\ell)} = N_{\text{spc}}^{(\ell)} - N_{\text{spc}}^{(\ell-1)} \). Otherwise, go to S1.

**S4** If \( d_{\text{spc}}^{(\ell)} < \theta_{d} \), increase \( c_{d} \) by 1 \( (c_{d} \leftarrow c_{d} + 1) \). Otherwise, reset \( c_{d} \) to zero \( (c_{d} \leftarrow 0) \) and go to S1.

**S5** If \( c_{d} < \theta_{\text{max}} \), go to S1.

**S6** If \( N_{\text{spc}}^{(\ell)} \leq \theta_{\text{spc}} \), STOP the decoding. Otherwise, reset \( c_{d} \) to zero, \( c_{d} \leftarrow 0 \) and go to S1.

In S3, the proposed decision rule watches the increment \( (d_{\text{spc}}^{(\ell)}) \) of the number of satisfied parity-check constraints at each iteration. In S4, if the increment is less than a threshold \( \theta_{d} > 0 \), it counts \( (c_{d}) \) how long the small/negative increment successively persists. If \( c_{d} \) becomes bigger than a threshold \( \theta_{\text{max}} \), it stops the decoding when \( N_{\text{spc}}^{(\ell)} \) is less than a threshold \( \theta_{\text{spc}} \). If \( N_{\text{spc}}^{(\ell)} \) is bigger than \( \theta_{\text{spc}} \), it assumes the slow convergence and waits for the change of the decoding behavior from the oscillation to the convergence. In S4, \( c_{d} \geq \theta_{\text{max}} \) is a sign for the stuck or the downfall in the oscillation. Although different choices of the thresholds \( (\theta_{d}, \theta_{\text{max}}, \theta_{\text{spc}}) \) will give us different prediction performances, once a good choice of the threshold is selected, it will be channel independent.

Theoretical analysis of good choices of \( (\theta_{d}, \theta_{\text{max}}, \theta_{\text{spc}}) \) seems intractable due to the nonlinear behaviors of the BP.
decoding at short block lengths. However, we can intuitively understand that $\theta_d$ depends on how fast the BP decoding converges, which is determined by the degree distribution since higher degree nodes have faster convergence. It is well-known that BP decoding has slow and fast phases [6]. $\theta_{\text{max}}$ determines whether the decoder is merely in the slow phase or stuck. The slow convergence in Fig. 1 is very unpredictable and to our best knowledge, $\theta_{\text{spc}}$ can be determined only by simulations as $VNR_{\text{off}}$ in [3].

3. Simulation Results

In [3], the performance of the stopping criterion is evaluated in terms of frame-error rate (FER), average number of iterations (ANI), and false-alarm rate (FAR). The FAR is defined as the rate of terminated decoding which would have been successfully decoded if a stopping criterion were not applied. Actually, ANI can be made arbitrary small, if a stopping criterion terminates all suspicious decoding at an early stage of the iterations, which however, results in unnecessary retransmissions in ARQ protocols. Thus, we evaluate the performances of FAR and ANI at the same time and find that the two performance measures are complementary. That is, if we set a required FAR low, then ANI becomes higher and vice versa.

The designed LDPC code in Fig. 1 is evaluated on an AWGN channel where the stopping criteria are incorporated into the decoding and the maximum number of iterations ($N_{\text{max}}$) is set to 50. In this experiment, we evaluate ANIs and FARs at different $E_b/N_0$ points, and the results are depicted in Fig. 2 where $\theta_d$, $\theta_{\text{max}}$, and $\theta_{\text{spc}}$ are selected from numerous observations of $N_{\text{spc}}$. That is, we perform many decodings and observe the evolutions of $N_{\text{spc}}$ during iterations in which we can determine each parameter independently since each of them describes an independent behavior of the BP decoding. As aforementioned, the theoretical analysis of the parameters is intractable but we claim that the parameters depend only on the LDPC code structure and decoding algorithm. Thus, once we have the parameters, it will be valid thereafter, which will be shown soon.

The simulation results clearly show that the FARs with the proposed criterion outperform those with the previous work. However, in the case of ANI, it seems that the previous work shows better performance, which may mislead readers into believing better performance of the previous work. As aforementioned, the smaller ANIs with the previous work is due to higher FARs. In the comparison between the decoding with/without the proposed stopping criterion, we can conclude that the proposed stopping criterion saves significant amount of iterations as compared to the conventional decoding algorithm together with lower FARs than those of the previous work.

Since the performances of the proposed and previous criteria depend on the choices of parameters, we are interested in the variations of FARs and ANIs with different choices of parameters. In this experiment, we try to find the best choice of parameters for each ANI which minimizes FAR. We depict the simulation results in Fig. 3, where $a:b:c$ indicates a range of numbers from $a$ to $c$ by step $b$. We also find the best choice of parameters for the previous work.

Again, we can see the complementary behavior of ANI and FAR in Fig. 3. However, the proposed stopping criterion shows better performances when the FAR is less than $10^{-3}$.

In Fig. 4, we compare the ANIs of the BP decoding with the proposed and previous stopping criteria on an AWGN and a fading channel where all coded symbols have independent channel gains which follow a Rayleigh distribution with unit power gain. The comparisons show that the proposed stopping criterion is channel independent. However, the results from the previous stopping criterion have much variations which is due to the channel dependent parameter, $MB$. 

![Fig. 2](image1.png)  
**Fig. 2** False alarm rate and average number of iterations.

![Fig. 3](image2.png)  
**Fig. 3** Performance of the proposed and previous stopping criteria with different choices of parameters.
4. Conclusion

We propose a stopping criterion made up of a simple measure of decoding behaviors and a channel independent decision rule to predict decoding failure of LDPC codes. In addition to the computational advantage of the proposed measure, the proposed decision rule is channel independent which is very important in practical situations. In performance comparisons we show that the proposed stopping criterion significantly lowers ANI as compared to the conventional BP decoding together with better FARs than those of the previous work. Such lower FARs and smaller number of iterations provide better efficiency of ARQ protocols.

References