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Networked Control Systems: A Model-Based Approach

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Abstract

In this paper a class of networked control systems called Model-Based Networked Control Systems (MB-NCS) is considered. This control architecture uses an explicit model of the plant in order to reduce the network traffic while attempting to prevent excessive performance degradation. MB-NCS can successfully address several important control issues in an intuitive and transparent way. In this paper the main results of this approach are described with examples. Specifically, conditions for the stability of state and output feedback systems are derived. In addition, delay compensation, constant and time varying update times, non-linear plants, quantization, and performance measures are considered.

1 Introduction

A networked control system (NCS) is a control system in which a data network is used as feedback media. NCS is an important area see for example [10] and [9, 11, and 12]. The use of networks as media to interconnect the different components in an industrial control system is rapidly increasing. The use of networked control systems poses, though, some challenges. One of the main problems to be addressed when considering a networked control system is the size of bandwidth required by each subsystem. It is clear that the reduction of bandwidth necessitated by the communication network in a networked control system is a major concern. This can perhaps be addressed by two methods: the first method is to minimize the transfer of information between the sensor and the controller/actuator; the second method is to compress or reduce the size of the data transferred at each transaction. Since a shared characteristic among popular industrial networks are the small transport time and big overhead, using less bits per packet has small impact over the overall bit rate. *So reducing the rate at which packets are transmitted brings better benefits than data compression in terms of bit rate used*. In this paper, we consider the problem of reducing the packet rate of a NCS using a novel approach called Model-Based NCS (MB-NCS). The MB-NCS architecture makes explicit use of knowledge about the plant dynamics to enhance the performance of the system. Model-Based Networked Control Systems (MB-NCS) were introduced in [5].

In Section 2 the basic MB-NCS setup is introduced. Stability of MB-NCS' with no quantization and periodic transmissions are considered. In Section 3 a performance measure is introduced for the previously presented MB-NCS. The stability of MB-NCS with stochastic transmission times is studied in Section 4. Static quantization schemes are considered in Section 5. Finally, stability for a class of nonlinear systems is studied in Section 6.

2 Stability of Linear MB-NCS with Constant Update Times

2.1 State Feedback Continuous MB-NCS

We consider the control of a continuous linear plant where the state sensor is connected to a linear controller/actuator via a network. In this case, the controller uses an explicit model of the plant that approximates the plant dynamics and makes possible the stabilization of the plant even under slow network conditions.

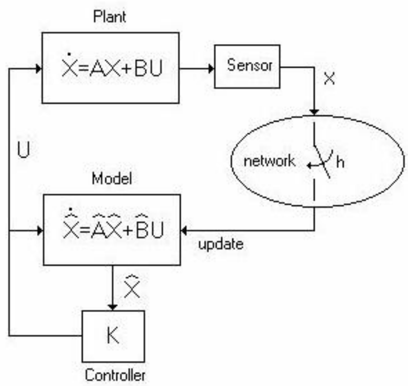


Figure 1: Proposed configuration of networked control system.

We will concentrate on characterizing the transfer time between the sensor and the controller/actuator, which is the time between information exchanges. The plant model is used at the controller/actuator side to recreate the plant behavior so that the sensor can delay sending data since the model can provide an approximation of the plant dynamics. *The main idea is to perform the feedback by updating the model's state using the actual state of the plant that is provided by the sensor. The rest of the time the control action is based on a plant model that is incorporated in the controller/actuator and is running open loop for a period of h seconds.* The control architecture is shown in Figure 1.

Our approach is novel in that it incorporates a model of the plant, the state of which is updated at discrete intervals by the plant's state. We present a necessary and sufficient condition for stability that results in a maximum transfer time.

If all the states are available, then the sensors can send this information through the network to update the model's vector state. For our analysis we will assume that the compensated model is stable and that the transportation delay is negligible. We will assume that the frequency at which the network updates the state in the controller is constant. The idea is to find the smallest frequency at which the network must update the state in the controller, that is, an upper bound for h, the update time. Usual assumptions in the literature include requiring a stable plant or in the case of a discrete controller, a smaller update time than the sampling time. Here we do not make any of these assumptions and the plant may be unstable.

