Application of Data-Driven Loop-Shaping Method to Multi-Loop Control Design of Benchmark PID 2012

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Abstract: In this paper, an MIMO boiler control problem of Benchmark PID 2012 is studied, and a multi-loop PID control system is designed by our data-driven loop-shaping method. Our method is applicable to non-minimum phase or time-delay SISO plants very easily especially for stable plants, and a good disturbance rejection can be attained with a specified stability margin. The boiler model has non-minimum phase and integrating properties. A two step design procedure is newly presented so that our design method can be applied to integrating processes. The multi-loop controller consists of one PID and one PID-PD controllers, and each PID controller is designed by using a one-shot input-output response around the operating point. The control performance is considerably improved compared with a conservative initial PID gain.

Keywords: PID control; Data models; Control system design; Filtering techniques.

1. INTRODUCTION

PID control is widely used for many industrial plants, and it is important to set the PID gains at the appropriate value for energy saving and high-quality products. However, it is said that half of the PID gains are not well-tuned, and it is significant to develop an easy to use design method that can be applied to wide range of plants.

In model-based control design, plant modeling is usually costly or modeling error brings about inherent difficulty. Therefore, it is expected that a data-driven design method that does not require plant modeling has the potential to give a solution to this problem. There are a few data-driven methods for off-line control; IFT (iterative feedback tuning) (Hjalmarsson et al. [1999]), VRFT (virtual reference feedback tuning) (Campi et al. [2002]), FRIT (Fictitious Reference Iterative Tuning) (Kaneko et al. [2005]), and so on.

Loop shaping is recognized as a useful and primary design criterion in the classical and also robust control designs (Skogestad & Postlethwaite, [1996]). We have applied the idea of the unfalsified control given in the reference (Safonov & Tsao, [1997]) to the mixed sensitivity control problem and its variants, and developed a data-driven loop shaping design method of PID control. First, we give a parameter plane method, and then a numerical optimization method (Saeki, [2008], Saeki & Kishi, [2011]). In our method, many fictitious plant responses are generated by filtering a single plant response with many bandpass filters, i.e. a filter bank. This filter bank method is essential for the improvement of the efficiency of controller falsification together with the method of the virtual reference input.

Our method has the following features.

One-shot plant response data that may be measured in the normal operating condition can be used for design, if the plant is sufficiently excited at the steady state and the influence of the disturbance to the response data is negligibly small during the experimental phase for the tuning of the controllers. A plant step response in the open-loop operation, or a plant response for a step reference input in the closed-loop operation can be used for design. When the plant dynamic characteristics vary depending on operating points, a more robust PID control can be designed immediately by using the plant response data measured around those operating points.

Our method can be applied to a wide range of plants without care to time-delay or unstable-zeros of the plant. Note that those design methods that adopt a model matching performance criterion such as VRFT and FRIT need the identification of these properties in order to set the appropriate reference model (Campestrini et al. [2011]).

As the iteration being used in the $H_{\infty}$ control, in our data-driven method for the mixed sensitivity control problem, it is also necessary to solve the optimization problem iteratively for various values of a parameter of the performance index. This problem can be mitigated considerably in MIGO (M constrained Integral Gain Optimization), which has been studied for a transfer function model (Åström et al. [1998], Åström & Hägglund [2006]).

Therefore, we have developed a data-driven version of MIGO for the problems of maximizing the integral gain subject to the maximum sensitivity constraint or the open-loop stability constraint. As a result, selection of the constraint becomes very easy in these two methods. The former constraint is non-convex, and a local solution is obtained (Saeki, [2008]). The latter constraint is linear, and the global solution can be easily obtained (Saeki &
Kishi. [2011]). The latter method is the easiest to use among our methods, though the applicable plant is limited to stable plants.

In this paper, we will apply this method to a multi-loop PID control design for the boiler benchmark problem (Moriglia. [2011]). The main control difficulties of this problem are caused by the coupling, the non-minimum phase, the integration and the load disturbance (Moriglia. [2011]). We will design each PID gain using the plant response data of the feedback system controlled by the initial controller conservatively tuned. We will show that our method can treat non-minimum phase property without any difficulty. In order to apply our simplest method to integrating plants, we will propose a two step design procedure of a PID-PD controller, and apply it to the benchmark. We will evaluate the control performance and compare it with that of the initial controller.

