Block Relay for Physical-Layer Network Coding

Ning Xu, Shengli Fu  
Dept. of Electrical Engineering  
University of North Texas, USA  
Email: ningxu,fu@unt.edu

Dong Wang  
Wireless Comm. & Networking Dept.  
Philips Research North American  
Email: dong.wang@philips.com

Yabo Li  
Dept. of Info. Sci & EE  
Zhejiang University, China  
Email: yaboli@zju.edu.cn

Abstract—In this paper, we study the transmission strategy in a two-way relay channel. In particular, we propose a new class of $\log_{M_1}(M_2)$-MA (multi-access) block relay protocols where an $M_1$-ary modulation is used in the MA phase, and an $M_2$-ary in the broadcast phase. Lower-order (e.g., BPSK) and higher-order (e.g., 16-QAM, 32-PSK) modulations are used respectively for the multi-access phase and the broadcast phase, in contrast to the conventional method of using the same modulation type (of either higher or lower order) throughout. To achieve this, the relay buffers the received symbols from consecutive multi-access phases and performs signal combining in conjunction with the decode-and-forward protocol. We derive the theoretical performance bounds for the Rayleigh fading channels and closed forms for the AWGN channels. Numerical results confirm the advantages of the proposed scheme with an improved throughput in lower signal-to-noise ratio range.

Index Terms—Network coding, modulation, packet error rate, throughput, two-way relay channel

I. INTRODUCTION

The topic of two-way relay channels (TWRC) has attracted significant attention recently. In a TWRC system, terminals as transceiving entities, exchange information through the relay. To achieve the spectral efficiency [1], terminals send symbols to the relay simultaneously first, and then the relay broadcasts the processed symbols back to the terminals. By coordinating the terminals for simultaneous transmission and using the additive nature of the electromagnetic waves, we can achieve equivalent network coding operations at the physical layer [2]. This technique is the physical layer network coding (PLNC). Instead of treating the received symbols from other terminals as interference, PLNC turns it into a capacity boost.

To our best knowledge, in most existing literatures, the same modulation is assumed throughout all stages of the PLNC transmission. There are several recent works on improving the throughput by using more elaborate modulations [3]–[9]. Liu et al. discussed the superimposed XOR, a hybrid of the bitwise XOR and the symbol-level superposition coding, for asymmetric channels [3]. It addressed the issue of applying PLNC in a system where terminals transmit information at different rates. In [4], [5], Koike-Akino et al. investigated a modulation optimizing scheme by way of sphere packing on the denoising map of the relay. [6] proposed non-uniform constellations to lower the complexity of PLNC with high-order PAMs by using binary codes. [7] investigated the constellation design of the differential modulation for PLNC. It proves to be beneficial especially for systems with higher data rate requirements. Yun et al. presented a physical-layer retransmission scheme that applies coding on the packet level to reduce retransmission cost [8]. This approach piggybacks a new packet on a retransmitted packet by using higher modulations, and recovers both by exploiting previously received packets. A novel mapping codebook-based physical network coding scheme is proposed in [9] for asymmetric two-way relay channels where source nodes exchange data in different flow rates. A mapping codebook contains several subcodebooks to be adaptively selected based on the signal phase difference to improve the system performance.

Our work tackles the problem from a different angle by exploiting the use of constellations of different orders during different phases. We propose using higher-order constellations (e.g., 16-QAM, 32-PSK) during the broadcast stage while lower-order (e.g., BPSK) during the multi-access stage and investigate its impact on the overall throughput. The rest of the paper is organized as follows. Section II gives a detailed description of the system model. In Section III, the theoretical performance in terms of throughput is discussed, for both fading and Gaussian channels. Numerical results are presented in Section IV, followed by the conclusion in Section V.