Consider the control system of Figure 1 where plant is given by $\dot{x} = Ax + Bu$, the plant model by $\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u$, and the controller by $u = K\hat{x}$. The state error is defined as $e = x - \hat{x}$, and represents the difference between the plant state and the model state. The modeling error matrices $\tilde{A} = A - \hat{A}$ and $\tilde{B} = B - \hat{B}$ represent the difference between the plant and the model. Also define the error $e(t) = x(t) - \hat{x}(t)$. A necessary and sufficient condition for stability of the state feedback MB-NCS will now be presented.

Theorem #1 [7]

The State Feedback MB-NCS is globally exponentially stable around the solution $z = \begin{bmatrix} x & e \end{bmatrix}^T = \mathbf{0}$ *if and only if the eigenvalues of* $\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ *are strictly inside the unit circle.*

It can be shown (as in [4]) that the eigenvalues of $M = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ are inside the unit circle if and only if the eigenvalues of $N = e^{(\hat{A}+\hat{B}K)h} + \Delta$ with $\Delta = e^{Ah}\int_0^h e^{-A\tau}(\tilde{A} + \tilde{B}K)e^{(\hat{A}+\hat{B}K)\tau}d\tau$ are inside the unit circle. Observe that the eigenvalues of the compensated model appear in the first term of N and that the second term Δ can be made small by having small update times h or small modeling error. A detailed proof for Theorem **1** can be found in [5].

Example

In real applications uncertainty can frequently be expressed as tolerances over the different measured parameter values of the plant. This can be mapped into structured or parametric uncertainties on the state space matrices. Next an example is given on how the Theorem **1** can be applied if two entries on the A matrix of the model can vary within a certain interval.

$$\text{model:} \quad \hat{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \hat{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

$$\text{plant:} \quad A = \begin{bmatrix} 0 & 1+\tilde{a}_{12} \\ 0+\tilde{a}_{21} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

$$\text{with } \tilde{a}_{12} = \left[-0.5, 0.5\right],\ \tilde{a}_{21} = \left[-0.5, 0.5\right]$$

$$\text{controller:} \quad K = [-1, -2].$$

The system will now be tested for an update time of h=2.5 seconds. The following contour plot in Figure 2 represents the maximum eigenvalue magnitude for the test matrix M as a function of the (1,2) and (2,1) entries possible values. Here the contour at height equal to one is the relevant to stability. It is easy to isolate the stable and unstable regions in the uncertainty parameter plane. The stable region is between the lines labeled as 1.

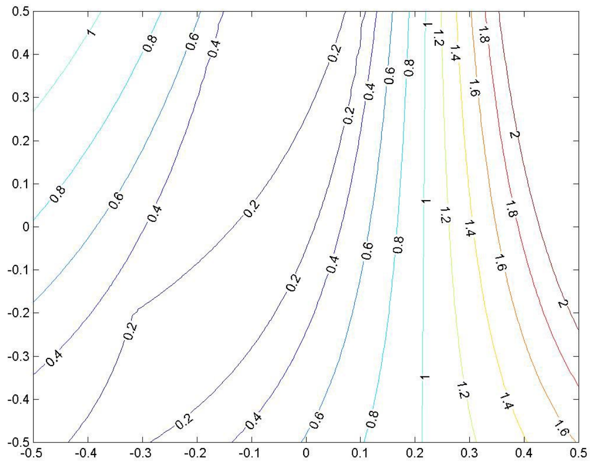


Figure 2. Contour Plot Maximum Eigenvalue Magnitude vs Model Error

2.2 Output Feedback Continuous MB-NCS

We now extend our approach to include plants where the state is not directly measurable. In this case, in order to obtain an estimate of the plant state vector, a state observer is used. It is assumed that the state observer is collocated with the sensor. Again, we use the plant model, $\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u$, to design the state observer. See Figure 3. The observer has the form of a standard state observer with gain L. In summary, the system dynamic equations are for $t \in [t_k, t_{k+1})$:

Plant:	$\dot{x} = Ax + Bu,$	$y = Cx + Du$
Model:	$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u,$	$y = \hat{C}\hat{x} + \hat{D}u$
Controller:	$u = K\hat{x}$	
Observer:	$\dot{\bar{x}} = (\hat{A} - L\hat{C})\bar{x} + \begin{bmatrix} \hat{B} - L\hat{D} & L \end{bmatrix}\begin{bmatrix} u & y \end{bmatrix}^T$	

We now proceed in a similar way as in the previous case of full feedback. Namely, there will be an update interval h, after which the observer updates the controller's model state $\hat{x}$ with its estimate $\bar{x}$. We will also define an error e that will be the difference between the controller's model state and the observer's estimate: $e = \bar{x} - \hat{x}$. Also we will define the modeling error matrices in the same way as before:

$\tilde{A}=A-\hat{A},\quad \tilde{B}=B-\hat{B},\quad \tilde{C}=C-\hat{C},\quad \tilde{D}=D-\hat{D}$. Define $z=\begin{bmatrix}x & \bar{x} & e\end{bmatrix}^T$, and

$$\Lambda_o=\begin{bmatrix} A & BK & -BK \\ LC & \hat{A}-L\hat{C}+\hat{B}K+L\tilde{D}K & -\hat{B}K-L\tilde{D}K \\ LC & L\tilde{D}K-L\hat{C} & \hat{A}-L\tilde{D}K \end{bmatrix}.$$

Theorem #2 [7]

The Output Feedback MB-NCS is globally exponentially stable around the solution $z=\begin{bmatrix}x & \bar{x} & e\end{bmatrix}^T=\mathbf{0}$ *if and only if the eigenvalues of* $M_o=\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0\end{bmatrix} e^{\Lambda_o h}\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0\end{bmatrix}$ *are inside the unit circle.*

2.3 State Feedback Continuous MB-NCS with Network Induced Delays

Previously we assumed that the network delays were negligible. This is usually true for plants with slow dynamics relative to the network bandwidth. When this is not the case the network delay cannot be neglected. In the following, we extend our results to include the case were transmission delay is present. We will assume that the update time h is larger than the delay time τ. As before we will assume that the update time h and delay τ are constant. We will present here the case of full state feedback systems.

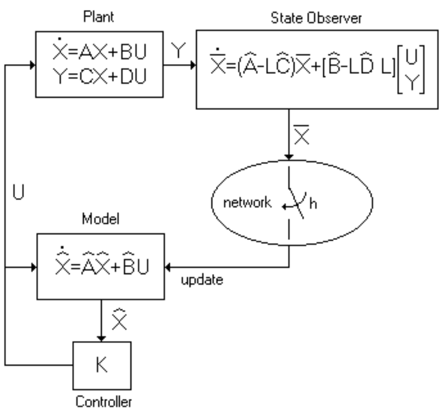


Figure 3. Proposed configuration of an output feedback networked control system.

So, at times kh-τ the sensor transmits the state data to the controller/actuator. This data will arrive τ seconds latter. So, at times kh the controller/actuator receives the state vector value $x(kh$-$\tau)$. The main idea is to use the plant model in the controller/actuator to calculate the present value of the state. After this, the state approximate obtained can be used to update the controller's model as in previous setups. The system is depicted in Figure 4.

The Propagation Unit uses the plant model and the past values of the control input $u(t)$ to calculate an estimate of actual state $\breve{x}(kh)$ from the received data $x(kh$-$\tau)$. This estimate is then used to update the model that with the controller will generate the control signal for the plant.

The system is described by the following equations:

Plant:	$\dot{x}=Ax+Bu$	
Model:	$\dot{\hat{x}}=\hat{A}\hat{x}+\hat{B}u$	
Controller:	$u=K\hat{x},\ \ t\in[t_k,t_{k+1})$	(1)
Propagation Unit:	$\dot{\breve{x}}=\hat{A}\breve{x}+\hat{B}u,\ \ t\in[t_{k+1}-\tau,t_{k+1}]$	
Update law:	$\begin{Bmatrix}\breve{x}\leftarrow x,\ t=t_{k+1}-\tau \\ \hat{x}\leftarrow\breve{x},\ t=t_{k+1}\end{Bmatrix}$	

We define the errors $\hat{e}=\breve{x}-\hat{x}$ and $\breve{e}=x-\breve{x}$. We also make the following definitions:

$$\begin{matrix}\tilde{A}=A-\hat{A}\\ \tilde{B}=B-\hat{B}\end{matrix},\ \Lambda_d=\begin{bmatrix}A+BK & -BK & -BK\\ \tilde{A}+\tilde{B}K & \hat{A}-\tilde{B}K & -\tilde{B}K\\ 0 & 0 & \hat{A}\end{bmatrix},\ z=\begin{bmatrix}x\\ \breve{e}\\ \hat{e}\end{bmatrix}$$

We will present now the necessary and sufficient conditions for this system to be exponentially globally stable.

