

Non-perturbative propagators and dimension 2 condensate in Yang-Mills theory*

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We have found ultraviolet asymptotic solutions of the Schwinger-Dyson equation for the gluon and ghost propagators which have simultaneously the perturbative logarithmic correction and the non-perturbative $1/p^2$ power correction. By including the perturbative corrections, the power correction reproduces exactly the leading OPE result suggesting the existence of dimension two condensate.

1. Introduction

Various condensates are related to non-perturbative properties of QCD. The importance of the condensates such as $\langle FF \rangle$ and $\langle \bar{q}q \rangle$ are well-known.

Though the dimension two operators such as mass terms are not BRST invariant in Yang-Mills (YM) theory, it was pointed out recently that these operators contain a gauge invariant physical part and that a special combination of them can be (on-shell) BRST invariant [1] [2] [3].

It was argued that the minimum of A^2 along the gauge orbit can have a definite physical meaning, and how to define the physical part non-perturbatively [1].

There is a BRST-invariant composite operator of mass dimension 2 [2] as a linear combination of A^2 and quadratic ghost, averaged in spacetime:

$$\mathcal{O} = \Omega^{-1} \int d^4x \operatorname{tr} [12A^\mu(x)A_\mu(x) + \lambda i\bar{C}(x)C(x)]. \quad (1)$$

In the Lorentz gauge, it reduces to the same form as the known Curci-Ferrari mass term. But we do not include it in the Lagrangian. Therefore our

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theory is usual YM.

It is easily seen that this operator is on-shell BRST invariant: $\delta_{BRST}\mathcal{O} = 0$. The A^2 part should be divided into the physical (transverse) part and other unphysical (longitudinal and scalar) part. When we take the vacuum expectation value, the $\bar{C}C$ part precisely cancels this unphysical part. Thus the operator has a gauge invariant expectation value, though the remaining physical part is nonlocal. Especially in the Landau gauge $\lambda \rightarrow 0$, $\langle \mathcal{O} \rangle \rightarrow \langle 12A^\mu(0)A_\mu(0) \rangle$.

Since these operators are not BRST invariant as local polynomials, they do not appear in OPE of usual gauge invariant quantities. But they may appear in OPE of BRST non-invariant quantities such as propagators.

In fact, this operator in the Landau gauge has been estimated in various methods. Boucaud et al.[3] have simulated the lattice propagator and used OPE fit to obtain $\langle A^2 \rangle \simeq (1.4\text{GeV})^2$. Verschelde et al. [4] discussed the effective potential of A^2 .

Here we take the Schwinger-Dyson (SD) equation approach. Recent investigations [5][6] of Euclidean pure $SU(N_c)$ Yang-Mills theory in the

ters will be automatically determined by considering subleading solutions and OPE consistency.

We put these into SD equations, and get infinite series of algebraic equations with respect to coefficients $A, B, \omega, \gamma, \delta, c_n, d_n, a^{(1)}, b^{(1)}, \gamma_1, \delta_1$ and parameters a, b .

5. Result

We first find that without power corrections ($a^{(1)} = 0, b^{(1)} = 0$) the ansatz cannot satisfy the coupled SD equation for large p^2 , even if we include the logarithmic corrections $c_n, d_n \neq 0$.

A self-consistent solution is obtained when

$$\begin{aligned} \gamma &= -13/22, \delta = -9/44 & (5) \\ \omega &= \frac{11}{3}N_c\lambda\tilde{Z}_1AB^2 = \frac{11}{3}N_c\lambda = \beta_0 \quad (\lambda := \frac{g^2}{16\pi^2}) \end{aligned}$$

for arbitrary value of a and b .

These reproduce leading exponents and beta function in agreement with 1-loop perturbation.

The remaining coefficients are determined successively except for one degrees of freedom ($a+b$).

$$\begin{aligned} (a+b) &= -\frac{421}{126} + \frac{9152}{945}d_1, \\ \gamma + \gamma_1 &= -1.28 - \frac{9}{88}(a+b), \quad \delta_1 = \gamma_1 - 1, \\ b^{(1)} &= \frac{9N_c}{8}\lambda\tilde{Z}_1AB^2a^{(1)}, \\ c_1 &= \frac{35}{24}N_c\lambda + \frac{26}{9}d_1, \\ d_2 &= (4202070705N_c\lambda^3 - 958366464N_c\lambda^2d_1 \\ &\quad + 6683578880N_c\lambda d_1^2 - 39086260224d_1^3) \\ &\quad / (304128(645N_c\lambda - 43648d_1)), \\ c_2 &= \dots \\ &\vdots \end{aligned} \quad (6)$$

Coefficients A and B (overall normalization) will be determined if we fix the renormalization condition. $a^{(1)}$ and $b^{(1)}$ (coefficients of power correction) are not determined. Except for d_1 (or $a+b$), all the other coefficients and exponents are calculated by simple algebraic equations up to any finite orders. The logarithmic expansions seem to converge in UV region.

6. Comparison with OPE result

The additional logarithmic exponent of the power correction calculated from OPE is $-(1 -$

$\hat{\gamma}_0\beta_0) = -35/44 = -0.795$. Corresponding exponent from the SD solutions in bare vertex approximation is $\gamma + \gamma_1 = -0.935$ [9].

In the improved vertex case (6), only one unknown parameter ($a+b$) governs this exponent. We use the relation (6) to determine the parameter. When we set $a+b = -4.778$ (or $d_1 = -0.148$), the exponent $\gamma + \gamma_1$ becomes -0.795 which reproduces the leading OPE. Moreover, this value for $a+b$ leads to the ghost dominance solution in the IR region.

7. Conclusion

We have found the asymptotic SD solutions of the gluon and ghost propagators in $SU(N)$ Yang-Mills in Landau gauge for the ansatz using power and logarithmic expansion.

“Non-perturbative” $O(1/p^2)$ power corrections are necessary. This corresponds to dimension two vacuum condensates [1-3].

Contrary to the fermion SD case, the bare vertex approximation cannot reproduce the UV solution consistent with the UV perturbation and the UV OPE. We have found the consistent solution by considering the UV higher order corrections.

Using the same vertex ansatz (4) determined as above, we have obtained also the IR solutions which have the same form as the power-law solutions [6]: $F(p^2) \simeq A \cdot (p^2)^{1.54}$, $G(p^2) \simeq B \cdot (p^2)^{-0.77}$. Therefore the existence of non-perturbative power correction does not influence the IR solution. IR ghost dominance is realized, and the color confinement criterion of Kugo and Ojima is satisfied.

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