II. SYSTEM MODEL

In this section, we describe in detail the proposed $\log_{M_1}(M_2)$-MA (multi-access) block relay scheme, where an $M_1$-ary modulation is used in the MA phase, and an $M_2$-ary in the broadcast phase. To start off with an example, Fig. 1 shows three relaying protocols in a two-way relay system, where packets of $L = 4$ (we use 4 here for simplicity while a typical packet size can be 128 or more) bits are exchanged. Fig. 1(a) carries out the transmission bit by bit using the BPSK modulation. Utilizing the spectral efficient method [1], a single round of two-way communication can be accommodated in two time slots. Two terminals ($T_1, T_2$) utilize exactly one time slot to transmit their symbols ($X_1, X_2$) to the relay ($R$) simultaneously. This is typically known as the MA phase. The next time slot is dedicated for the relay to broadcast a new symbol ($\hat{X}_R$) back to $T_1$ and $T_2$. This is the broadcast or BC phase. The approach of obtaining $\hat{X}_R$ is discussed later in this section. Denoting the duration of a single time slot as $T$, the total time to exchange two packets is $2T \times 4 = 8T$.

Now in order to improve the throughput (we will come back to its formal definition in a moment) or in another way, to have more packets correctly received at the receiver side within a
certain period of time, an attempt is made by using a higher-order modulation in Fig. 1(b). In this approach, the four bits inside a packet can be encapsulated into a 16-QAM symbol to be transmitted at the same time, and the total transmission time is shortened to $2T$. However, this only becomes beneficial with sufficiently high signal-to-noise ratios (SNRs), as it is known from analysis in previous works, such as [10], [11], that higher-order modulations statistically incur more errors than the lower-order under the same SNR constraint. Especially, this disadvantage results in excessive retransmissions in the low SNR range, which potentially cancels out the advantage of the shorter transmission time per packet.

Fig. 1(c) gives an example of our proposed method $\log_{M_1} (M_2)$-MA where $M_1 = 2, M_2 = 16$ in this case. BPSK is used in the MA phase. BPSK symbol pairs are transmitted to the relay in $\log_2 16 = 4$ consecutive slots. The relay combines them into a 16-QAM symbol ($X_R$) and broadcasts it back to $T_1, T_2$ using one slot. The transmission time totals $4T + T = 5T$. This approach shortens the per-packet transmission time compared to Fig. 1(a), and achieves a better error rate compared to Fig. 1(b). It proves to be the best of the three in terms of throughput in the lower SNR range. We will get into the details in Section III.

$$U[m, t] = h_1[m, t]\sqrt{P_1}X_1[m, t] + h_2[m, t]\sqrt{P_2}X_2[m, t] + n_R[m, t],$$

where $U[m, t]$ is the received symbol, $X_k[m, t]$ is the transmitted symbol from $T_k$, $P_k$ is the transmission power of $T_k$, $h_k[m, t]$ is the channel coefficient of link $T_k \rightarrow R$, and $n_R[m, t]$ is the additive white Gaussian noise (AWGN) with zero mean and $\sigma^2$ variance. Note that when $h_1 = h_2 = 1$, the system is an AWGN channel.

During each transmission, the channel state information is assumed known to the receiver. Additionally, we assume that transmitted symbols are backlogged at $T_1, T_2$ and $R$ respectively and can be fetched whenever polled. For simplicity, we only consider BPSK in the MA phase here. The discussion hereafter generally can also be extended to combining other types of modulation.

The PLNC protocol is employed for processing at the relay $R$. Instead of decoding $X_1[m, t], X_2[m, t]$ individually, we obtain the combination $X_1[m, t] + X_2[m, t]$, and map it back to an information bit. For more detailed discussions on PLNC, we recommend [1], [2] for references. After obtaining $\log_{M_1} (M_2)$ consecutive bits from the pairs of $X_1[1, t] + X_2[1, t], \ldots, X_1[\log_{M_1} (M_2), t] + X_2[\log_{M_1} (M_2), t]$, $X_R[t]$ is formed by combining them bitwisely into a $M_2$-ary symbol. In the BC slot that follows, $R$ broadcasts $X_R[t]$ to $T_1$ and $T_2$. Once again, take the log$_2$ 16-MA in Fig. 1(c) as an example. As in [2], $X_1[m, t] + X_2[m, t]$ can be -2, 0, 2 for BPSK. -2 and 2 are mapped to 0 while 0 is mapped to 1. Suppose, after decoding in $\log_{M_1} (M_2) = \log_2 16 = 4$ consecutive MA slots, we have $X_1[m, t] + X_2[m, t]$ being -2, 2, 0, 0 respectively. The information bits are 0, 0, 1, 1. Thus we have the symbol 0011 with the $m$-th bit taken from the $m$-th slot. We modulate it using 16-QAM, and proceed to the BC phase transmission.