Theorem #3 [7]

The State Feedback MB-NCS with networked induced delay τ *is globally exponentially stable around the solution* $z=\begin{bmatrix}x & \breve{e} & \hat{e}\end{bmatrix}^T=\mathbf{0}$ *if and only if the eigenvalues of* $M_T=\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0\end{bmatrix} e^{\Lambda_T\tau}\begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & I\end{bmatrix} e^{\Lambda_T(h-\tau)}$ *are inside the unit circle.*

It is interesting to note that the results on Theorem #**3** can be seen as a generalization of Theorem #**1**. This can be shown by driving τ to zero.

3 A Performance Index for Linear MB-NCS with Constant Update Times

The performance characterization of Networked Control Systems under different conditions is of mayor concern. It is clear that, since the MB-NCS is h-periodic, there is no transfer function in the normal sense whose H2 norm can be calculated [1]. For LTI systems the H2 norm can be computed by obtaining the 2-norm of the impulse response of the system. We will extend this definition to specify an H2 norm, or more properly, to define an H2-like performance index [1]. We will call this performance index Extended H2 Norm. We will study the extended H2 norm of the MB-NCS with output feedback studied in the previous section and shown in Figure 5. A disturbance signal w and a performance or objective signal z are included in the setup.

We will start by defining the system dynamics:

Plant Dynamics:

$$\dot{x} = Ax + B_1 w + B_2 u$$
$$z = C_1 x + D_{12} u$$
$$y = C_2 x + D_{21} w + D_{22} u$$

Observer Dynamics:

$$\dot{\bar{x}} = (\hat{A} - L\hat{C}_2)\bar{x} + (\hat{B}_2 - L\hat{D}_{22})u + Ly \qquad (2)$$

Model Dynamics:

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}_2 u$$

Controller:

$$u = K\hat{x}$$

Define the following:

$$\Lambda = \begin{bmatrix} A & B_2K & -B_2K \\ LC_2 & \hat{A} - L\hat{C}_2 + \hat{B}_2K + L\tilde{D}_{22}K & -\hat{B}_2K - L\tilde{D}_{22}K \\ LC_2 & L\tilde{D}_{22}K - L\hat{C}_2 & \hat{A} - L\tilde{D}_{22}K \end{bmatrix}$$

$$M(h) = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} e^{\Lambda h},\ B_N = \begin{bmatrix} B_1 \\ LD_{21} \\ LD_{21} \end{bmatrix},\ C_N = \begin{bmatrix} C_1 \\ D_{12}K \\ -D_{12}K \end{bmatrix}^T$$

Theorem 4 [13]

The Extended H2 Norm, $\|G\|_{xh2}$, of the MB-NCS described in Equation (2) is given by $\|G\|_{xh2} = B_N{}^T X B_N$ where X is the solution of the discrete Lyapunov equation $M(h)XM(h)^T - X + W_o(0,h) = 0$ and $W_o(0,h)$ is the observability Gramian computed as $W_o(0,h) = \int_0^h e^{\Lambda^T t} C_N{}^T C_N e^{\Lambda t} dt$.

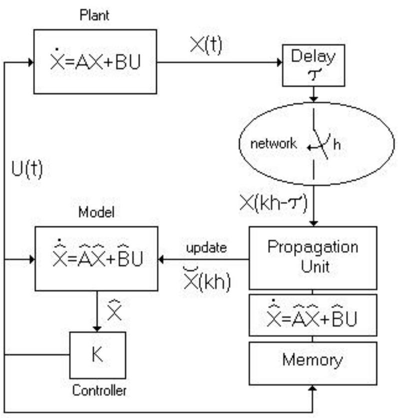


Figure 4. Proposed configuration of a state feedback networked control system in the presence of network delays.

Example

We now present an example using a double integrator as the plant. The plant dynamics are given by: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B_1 = [0.1 \quad 0.1]^T$, $B_2 = [0 \quad 1]^T$, $C_1 = [0.1 \quad 0.1]$, $C_2 = [1 \quad 0]$, $D_{12} = 0.1$; $D_{22} = 0.1$; $D_{22} = 0$. We will use the state feedback controller $K = [-1, \quad -2]$. A state estimator with gain $L = [20 \quad 100]^T$ is used to place the state observer eigenvalues at –10. We will use a plant model with the following parameters: $\hat{A} = \begin{bmatrix} 0.1634 & 0.8957 \\ \text{-}0.1072 & \text{-}0.1801 \end{bmatrix}$, $\hat{B}_2 = \begin{bmatrix} \text{-}0.1686 \\ 1.0563 \end{bmatrix}$, $\hat{C}_2 = [0.8764 \quad 0.1375]$, and $\hat{D}_{22} = \text{-}0.1304$. In Figure 6 we plot the extended H2 norm of the system as a function of the updates times. Note that as the update time of the MB-NCS approaches zero, the value of the Extended H2 norm approaches the H2 norm of the non-networked compensated system. Also note the performance degradation as the update time *h* is increased.

