Error-rate analysis of switched diversity receivers in Weibull fading

N.C. Sagias, G.K. Karagiannidis and G.S. Tombras

Novel expressions for the average symbol error probability (ASEP) in switched and stay combining receivers operating over Weibull fading channels have been derived. Closed-form expressions for the optimum switching threshold in the sense of maximum ASEP have also been obtained. The single-branch receiver performance in the Weibull channel is investigated as a special case.

Introduction: Diversity is an effective and widely used technique for mitigating the effects of multipath fading [1]. Among the well-known diversity techniques, switched and stay combining (SSC) is simple and cheap, though less efficient. In a dual-branch SSC receiver, if the instantaneous signal-to-noise ratio (SNR) of the first branch falls below a predefined switching threshold, the second branch is immediately selected, regardless of whether or not the SNR of that branch is above or below the predetermined threshold.

The Weibull fading channel model is a flexible model providing good fit to experimental wireless channel measurements for both indoor [2] and outdoor [3] environments. Considering the performance of diversity receivers over Weibull fading channels, in [4] a study on dual-branch selection combining (SC) receiver performance over correlative Weibull fading has been presented. Another contribution related to the performance analysis of SSC receivers in Weibull fading has been presented in [5]. In [5], useful formulas for statistical parameters of the dual-branch SSC output SNR, such as the cumulative distribution function (cdf), the probability density function (pdf) and the moments have been derived. Capitalising on these expressions, the average output SNR, the amount of fading (AoF), the outage probability, the optimum switching threshold for maximum SNR and the switching rate (SR) of the combiner have been also obtained in closed forms. However, the error performance analysis was not presented in [5] due to space limitations.

In this Letter, the symbol error rate performance of dual-branch SSC receivers in a Weibull fading environment is studied. Analytical expressions for the average symbol error probability (ASEP) for several binary and multilevel modulation schemes are extracted and the common switching threshold for maximum ASEP is obtained in closed form. The single-branch receiver performance in the Weibull channel is also investigated. Finally, selected numerical examples are presented to point out the effect of the fading severity and the switching threshold on the receiver's performance.

System and channel model: We consider a dual-branch SSC receiver operating in a flat Weibull fading environment. The pdf and the cdf of the instantaneous SNR per symbol in the ℓth input branch γ_ℓ, $\ell=1$ and 2, are [4]

$$f_{\gamma_\ell}(\gamma_\ell)=\frac{\beta_\ell}{2a_\ell\bar{\gamma}_\ell}\left(\frac{\gamma_\ell}{a_\ell\bar{\gamma}_\ell}\right)^{(\beta_\ell/2)-1}\exp\left[-\left(\frac{\gamma_\ell}{a_\ell\bar{\gamma}_\ell}\right)^{\beta_\ell/2}\right] \tag{1}$$

and

$$F_{\gamma_\ell}(\gamma_\ell)=1-\exp\left[-\left(\frac{\gamma_\ell}{a_\ell\bar{\gamma}_\ell}\right)^{\beta_\ell/2}\right] \tag{2}$$

respectively, where $\bar{\gamma}_\ell$ is the corresponding average input SNR, $a_\ell=1/\Gamma(1+2/\beta_\ell)$, with $\Gamma(\cdot)$ being the gamma function ([6], equation (8.310/1)) and β_ℓ is the Weibull fading parameter ($\beta_\ell>0$). As β_ℓ increases, the severity of the fading decreases, while for $\beta_\ell=2$, (1) reduces to the well-known Rayleigh pdf. To the best of the authors' knowledge, the moment generating function (mgf) of (1) $\mathcal{M}_{\gamma_\ell}(s)\triangleq E\langle\exp(-s\gamma_\ell)\rangle(E\langle\cdot\rangle$ denotes expectation) cannot be analytically extracted using tables included in classical reference books, such as in [6]. The problem arises because a non-tabulated type of integral is present. However, this type of integral can be efficiently expressed in closed form using [7]. Thus, the mgf of the SSC output SNR can be obtained as

$$\mathcal{M}_{\gamma_\ell}(s)=\frac{\beta_\ell}{2}\frac{k^{1/2}l^{(\beta_\ell-1)/2}(a_\ell\bar{\gamma}_\ell s)^{-(\beta_\ell/2)}}{(2\pi)^{((k+l)/2)-1}}\mathrm{G}^{k,l}_{l,k}\left[\frac{l^l k^{-k}}{(a_\ell\bar{\gamma}_\ell s)^l}\middle|\begin{matrix}\frac{1-(\beta_\ell/2)}{l},\frac{2-(\beta_\ell/2)}{l},\ldots,\frac{l-(\beta_\ell/2)}{l}\\ 0,\frac{1}{k},\ldots,\frac{k-1}{k}\end{matrix}\right] \tag{3}$$