The received symbol at $T_k$ at the end of the $t$-th BC slot
\[ Y_{k}[t] = h_{k}[t] \sqrt{P_{R} X_{R}[t]} + n_{k}[t], \]  
where \( P_{R} \) is the transmission power of \( R \). Eventually, the \( m \)-th bit from the other terminal can be obtained from both \( X_{k}[t] \) and the \( m \)-th bit of the decoded \( Y_{k}[t] \).

For a fair comparison, we allow a per-packet power constraint. To be specific, we allow a constant total power for the terminals and the relay altogether to transmit a packet from end to end. This total power per packet can be written as \( \frac{3}{2}LP \). Note that, every round of MA and BC communication, two packets are exchanged. Here we used \( \frac{3}{2} \) instead of 1 to avoid introducing too many fractions in the analysis. If equally allocated, the power of \( T_{1}, T_{2}, R \) is \( LP \). Then the power for per-bit transmission is \( P \). For example, in Fig. 1(c), BPSK is used in the MA phase, and its symbol (one bit) has a power of \( P \). As for the BC phase, 16-QAM is used, and its symbol (four bits) power is \( 4P \). For the purpose of simplicity, equal allocation is assumed hereafter unless explicitly specified otherwise.

III. PERFORMANCE ANALYSIS

We choose the normalized end-to-end throughput as the primary performance metric for the proposed \( \log_{M_{1}} (M_{2}) \)-MA block relay scheme. Previously, we investigated the performance of PLNC, in particular when using BPSK modulations [13], [14]. Here, we focus on the case of \( M_{1} = 2 \) when BPSK is used in the MA phase and higher-order modulations in the BC phase. Whenever not causing confusion, we ignore the subscript of \( M_{2} \) and use \( \log_{2} M \) instead. In this section, the theoretical performance bounds are derived for the Rayleigh fading channels and the exact closed forms are obtained for the AWGN channels. Furthermore, we discuss the optimal power allocation between the terminals and the relay under a global power constraint so as to maximize the overall throughput.

Denote \( P_{p}, R_{p} \) as the end-to-end packet error rate (PER) and the end-to-end throughput, respectively. As mentioned in Section II, the throughput is the size of a packet over the average time of correctly transmitting and receiving it, assuming ideal retransmissions whenever an error occurs [12]. We use a slightly different approach here, as the data rates in our investigated systems are modulation-dependent. Instead, we use the rate of BPSK as the nominal data rate, which is denoted as \( R_{s} = L/T_{s} \) (again \( T_{s} \) is the packet duration using BPSK). The packet duration using \( M \)-ary modulations is \( T_{s} \log_{2} M \). With ideal retransmission, the average time to transmit a packet from \( T_{1} \) to \( T_{2} \) using \( \log_{2} M \) is \( T_{s} + T_{r} / \log_{2} M \) \( 1 - P_{p} \) [12]. The same is true for the link \( T_{2} \rightarrow T_{1} \) due to the system symmetry. Therefore, the throughput can be written as

\[
R_{p} = 2L/( \log_{2} M + 1 \cdot T_{s} (1 - P_{p}) ),
\]

Note that, we used \( 2L \) rather than \( L \) in Eq. (3) because transmissions \( T_{1} \rightarrow T_{2} \) and \( T_{2} \rightarrow T_{1} \) take place simultaneously. Now we can express the throughput \( R_{p} \) in terms of the BPSK data rate \( R_{s} \). As the system is symmetric, and derivation for either side is sufficient. Without loss of generality, we consider \( T_{2} \rightarrow T_{1} \). We only need to derive \( P_{p} \) and \( R_{p} \) can be computed through Eq. (3).