2. DATA-DRIVEN LOOP-SHAPING METHOD

2.1 Design problem

Let us consider a feedback system described by

\[ y = P(u + d) \]
\[ u = K(r - y) \]

where \( u \) and \( y \) are the plant input and output, \( r \) is the reference input, and \( d \) is the disturbance. \( P \) is a single-input-single-output linear time-invariant plant and \( K \) is a PID controller. The transfer functions of \( P \) and \( K \) are \( P(s) \) and

\[ K(s) = K_P + K_I \frac{1}{s} + K_D \frac{s}{1 + bs} \]

respectively, where \( b \) is a time constant. We also represent the PID gain as \( J = [K_P, K_I, K_D] \).

It is assumed that a plant transient response around an operating point can be measured by applying an external input to the plant at the steady state. Since we design the controller directly from the plant response \( u(t), y(t), t \in [0, T]\), the influence of disturbance \( d(t) \) on the plant response must be negligibly small during the experimental phase for the tuning of the controllers. If the measured data is biased, namely, \( y(0) \neq 0 \) or \( u(0) \neq 0 \), it is unbiased as \( y(t) - y(0), u(t) - u(0) \) for \( t \in [0, T] \) in our design procedure.

It is also required that the plant modes are sufficiently excited by the external input. It is preferable that the plant responses can be obtained in the normal operating conditions. As examples of suitable external inputs, a step or rectangle plant input in the open-loop operation, or a step or rectangle reference input in the closed-loop operation can be enumerated.

We would like to design a PID controller that gives a good disturbance rejection with a sufficient stability margin by using the finite-time plant response.

2.2 Design method

In this subsection, we will give the problem formulation, and explain the design procedure. This result is given in the reference (Saeki & Kishi. [2011]).

First, explain the performance index for disturbance attenuation. Let \( y(t) \) be the response of the feedback system (1) and (2) for the unit step disturbance input. Then, the next equality holds.

\[ \int_0^\infty y(t) dt = \frac{1}{K_I} \]

Hence, it is expected that a good disturbance rejection is attained by maximizing \( K_I \), and \( K_I \) can be used as the performance index for disturbance rejection (Åström et al. [1998]).

Next, explain the stability margin constraint. Consider the next open-loop stability condition.

\[ \text{Re} \{P(j\omega)K(j\omega)\} > a, \omega \in R \]

From the Nyquist stability criterion, this condition is only applicable to stable plants and \(-1 < a < 0\) needs to be satisfied in order to guarantee the closed-loop stability. From (5), the next time-domain condition can be derived as a necessary condition.

\[ K_P \langle u, y \rangle_T + K_I \langle u, y \rangle_T + K_D \langle u, y \rangle_T \geq a \langle u, u \rangle_T \]

where \( \langle u, v \rangle_T = \int_0^T w(t)v(t) dt \) for signals \( w(t), v(t) \in R, t \in [0, \infty) \), and \( y_I \) and \( y_D \) are the filtered outputs generated by

\[ y_I = \frac{1}{s} y, \quad y_D = \frac{s}{1 + bs} y \]

with zero initial conditions.

The stability margin constraints can be obtained by applying many plant responses to the inequality (6). However, many experiments are necessary to get these data. Instead of using the actual plant responses, we have proposed generating many fictitious responses by using a filter bank. Namely, first, set sampling frequencies \( \omega_i, \quad i = 1, 2, \ldots, n_\omega \) and give the corresponding bandpass filters \( F_i(s) \) that have a peak gain near \( \omega_i \). Then, generate \( n_\omega \) sets of signals \( u_F(t), y_F(t), t \in [0, T] \) by \( u_F = F_i u, \quad y_F = F_i y \), and apply these signals to (6).

We have used the next bandpass filter for \( \alpha > 0 \), which has a peak gain at \( \omega = \omega_i (1 + \alpha^2)^{0.5} \).

\[ F_i(s) = \left( \frac{2\alpha\omega_i s}{(s + \alpha\omega_i)^2 + \omega_i^2} \right)^4 \]

The parameter \( \alpha \) is given by the algorithm: Calculate \( \alpha^* = 2\pi/T\omega_i \), then set \( \alpha = 0.05(\alpha^* < 0.05), \alpha^*(0.05 \leq \alpha^* \leq 1), \) or \( 1(\alpha < \alpha^*) \).