where k and l are positive integers so that $l/k=\beta_\ell/2$ and G[$\cdot$] is the tabulated Meijer's G-function ([6], equation (9.301)), available in most of the well-known mathematical software packages, such as Mathematica, Matlab or Maple. Depending on the value of β_ℓ, a couple of minimum values of k and l can be properly chosen (e.g. for $\beta_\ell=3.5$ we have to choose $k=4$ and $l=7$). For the special case where β_ℓ is an integer, $k=2$ and $l=\beta_\ell$. Note that, using (3) and the well-known mgf-based approach ([1], chap. 1), the ASEP in Weibull fading of the single-branch receiver can be obtained for several binary signalling constellations, such as (differential) phase shift keying ((D)PSK), (non-coherent) frequency shift keying ((N)FSK), FSK and M-ary signalling constellations, such as amplitude modulation (M-AM), M-PSK, quadrature amplitude modulation (M-QAM) and M-DPSK.

Error-rate analysis of SSC: Similarly to the ASEP analysis for the single-branch receivers, we evaluate the average error rates of SSC receivers for several well-known modulation schemes using the mgf approach. The pdf of the SSC instantaneous output SNR per symbol γ_{out} is given by [5]

$$f_{\gamma_{out}}(\gamma_{out})=\begin{cases}\frac{P_1P_2}{P_1+P_2}[f_{\gamma_1}(\gamma_{out})+f_{\gamma_2}(\gamma_{out})], & \gamma_{out}\le\gamma_\tau\\ \frac{P_2(1+P_1)}{P_1+P_2}f_{\gamma_1}(\gamma_{out})+\frac{P_1(1+P_2)}{P_1+P_2}f_{\gamma_2}(\gamma_{out}), & \gamma_{out}>\gamma_\tau\end{cases} \tag{4}$$

where γ_τ is the common switching threshold and $P_\ell=F_{\gamma_\ell}(\gamma_\ell=\gamma_\tau)$. Using (3) and (4), the corresponding mgf of the SSC instantaneous output SNR can be obtained as

$$\begin{aligned}\mathcal{M}_{\gamma_{out}}(s)=&\frac{P_2(1+P_1)}{P_1+P_2}\mathcal{M}_{\gamma_1}(s)+\frac{P_1(1+P_2)}{P_1+P_2}\mathcal{M}_{\gamma_2}(s)\\&-\frac{P_2}{P_1+P_2}\int_0^{\gamma_\tau}\exp(-s\gamma)f_{\gamma_1}(\gamma)d\gamma\\&-\frac{P_1}{P_1+P_2}\int_0^{\gamma_\tau}\exp(-s\gamma)f_{\gamma_2}(\gamma)d\gamma\end{aligned} \tag{5}$$

Equation (5) includes finite integrals, which can be easily evaluated via numerical integration. The main advantage of the mgf-based approach is that, for the binary non-coherent NFSK and DPSK modulation schemes, the ASEP $\bar{P}_e$ can be calculated directly using simple expressions for the mgf. For example, for DPSK $\bar{P}_e=0.5\mathcal{M}_{\gamma_{out}}(1)$. Furthermore, for other modulation schemes, such as M-AM, M-PSK, M-QAM, FSK and M-DPSK, single integrals with finite limits and integrands composed of elementary (exponential and trigonometric) functions can be readily evaluated via numerical integration. The aforementioned expressions for the ASEP, using the mgf-based approach, are summarised in [1], chap. 8. Note that, for $\beta_1=\beta_2=2$, (5) reduces to the mgf of the well-known Rayleigh channel model ([1], equation (9.273)), while for independent and identically distributed (i.i.d.) input branches ($\beta=\beta_\ell$, $a=a_\ell$ and $\bar{\gamma}=\bar{\gamma}_\ell$) simplifies to

$$\mathcal{M}_{\gamma_{out}}(s)=(1+P)\mathcal{M}_\gamma(s)-\int_0^{\gamma_\tau}\exp(-s\gamma)f_\gamma(\gamma)d\gamma \tag{6}$$

where $P=P_\ell$, $\mathcal{M}_\gamma(\cdot)=\mathcal{M}_{\gamma_\ell}(\cdot)$ and $f_\gamma(\cdot)=f_{\gamma_\ell}(\cdot)$.