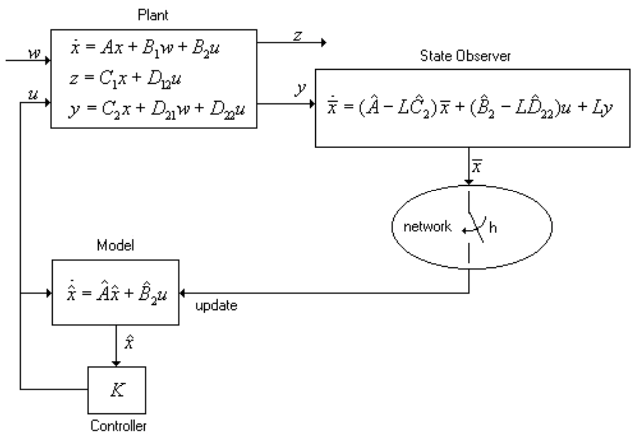


Figure 5. MB-NCS with disturbance input and objective signal output.

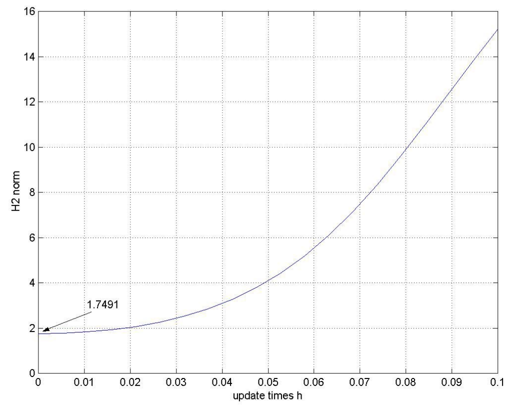


Figure 6. Extended H2 norm of the system as a function of the update times

4 Stability of MB-NCS with Time-Varying Update Times

In this section we relax our assumption that the update times *h(k)* are constant. Here we study the state feedback MB-NCS shown in Figure 1. We define the update times as the times between transmissions or model updates: $h(k)=t_{k+1}-t_k$. Note that a MB-NCS can have update times that generate test matrices with eigenvalues inside of the unit circle and still be unstable if a particular sequence of update times is given. We will assume that the transmission time delay is negligible.

4.1 Lyapunov Stability for MB-NCS

Here we present a condition under which a MB-NCS is stable based on the well-known Lyapunov second method for determining the stability of a system. We will assume that the properties of *h(k)* are unknown but *h(k)* is contained within some interval. This criterion provides a first approach to stability for time-varying transmission times NCS.

Theorem 5 [6, 8]

The State Feedback MB-NCS is Lyapunov Asymptotically Stable for $h \in [h_{\min},\ h_{\max}]$ *if there exists a symmetric positive definite matrix X such that* $Q = X - MXM^T$ *is positive definite for all* $h \in [h_{\min},\ h_{\max}]$, *where* $M = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$.

Theorem **5** may be used to derive an interval $[h_{\min},\ h_{\max}]$ for *h* for which stability is guaranteed. It is clear that the range for *h*, that is the interval $[h_{\min},\ h_{\max}]$, will vary with the choice of *X.* Another observation is that the interval obtained this way will always be contained in the set of constant update times for which the system is stable (as derived using Theorem **1**). That is, an update time contained in the interval $[h_{\min},\ h_{\max}]$ will always be a stable constant update time.

Next, we will study two types of stochastic stability: almost sure stability and mean square stability for update times that are independent and identically distributed. The results here presented and the case of Markov-driven update times can be found in [6, 8].

4.2 Almost Sure Stability for MB-NCS with Independent Identically Distributed Transmission Times

Here we will assume that the update times *h(k)* are independent identically distributed (iid) with probability distribution function *F(h)*. We will use the definition of almost sure asymptotic stability [2] that provides a stability criterion based on the sample path and resembles more the deterministic stability definition [3]. We now define Almost Sure or Probability-1 Asymptotic stability and now present the conditions under which the state feedback MB-NCS with iid update times is almost sure stable.