Thus, our problem is formulated as a convex problem; find the PID gain that maximizes \( K_I \) subject to the linear constraints

\[ K_P \langle u, y \rangle_T + K_I \langle u, y \rangle_T + K_D \langle u, y \rangle_T \geq a \langle u, u \rangle_T \]

where \( u = u_F, \quad y = y_F, \) for \( i = 1, 2, \ldots, n_\omega \).
Design procedure

**Step 1)** Set $\omega_i, i = 1, 2, \ldots, n_\omega$ and $a \in (-1, 0)$.

**Step 2)** Measure a plant response $u(t), y(t), t \in [0, T]$ by giving an external input to the plant at the steady state, and unbias the data.

**Step 3)** Generate responses $u_F, y_F$ using the filter bank $F_i(s), i = 1, 2, \ldots, n_\omega$, and obtain the linear constraints (9).

**Step 4)** Obtain the optimal PID gain that maximizes $K_I$ subject to the constraints by linear programming.

In the discrete time case where the data is given by $u(i), y(i), i = 1, 2, \ldots, m$, the design procedure is modified. The continuous time filters (7) and (8) are discretized using zero order hold and the sampling interval $t_s$, and the integration for the inner product of (9) is approximated by summation.

Our design method has the following features.

- It can be applied to wide range of plants that contain time-delay or non-minimum phase plants. It is unnecessary to care whether the plant has time-delay or non-minimum phase zeros or not.
- It has only a few design parameters; the stability margin $a$ and the sampling frequencies. In addition, they can be easily chosen. We recommend to choose $a$ from $[-0.5, -0.3]$ for adequate stability margin. Information about the bandwidth of the plant and the observation noise is useful to determine the sampling frequencies.
- It does not require iterations.
- The optimization problem is convex, and it can be easily solved.

### 2.3 Two step design for integrating process

Since the water level process of the benchmark shows an integrating response, the above design procedure cannot be directly applied to this process.

In this subsection, we will present a solution to this problem; a two step design of an PID-PD controller. Assume that the integrating process is stabilizable by PD control. Consider the PID-PD controller described by

\[
\begin{align*}
\dot{u} &= u_2 - u_1 \\
u_1 &= K_{21}y + w \\
u_2 &= K_{22}(r - y)
\end{align*}
\]

where $K_{21}$ is a PD controller and $K_{22}$ is a PID controller. $K_{21}$ is used for the stabilization of the plant, and $K_{22}$ is used for the compensation of the stabilized local-feedback system $\dot{P} = P/(1 + PK_{22})$. $w$ is an external input to get the plant response.

We will examine a situation that $K_{21}$ and $K_{22}$ have already been implemented and that it is required to tune them by measuring the plant response in the closed-loop operation.

First, measure $u_1$ and $y$ by giving an external signal $w(t)$ with $r(t) = 0$, and design $K_{21}$ where the proportional gain is maximized. This is because the proportional gain for the integrating plant corresponds to the integral gain for the stable part of the plant. Next, measure $u_2$ and $y$ by giving an external signal $r(t)$ with $w(t) = 0$, and design $K_{22}$ where the integral gain is maximized.

### 3. APPLICATION TO BENCHMARK

#### 3.1 MIMO boiler plant model

Let us explain the MIMO boiler model of the reference (Morilla [2011]) briefly. The control system uses the same nonlinear model proposed by Pellegrinetti & Bentsman [1996]. The model has been developed in Simulink including some changes. The source code of the boiler block is protected and the concrete nonlinear model is not given, but the plant responses can be obtained by simulation.

The MIMO boiler model has 3 inputs and 3 outputs. The control inputs are steam pressure $u_1$ and water level $u_2$, and the disturbance $d$ is load flow. The plant outputs that can be used for control are fuel flow $y_1$ and water flow $y_2$, and oxygen level can be used as quality performance variable.

The input variables need to be in the range $0 \sim 100\%$, and additionally a rate limit of $\pm 1\%/s$ has been incorporated for the fuel flow. The model always starts in the same operating point given by: Fuel flow $\approx 35.21\%$, Water flow $\approx 57.57\%$, Load level $\approx 46.36\%$, Steam pressure $= 60\%$, Water level $= 50\%$, and Oxygen level $= 50\%$.