A. Rayleigh Fading Channels

We start with the Rayleigh fading channels. As mentioned in Section II, there are \( \log_{2} M \)-MA slots associated with every BC slot. Also, a packet can be exactly divided into \( M \)-ary modulated symbols. The PER can be expressed in terms of the symbol error rate (SER). Furthermore, in Gray-coded MPSK, the bit error rate (BER) follows the independent and identical distribution (i.i.d.) regardless of the bit position [11], [15]. For the square QAM, the BER can be computed given the bit position [10]. Thus, we can further break down PER and express it through the BERs of its every bit in the MA and BC phases.

Following Eq. (1), we use \( t \) \( (1 \leq t \leq L / \log_{2} M) \) as the index of a BC slot, and for a MA slot associated with it, \( t \) and \( m \) \( (1 \leq m \leq L / \log_{2} M) \). Given that a bit of a BC symbol is obtained from decoding the combination of two corresponding MA symbols, we also use \( m \) to index the bits in a BC symbol. Let \( P_{ma}[m,t] \) be the BER of the \( m \)-th bit, \( t \)-designated MA slot, and \( P_{bc}[m,t] \) be the BER of the \( m \)-th bit of the \( t \)-th received symbol in the BC phase. Then we have

\[
P_{p} = 1 - \prod_{t=1}^{L / \log_{2} M} \prod_{m=1}^{L / \log_{2} M} [P_{ma}[m,t]P_{bc}[m,t]]^{(1-P_{ma}[m,t])[1-P_{bc}[m,t]]}. \tag{4}
\]

For all \( t \)'s, \( P_{ma}[m,t] \) is subject to i.i.d., and so is \( P_{bc}[m,t] \). We can omit the parameter \( t \) and simplify Eq. (4) to

\[
P_{p} = 1 - \prod_{m=1}^{L / \log_{2} M} [P_{ma}[m]P_{bc}[m]]^{(1-P_{ma}[m])[1-P_{bc}[m]]^{\log_{2} M}}. \tag{5}
\]

All \( P_{ma}[m] \) follows the i.i.d. As our previous works [13] indicate, \( P_{ma}[m] \) can be tightly upper bounded by \( \hat{P}_{ma} \) given in

\[
\hat{P}_{ma} = \frac{1}{2(\gamma_{ma} + 1)}, \tag{6}
\]

where \( \gamma_{ma} \) is the SNR per symbol. For BPSK, \( \gamma_{ma} = P / N_{0} \) \( (N_{0} \) is the noise spectral density). As mentioned in Section II, \( T_{1}, T_{2}, R \) each has a power of \( LP \) for \( L \)-bit packet. In Eq. (5),

\[
P_{ma}[m]P_{bc}[m] + (1 - P_{ma}[m])(1 - P_{bc}[m]) \]

\[
= 1 - P_{bc}[m] - (1 - 2P_{bc}[m])P_{ma}[m] \]

\[
\leq 1 - P_{bc}[m] - (1 - 2P_{bc}[m])\hat{P}_{ma} \]

\[
= \hat{P}_{ma}P_{bc}[m] + (1 - \hat{P}_{ma})(1 - P_{bc}[m]), \tag{7}
\]
as \(1 - 2P_{bc}[m]\) is always non-negative. Then the upper bound of \(P_p\), denoted as \(\hat{P}_p\), can be expressed as

\[
\hat{P}_p = 1 - \prod_{m=1}^{\log_2 M} \left[ \hat{P}_{ma} P_{bc}[m] \right] + \left(1 - \hat{P}_{ma}\right) \left(1 - P_{bc}[m]\right)^{\frac{L}{\log_2 M}}.
\]