The common optimum switching threshold γ_τ^* for maximum average error probability can be obtained as

$$\left.\frac{d\bar{P}_e}{d\gamma_\tau}\right|_{\gamma_\tau=\gamma_\tau^*}=0 \tag{7}$$

Despite the fact that closed-form expressions for γ_τ^* with specific signalling constellation and i.i.d. input branches can be easily obtained, a unified expression, including all the modulation schemes under consideration, cannot be extracted. Thus, as an example, a closed-form expression of γ_τ^* is given for DPSK and NFSK signalling with i.i.d. input branches. Using (6) and (7) and after some straightforward mathematical manipulations, γ_τ^* can be obtained in closed form as

$$\gamma_\tau^* = -\frac{1}{B}\ln[\mathcal{M}_\gamma(B)] \qquad (8)$$

where for DPSK $B=1$ and for NFSK $B=0.5$. If the average input SNRs are not identically distributed, the optimum switching threshold for maximum ASEP can be derived using numerical methods, available in most of the well-known mathematical software packages.

Numerical results: As an indicative example, the average bit error probability (ABEP) of DPSK signalling is presented in Fig. 1, using (6) for several values of β and for i.i.d. input branches. Three curves with different values of β for the ABEP are plotted against average input SNR (bottom axis) operating the SSC with the common optimum switching threshold. The performance evaluation results obtained show that the bit error performance improves with an increase of β. In the same Figure, the ABEP is also plotted against the common switching threshold (top axis) for $\bar{\gamma}=5$ dB and several values of β. It is easily recognised that in each curve a minimum, given by (8), appears. Furthermore, as the fading severity increases (i.e. β decreases) the optimum switching threshold decreases.

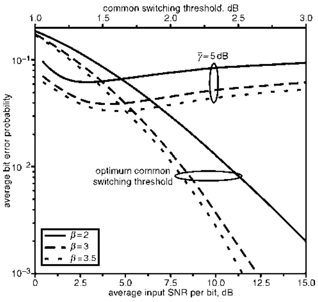

Fig. 1 *ABEP performance of DPSK signalling against common switching threshold (top axis) and average input SNR per bit (bottom axis) for several values of β*

Conclusions: Useful expressions for the ASEP of SSC receivers in a Weibull fading environment have been derived. The optimum common switching threshold for maximum ASEP is obtained in closed form for DPSK and NFSK signalling. The single-branch receiver error-rate performance over Weibull channels has also briefly been described.

© IEE 2004 5 February 2004

Electronics Letters online no: 20040479
doi: 10.1049/el:20040479

N.C. Sagias and G.S. Tombras (*Laboratory of Electronics, Physics Department, University of Athens, Panepistimiopolis, 15784, Athens, Hellas, Greece*)

E-mail: nsagias@space.noa.gr

G.K. Karagiannidis (*Institute for Space Applications and Remote Sensing, National Observatory of Athens, Metaxa & V. Pavlou, P. Penteli, 15236 Athens, Hellas, Greece*)

References

1 Simon, M.K., and Alouini, M.-S.: 'Digital communication over fading channels' (Wiley, New York, 2000, 1st edn.)

2 Hashemi, H.: 'The indoor radio propagation channel', *Proc. IEEE*, 1993, **81**, pp. 943–968

3 Adawi, N.S., *et al.*: 'Coverage prediction for mobile radio systems operating in the 800/900 MHz frequency range', *IEEE Trans. Veh. Technol.*, 1988, **37**, (1), pp. 3–72

4 Sagias, N.C., Karagiannidis, G.K., Zogas, D.A., Mathiopoulos, P.T., and Tombras, G.S.: 'Performance analysis of dual selection diversity in correlated Weibull fading channels', *IEEE Trans. Commun.*, (to be published)

5 Sagias, N.C., Zogas, D.A., Karagiannidis, G.K., and Tombras, G.S.: 'Performance analysis of switched diversity receivers in Weibull fading', *Electron. Lett.*, 2003, **39**, (20), pp. 1472–1474

6 Gradshteyn, I.S., and Ryzhik, I.M.: 'Table of integrals, series, and products' (Academic, New York, 1994, 6th edn.)

7 Adamchic, V.S., and Marichev, O.I.: 'The algorithm for calculating integrals of hypergeometric type functions and its realization in REDUCE system'. Proc. Int. Conf. on Symbolic and Algebraic Computation, Tokyo, Japan, 1990, pp. 212–224