The open-loop features can be observed by the step test. The steam pressure response is stable for the three inputs. The oxygen level is only slightly affected by the fuel flow. The water level shows non-minimum phase behavior for the fuel flow and the load level in addition to an integrating response for the three inputs. The main control difficulties
in this multi-variable process are caused by the coupling, the non-minimum phase, the integration and the load disturbance.

### 3.2 Controller structure

We will design a multi-loop PID controller by applying our data-driven method that uses (6). We use a discrete time PID controller described by

\[
K(z) = K_P \left(1 + \frac{t_c}{T_d} z^{-1} + \frac{T_d}{t_c} (b + 1) z^{-1} - b\right) \tag{14}
\]

where \( t_c \) is the sampling period. This is a discrete time approximation of (3) and the PID gain is given by

\[
f = [K_P, K_1, K_D] = [K_P, K_P/T_1, K_P T_D] \tag{15}
\]

The block diagram of the multi-loop PID control system is shown in Fig. 2.

![Boiler control system diagram](image)

**Fig. 2.** Boiler control system

Since the transfer function from \( u_1 \) to \( y_1 \) is stable, we will design a PID controller for \( K_1 \). In order to bound the magnitude of \( u_1 \) in the range of \( 0 - 100 \% \) and incorporate the rate limit \( \pm 1 \% \), the PID controller \( K_1 \) has a saturation function \( \phi \) and a rate limiter \( \psi \) with anti-windup compensation. Namely, \( K_1 \) is described by

\[
u_1 = \phi(\psi(\tilde{u})) \tag{16}
\]

\[
\tilde{u} = K_P \left(e_1 + \frac{T_D}{t_c} \frac{z-1}{(b+1)z-b} e_1 \right. \\
\left. + \frac{t_c}{T_d} \frac{z-1}{z-1} (e_1 + v)\right) \tag{17}
\]

\[
v = \Lambda (u_1 - \tilde{u}) \tag{18}
\]

where we set the anti-windup compensator gain as \( \Lambda = 1/|K_P| \).

Since the transfer function from \( u_2 \) to \( y_2 \) has integrating property, we will design the above mentioned PID-PD controller. In order to bound the magnitude of \( u_2 \) in the range \( 0 - 100 \% \), \( u_2 \) is bounded by a saturation function and the PID controller \( K_{22} \) also has a saturation function \( \phi \) with anti-windup compensation. \( K_{22} \) is described in the same way as \( K_1 \), where the rate limiter \( \psi \) is removed. \( K_{21} \) is a linear PD controller.

### 3.3 Controller design

Suppose that the plant operates at the initial operating point mentioned above where the PID gains of \( K_1 \), \( K_{21} \), \( K_{22} \) are set at \( f_{10} = [1, 0.01, 0] \), \( f_{210} = [1, 0, 1] \), \( f_{220} = [1, 0.01, 0] \), respectively. These gains are not well-tuned. We will examine the sequential tuning of the PID gains. Since the plant is integrating and non-minimum phase in the second control-loop, we will tune the second loop first to get a sufficient stability margin, and next, the first loop to improve the setpoint response to \( r_1 \). Application of our method to such a simultaneous tuning problem of the BLT tuning method is also possible (Johnson & Moradi [2005]), but it is our future work.

The same design parameters are used in each design. The time constant is \( b = 0.01 \) and the sampling period is \( t_c = 10s \). The stability margin parameter is \( a = -0.5 \). The appropriate value of \( a \) usually lies in the interval \([-0.5, -0.3]\) from our experience. The sampling frequencies \( \omega_i, i = 1, 2, \ldots, n_d \), are logarithmically equally spaced 100 points between \( 10^{-3} \) and the Nyquist frequency 3.14 rad/s.