For the square \(M\)-QAM modulations, [10] derived the error probability for AWGN channels. For fading channels, the conditional error probability of the BC phase given the fading amplitude \(r\) can be computed as

\[
P_{bc}[m|r] = \frac{1}{\sqrt{M}} \sum_{n=0}^{\infty} \left\{ (-1)^{2^{m-1}} \left[ 2^{m-1} - \left[ \frac{i \cdot 2^{m-1}}{\sqrt{M}} + \frac{1}{2} \right] \right) \cdot \text{erfc} \left( \frac{2 + 1}{2(M - 1)} \right) \right\}.
\]

where \(\kappa = (1 - 2^{-m})\sqrt{M} - 1\), \(\gamma_{bc} = P \log_2 M / N_0\) per symbol and \(\text{erfc}(\cdot)\) is the complementary error function. Given the probability distribution function of \(h_1[m,t]\) being \(P_{h_1}(r) = 2re^{-r^2}(r > 0)\), \(P_{bc}[m]\) can be obtained by integrating \(P_{bc}[m|r]\) with regard to \(r\),

\[
P_{bc}[m] = \int_0^\infty P_{bc}[m|r] P_{h_1}(r) \text{d}r.
\]

With the help of [16, Eq. (6.283)] and the variable substitution, we rewrite Eq. (10) as

\[
P_{bc}[m] = \frac{1}{\sqrt{M}} \cdot \left(1 - 2^{-m}\right) \sqrt{M} - 1 \sum_{n=0}^{\infty} \left\{ (-1)^{2^{m-1}} \left[ 2^{m-1} - \left[ \frac{i \cdot 2^{m-1}}{\sqrt{M}} + \frac{1}{2} \right] \right) \cdot \left(1 - \frac{1}{\sqrt{1 + \frac{2(M - 1)}{2(2^m - 1)} \gamma_{bc}}} \right) \right\}.
\]

Then \(\hat{P}_p\) can be computed by Eq. (8). Furthermore, \(R_p\) can be lower bounded as

\[
R_p \geq \hat{R}_p = R_s(1 - \hat{P}_p) \frac{2 \log_2 M}{\log_2 M + 1}.
\]

As for Gray-coded \(M\)-PSK, the average \(P_{bc}[m]\) is identical regardless of the bit position \(m\) and can be expressed as [11]

\[
P_{bc}[m] = P_{bc} = \frac{1}{\log_2 M} \cdot \sum_{k=1}^{M-1} \left(2 \left| \frac{k}{M} - \left[ \frac{k}{M} \right] \right| + 2 \sum_{i=2}^{\log_2 M} \left| \frac{k}{2^i} - \left[ \frac{k}{2^i} \right] \right| \right) p_b(k),
\]

where

\[
p_b(k) = \begin{cases} 
1 - I \left( \frac{\pi}{M} \right) - I \left( \frac{(k + 1) \pi}{M} \right), & k = 0, \\
I \left( \frac{(k - 1) \pi}{M} \right) - I \left( \frac{(k + 1) \pi}{M} \right), & k = 1, \ldots, M - 1, \\
I \left( \frac{(-1) \pi}{M} \right), & k = M.
\end{cases}
\]

and \(\lfloor \cdot \rfloor\) is the operation of rounding to the closest integer, and \(p_b(k) = p_b(M - k)\) for \(k = \frac{M}{2} + 1, \ldots, M - 1\). \(I(\cdot)\) is defined as

\[
I(\theta) = 1 - \frac{\theta}{\pi} - \frac{1}{\pi} \arctan \frac{\cot \theta}{\sqrt{1 + \frac{1}{\gamma_{bc} \sin^2 \theta}}}.
\]

Thus Eq. (8) becomes

\[
\hat{P}_p = 1 - \left[ \hat{P}_{ma} P_{bc} + (1 - \hat{P}_{ma})(1 - P_{bc}) \right]^{\frac{L}{\log_2 M}}.
\]