Definition 1

The equilibrium $z = 0$ *of a system described by* $\dot{z} = f(t, z)$ *with initial condition* $z(t_0) = z_0$ *is almost sure (or with probability-1) asymptotically stable at large (or globally) if for any* $\beta > 0$ *and* $\varepsilon > 0$ *the solution of* $\dot{z} = f(t, z)$ *satisfies*

$$\lim_{\delta \to \infty} P\left\{ \sup_{t \geq \delta} \left\| z(t, z_0, t_0) \right\| > \varepsilon \right\} = 0 \ \text{whenever} \ \|z_0\| < \beta. \qquad (3)$$

Theorem 6 [6, 8]

The State Feedback MB-NCS with update times h(j) independent identically distributed random variable with probability distribution F(h) is globally almost sure (or with probability-1) asymptotically stable around the solution $z = [x \quad e]^T = \mathbf{0}$ *if* $N = E\left[\left(e^{2\bar{\sigma}(\Lambda)h} - 1\right)^{1/2}\right] < \infty$ *and the expected value of the maximum singular value of the test matrix M,* $E\left[\|M\|\right] = E\left[\bar{\sigma}_M\right]$, *is strictly less than one, where* $M = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$.

Note that the condition on the matrix *N* ensures that the probability distribution function for the update times *F(h)* assigns smaller occurrence probabilities to increasingly long update times, that is *F(h)* decays rapidly. In particular we observe that *N* can always be bounded if there exists h_m such that *F(h)*=0 for *h* larger than h_m.

4.3 Mean Square Stability for MB-NCS with Independent Identically Distributed Transmission Times

We now define a different type of stability, namely Mean Square Asymptotic Stability:

Definition 2

The equilibrium $z = 0$ *of a system described by* $\dot{z} = f(t, z)$ *with initial condition* $z(t_0) = z_0$ *is mean square stable asymptotically stable at large (or globally) if the solution of* $\dot{z} = f(t, z)$ *satisfies*

$$\lim_{t \to \infty} E\left[\left\| z(t, z_0, t_0) \right\|^2 \right] = 0 \qquad (4)$$

A system that is mean square stable will have the expectation of system states converging to zero with time in the mean square sense. This definition of stability is attractive since many optimal control problems use the squared norm in their formulations. We present the conditions under which the state feedback MB-NCS previously introduced is mean square stable.

Theorem 7 [6, 8]

The state feedback MB-NCS with update times h(j) independent identically distributed random variable with probability distribution F(h) is globally mean square asymptotically stable around the solution $z = [x \quad e]^T = \mathbf{0}$ *if* $K = E\left[\left(e^{\bar{\sigma}(\Lambda)h}\right)^2\right] < \infty$ *and the maximum singular value of the expected value of* M^TM, $\left\| E\left[M^T M\right] \right\| = \bar{\sigma}\left(E\left[M^T M\right]\right)$, *is strictly less than one, where* $M = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$.

Note the similarity between the conditions given by Theorems 6 and 7. In Theorem 6 we require the expectation of the maximum singular value of *M* to be less than one. For the stability in Theorem 7 the maximum singular value of the expectation of M^TM should be less than one.

5 Stability of Linear MB-NCS with Quantization

Here we extend our stability results to consider the case where quantization errors occur. We will assume that the transmission times are constant. We will also assume that the compensated networked system *without* quantization is stable satisfying the following:

$$\left(e^{(\hat{A}+\hat{B}K)^T h}+\Delta(h)^T\right)P\left(e^{(\hat{A}+\hat{B}K)h}+\Delta(h)\right)-P=-Q_D \qquad (5)$$

with Q_D symmetric and positive symmetric.

5.1 State Feedback MB-NCS with Uniform Quantization

Define the Uniform Quantizer as a function $q:\mathbb{R}^n \to \mathbb{R}^n$ with the following property $\left\|z-q(z)\right\| \le \delta,\ z\in\mathbb{R}^n, \delta>0$.