For the BC phase, we also consider the square \(M\)-QAM and \(M\)-PSK modulations. In the square \(M\)-QAM, \(P_{bc}[m]\) of an AWGN channel equals Eq. (9) by setting \(r = 1\) [15]. Given the MA slots being identical and the symmetry along the two dimensions of the square \(M\)-QAM constellation, from Eq. (4) we have

\[
P_p = 1 - \left[ \prod_{m=1}^{\log_2 M} [P_{ma} P_{bc}[m] + (1 - P_{ma})(1 - P_{bc}[m])] \right]^{\frac{L}{\log_2 M}}.
\]

As for \(M\)-PSK using Gray coding, \(P_{bc}[m]\) or \(P_{bc}\) takes the same form as Eq. (13) with a different definition of \(p_b(k)\) [17]. Now

\[
p_b(k) = \frac{1}{2\pi} \left( \int_0^{\pi(1 - k \frac{\pi}{M})} \text{exp}(-\gamma_{bc} \sin^2 \theta) (2k - 1) \pi \text{d}\theta - \int_0^{\pi(1 - k \frac{\pi}{M})} \text{exp}(-\gamma_{bc} \sin^2 \theta) (2k + 1) \pi \text{d}\theta \right),
\]

regardless of the bit position \(m\). Thus, Eq. (4) is simplified to

\[
P_p = 1 - [P_{ma} P_{bc} + (1 - P_{ma})(1 - P_{bc})]^L.
\]

Computation of the overall throughput \(R_p\) is performed using Eq. (3).

C. Beneficial SNR Range

Two factors affect the end-to-end throughput, as shown from Eq. (3), namely the PER and the modulation type \(M\). Continuing with the previous section, we still only consider BPSK in the MA phase, and general modulation types in the BC phase. \(\frac{\log_2 M}{\log_2 M + 1}\) increases as \(M\) grows larger. However, for higher-order modulations, their error performance is
comparatively poorer, resulting in a worse end-to-end PER. For smaller SNRs, the error probabilities are rather large for high-order modulations, which is not compensated by the gain from \( \frac{P}{\log_2 M} \). Thus, lower-order modulations prove to be beneficial than the higher-order in the low SNR range. As the SNR becomes sufficiently large, few errors occur in the transmission, and the PER approaches zero. Then the gain from \( \frac{P}{\log_2 M} \) dominates in Eq. (3). Asymptotically, the higher-order modulations are advantageous.

Substitute the \( P_p \) in Eq. (3) with Eq. (17), and we have for the AWGN channels

\[
R_p = 2R_s \cdot \frac{\log_2 M}{\log_2 M + 1} \prod_{m=1}^{\log_2 M} [P_{ma}P_{bc}[m] + (1 - P_{ma})(1 - P_{bc}[m])]^{\frac{P}{\log_2 M}}.
\]

(19)

For the fading channels, it is \( \hat{P}_{ma} \) instead of \( P_{ma} \). Denote the factor other than \( 2R_s \) as \( F(M) \). For two systems using \( M' \)-ary and \( M'' \)-ary modulations respectively for their BC phases, the “inflection” SNR value (dividing the regions of one scheme’s beneficial and not compared to the other) can be obtained by solving \( F(M') = F(M'') \). As this equation does not generally have a tractable analytical solution, we do not go into the details of solving a particular pair of schemes numerically.

IV. NUMERICAL RESULTS

In this section, we present the numerical results. The end-to-end throughput is the primary metric of the system performance. It is measured in terms of the nominal data rate \( R_s \) of the BPSK modulation. The throughput values are plotted against SNR which is defined as \( P/N_0 \).

For the notation \( P \) (per-bit transmission power), here we briefly revisit some assumptions of the transmission power in Section II. We assume \( 3LP \) for the exchange of two packets (two packets travel via links \( T_1 \rightarrow T_2, T_2 \rightarrow T_1 \) simultaneously). \( L \) is the packet length. Thus, the power allocation per bit for the terminals \( T_1, T_2 \) and the relay \( R \) together is \( 3P \). Take Fig. 1 (\( L = 4 \)) as an example. Using BPSK throughout both phases (Fig. 1(a)), the packet is transmitted bit by bit, and each BPSK symbol/bit has a power of \( P \). It takes eight time slots (four for MA, BC respectively) for the exchange of two packets. The total power consumed is \((2P + P) \times 4 = 12P = 3LP\). As for using 16-QAM only, as in Fig. 1(a), the power of each symbol (four bits) is \( 4P \), and the transmission is completed in two time slots. The power consumption is \((4P + 4P) + 4P = 12P = 3LP\). Similarly, for the \( \log_2 16 \)-MA scheme (Fig. 1(c)), BPSK is used in the MA phase (\( P \) per symbol, four time slots), and 16-QAM for the BC phase (\( 4P \) per symbol, one slot). Therefore, the total power consumption is \((P + P) \times 4 + 4P \times 1 = 12P = 3LP\). From the above analysis, for all these three schemes, the transmission power of the terminal/relay is \( P \) per bit.

Fig. 2 compares the simulation results with the upper bounds obtained for each \( \log_2 M \)-MA scheme in fading channels, respectively. QPSK (4-QAM) is used for \( \log_2 4 \)-MA and 16-QAM for \( \log_2 16 \)-MA. As seen from the figure, the theoretical bounds merge tightly with the numerical values. Thus, the bound performance suffices for asymptotic analysis in place of the exact closed forms. The same pattern can be observed here that modulations of higher-order tend to have a better asymptotic throughput while proving less efficient in low SNRs. Fig. 3 performs the same comparison on the same three schemes in the AWGN channels. We see that the exact theoretical values are validated by the numerical results.

In Fig. 4, the throughput of the proposed \( \log_2 16 \)-MA scheme using 16-PSK is plotted for packet lengths 64, 128 and 256. For comparison, we choose BPSK and 16-PSK as two extreme cases among the conventional methods (whose order not larger than 16). BPSK sees a lower error probability and longer transmission time per packet, whereas 16-PSK has a higher error rate and shorter time to accommodate a packet. Our proposed \( \log_2 16 \)-MA approach makes comprise between PER and the per-packet time consumption. This proves to be beneficial in the low SNR range. For example, when the packet size \( L = 64 \), from around 26 dB SNR and below, \( \log_2 16 \)-MA surpasses 16 – 16. As the SNR becomes smaller, the throughputs of all three schemes approach zero, and \( B - B \) gains a very slim margin over \( \log_2 16 \)-MA. Overall, \( \log_2 16 \)-MA can be seen as the best scheme in the below 26 dB SNR region. For even larger packet sizes, we see the right end of this beneficial region expands towards higher SNR; it is 28 dB when \( L = 128 \) and 32 dB when \( L = 256 \).

V. CONCLUSIONS

In this paper, we proposed a new class of \( \log_2 M \)-MA block relay for TWRC systems. Instead of forwarding the received symbols for each MA phase, the relay buffers the symbols from multiple MA phases and forward the combination through a higher modulation. In particular, we focused on the case of \( M_1 = 2 \) using BPSK in the MA phase. We obtained the performance bounds for \( \log_2 M \)-MA block relay
Fig. 3. The end-to-end throughput performance of $\log_2 M$-MA (QPSK/4-QAM for $M = 4$, 16-QAM for $M = 16$) in AWGN channels when $L = 128$.

Fig. 4. The end-to-end throughput comparison between $\log_2 16$-MA (16-PSK) and the conventional methods using BPSK ($B$–$B$ in the figure legend), 16-PSK ($16$–$16$) throughout in fading channels. The packet sizes $L$ are 64, 128 and 256.

in fading channels and the exact closed forms for AWGN channels. Numerical results confirmed the advantages of the proposed scheme in terms of higher throughput in low SNRs.

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