Theorem 8 [13]

The plant state of the State Feedback MB-NCS satisfying (5) *and using the Uniform Quantizer will enter and remain in the region* $\|x\|\le R$ *defined by:*

$$R=\left(e^{\bar{\sigma}(\hat{A}+\hat{B}K)h}+\Delta_{\max}(h)\right)r+\left(e^{\bar{\sigma}(A)h}+\Delta_{\max}(h)\right)\delta$$

$$\textit{where } r=\sqrt{\frac{\lambda_{\max}\left(\left(e^{Ah}-\Delta(h)\right)^T P\left(e^{Ah}-\Delta(h)\right)^T\right)\delta^2}{\lambda_{\min}(Q_D)}}$$

$$\textit{and } \Delta_{\max}(h)=\int_0^h e^{\bar{\sigma}(A)(h-\tau)}\bar{\sigma}\left(\tilde{A}+\tilde{B}K\right)e^{\bar{\sigma}(\hat{A}+\hat{B}K)\tau}d\tau$$

5.2 State Feedback MB-NCS with Logarithmic Quantization

Define the Logarithmic Quantizer as a function $q:\mathbb{R}^n \to \mathbb{R}^n$ with the following property $\left\|z-q(z)\right\| \le \delta\|z\|,\ z\in\mathbb{R}^n, \delta>0$.

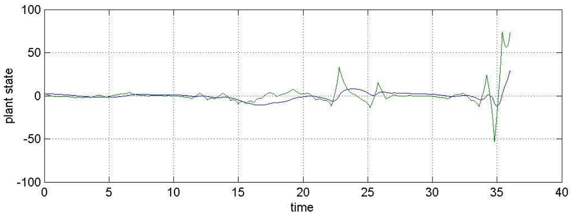


Figure 7. Plant and Model state time response for $q1$.

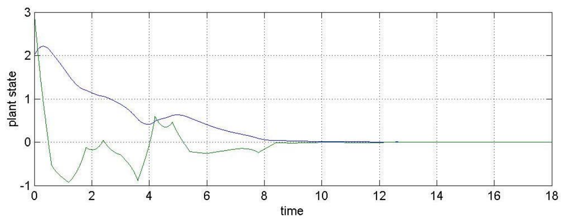


Figure 8. Plant and Model state time response for $q2$.

Theorem 9 [13]

The State Feedback MB-NCS satisfying (5) *and using the Logarithmic Quantizer is exponentially stable if:*

$$\delta<\sqrt{\frac{\lambda_{\min}(Q_D)}{\lambda_{\max}\left(\left(e^{Ah}-\Delta(h)\right)^T P\left(e^{Ah}-\Delta(h)\right)\right)}}$$

Example

For our example we will use the following plant model:

$$\dot{\hat{x}}=\begin{bmatrix}0 & 1\\ 1 & 3\end{bmatrix}\hat{x}+\begin{bmatrix}0\\ 1\end{bmatrix}u \qquad (6)$$

The model will be a perturbed version of our plant:

$$\dot{\hat{x}}=\begin{bmatrix}-0.0689 & 0.9757\\ 1.0396 & 3.0720\end{bmatrix}\hat{x}+\begin{bmatrix}0.0707\\ 1.0187\end{bmatrix}u \qquad (7)$$

Both are unstable plants. The controller is designed using the plant model: $u=\begin{bmatrix}-2 & -5\end{bmatrix}\hat{x}$. This controller places both eigenvalues of the compensated plant model at –1. Using update time of h=0.6 s, we will test two logarithmic quantizer functions $q1$ with a mantissa word length of 12 bits and $q2$ with mantissa word length of 13 bits. Their relative errors for the two dimensional space they will work on are for $q1$: 0.33 and for $q2$: 0.20. Initializing the plant at [2;3], we observe from Figure 7 that the system working with quantizer $q1$ ($\delta=0.33$) is unstable, while with $q1$ ($\delta=0.20$) is stable (Figure 8). By using Theorem 9 and a Q_D=I we obtain a maximum relative error of 0.1241.

6 Stability of a Class of Non-Linear MB-NCS

We will determine sufficient conditions for the stability of a state feedback MB-NCS when the plant and controller are nonlinear. Let the plant, plant model and controller be given by:

$$\begin{aligned}&\text{plant:} && \dot{x}=f(x)+g(u)\\ &\text{model:} && \dot{\hat{x}}=\hat{f}(x)+\hat{g}(u)\\ &\text{controller:} && u=\hat{h}(\hat{x})\end{aligned} \qquad (8)$$

Also define $e=x-\hat{x}$ as the error between the plant state and the plant model state. From (8) we obtain:

$$\begin{aligned}\dot{x}&=f(x)+g\left(\hat{h}(\hat{x})\right)=f(x)+m(\hat{x})\\ \dot{\hat{x}}&=\hat{f}(\hat{x})+\hat{g}\left(\hat{h}(\hat{x})\right)=\hat{f}(\hat{x})+\hat{m}(\hat{x})\end{aligned} \qquad (9)$$

We will also assume that the plant model dynamics differ from the actual plant dynamics in an additive fashion:

$$\begin{aligned}\hat{f}(\varsigma)&=f(\varsigma)+\delta_f(\varsigma)\\ \hat{m}(\varsigma)&=m(\varsigma)+\delta_m(\varsigma)\end{aligned} \qquad (10)$$

So we can rewrite (9) as:

$$\begin{aligned}\dot{x}&=f(x)+m(\hat{x})\\ \dot{\hat{x}}&=f(\hat{x})+m(\hat{x})+\delta_f(\hat{x})+\delta_m(\hat{x})=f(\hat{x})+m(\hat{x})+\delta(\hat{x})\end{aligned} \qquad (11)$$

Luis Montestruque, Panos J. Antsaklis, “Networked Control Systems: A Model-Based Approach,” Proc. of the 12th IEEE Mediterranean Conference on Control and Automation, Kusadasi, Turkey, June 6-9, 2004.

We will now assume that f and δ satisfy the following local Lipschitz conditions for $x, y \in B_L$ with B_L a ball centered on the origin:

$$\begin{aligned} \|f(x) - f(y)\| &\le K_f \|x - y\| \\ \|\delta(x) - \delta(y)\| &\le K_\delta \|x - y\| \end{aligned} \tag{12}$$

At this point it is to be noted that if the plant model is accurate the Lipschitz constant K_δ will be small.

We will assume that the non-networked compensated plant model is exponentially stable when $\hat{x}(t_0) \in B_S$, with $\hat{x}(t) \in B_h$ for $t \in [t_0, t_0 + h)$ with B_S and B_h balls centered on the origin.

$$\|\hat{x}(t)\| \le \alpha \|\hat{x}(t_0)\| e^{-\beta(t-t_0)}, \text{ with } \alpha, \beta > 0. \tag{13}$$

Theorem #10 [13]

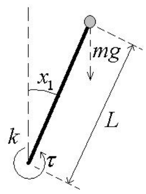
The non-linear MB-NCS with dynamics described by (8), that satisfies the Lipschitz conditions described by (12) and with exponentially stable compensated plant model satisfying (13) is asymptotically stable if:

$$\left(1 - \alpha\left(e^{-\beta h} + \left(e^{K_f h} - e^{-\beta h}\right)\left(\frac{K_\delta}{K_f + \beta}\right)\right)\right) > 0$$

Theorem **10** presents a sufficient condition for stability of a class of nonlinear systems. Note that if the model has the exact same dynamics as the plant, that is if $K_\delta = 0$, then the condition will be satisfied for arbitrarily large h.

Example

We use the inverted pendulum in Figure 9 as example.



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_2 \\ -\dfrac{g}{L}\sin(x_1) - \dfrac{k}{m}x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{mL^2} \end{bmatrix} \tau$$

Figure 9. Inverted Pendulum.

The parameters for the plant are g=10, L=10, k=0.1, and m=1.01. The model parameters are the same as the plant ones, except for the mass that is m=1.00. Finally the controller is given by: $\tau = [-316 \quad 316]x$. Using the following Lipschitz and exponential stability constants: $K_f = 1.0507$, $K_\delta = 0.0450$, $\alpha = 1.5$, and $\beta = 0.6$, Theorem **10** predicts stability for update times between 0.55 and 2.55 sec. Simulations show that the system is unstable for h greater that approximately 4.5 sec.

7 Conclusions

The MB-NCS control architecture presented in this paper represents a natural way of placing critical information about the plant on the network so to reduce the data traffic load. By making the sensor and actuator more “intelligent” the networked control system is able to predict the future behavior of the plant, and send the precise information at critical times so to ensure plant stability.

Model-based networked control systems are only one of the various approaches to networked control. We study these systems since the benefits and properties they have make it well suited for a variety of practical applications. Furthermore, the control structures presented appear to be amenable to detailed analysis. It is also to be noted that the results available in the literature for this kind of systems are limited.

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