First, design the PD controller \( K_{21} \). The controlled object is a feedback system composed of \( P \), \( K_1 \), \( K_{22} \), and \( u_{21} \) and \( y_2 \) are its input and output. We apply a step input \( w(t) = 5\% \) at \( t \geq 0 \) to the total system controlled by the initial gains \( f_{10}, f_{210}, f_{220} \), and measure the responses \( u_{21}(t) \) and \( y_2(t) \) (Fig. 3). Since the plant is integrating, the proportional gain \( K_P \) is maximized instead of \( K_1 \). Our design method gives

\[
f_{21} = [3.2957, 0, 0.3460] \tag{19}
\]

and the initial gain \( f_{210} \) is replaced with \( f_{21} \).

![Plant response to \( w = 5\% \) for the design of \( K_{21} \)](image)

**Fig. 3.** Plant response to \( w = 5\% \) for the design of \( K_{21} \)

Next, design the PID controller \( K_{22} \) where the controlled object is the feedback system composed of \( P \), \( K_1 \), \( K_{21} \), and \( u_{22} \) and \( y_2 \) are its input and output. We apply a step reference input \( r_2(t) = 5\% \), \( t \geq 0 \) to the total system controlled by the gains \( f_{10}, f_{21}, f_{220} \), and we measure the responses \( u_{22}(t) \) and \( y_2(t) \) (Fig. 4). Our design method gives

\[
f_{22} = [2.6556, 0.1148, 11.5022] \tag{20}
\]

and the initial gain \( f_{220} \) is replaced with \( f_{22} \). The anti-windup compensator gain is also replaced with the new gain by \( \Lambda = 1/|K_P| \).

![Plant response to \( r_2 = 5\% \) for the design of \( K_{22} \)](image)

**Fig. 4.** Plant response to \( r_2 = 5\% \) for the design of \( K_{22} \)
Lastly, design the PID controller $K_1$ where the controlled object is the feedback system composed of $P_2$, $K_{21}$, $K_{22}$, and $u_2$ and $y_1$ are its input and output. We apply a step reference input $r_1(t) = 5\%$, $t \geq 0$ to the total system controlled by the gains $f_{10}$, $f_{21}$, $f_{22}$, and measure the responses $u_1(t)$ and $y_1(t)$ (Fig. 5). Our design method gives

$$f_1 = [3.4917, 0.1112, 18.5671]$$

and the initial gain $f_{10}$ is replaced with $f_1$. The anti-windup compensator gain is also replaced with the new gain by $\Lambda = 1/[K_P]$.

Fig. 5. Plant response to $r_1 = 5\%$ for the design of $K_1$

![Graphs](a) $u_1$ (b) $y_1$

3.4 Evaluation by simulation

We will compare the control performance of the initial system with $f_{10} = [1, 0.01, 0]$, $f_{210} = [1, 0, 1]$, $f_{220} = [1, 0.01, 0]$ and the newly designed system with $f_1 = [3.4917, 0.1112, 18.5671]$, $f_{21} = [3.2957, 0, 0.3460]$, $f_{22} = [2.6556, 0.1148, 11.5022]$ by the three tests of the benchmark. Since the state variables of the integrator and the derivative filter that constitute the PID controllers take certain values at the steady state, these values are set as the initial condition for simulation.

In the first test, we evaluate the responses of the feedback system for the setpoint change. $r_1(t)$ is changed from 60% to 65% at $t = 100s$, then $r_2(t)$ is changed from 50% to 55% at $t = 2100s$. The results are shown in Fig. 6. The responses of the initial system are shown by dotted lines, which is shown by Reference case in the figures. The responses of the newly designed system are shown by solid lines, which is shown by Evaluated case.

![Graphs](a) $u_1$ (b) $y_1$ (c) $u_2$ (d) $y_2$

Fig. 6. Responses to step reference inputs (test 1)

![Graphs](a) $u_1$ (b) $y_1$ (c) $u_2$ (d) $y_2$

Fig. 7. Responses to step disturbance input (test 2)

Fig. 6(b) shows that the response $y_1(t)$ to $r_1(t)$ is improved considerably and the influence of $r_2(t)$ to $y_1(t)$ for $t > 2100$ is small. Fig. 6(d) shows that the response $y_2(t)$ to $r_2(t)$ is improved and the influence of $r_1(t)$ to $y_2(t)$ at $t = 100$ is not small. Responses to $r_1(t)$ and $r_2(t)$ are evaluated separately, and the results are listed in Table 1 and Table 2, respectively. The first three indexes are the Ratios of Integrated Absolute Error for the steam pressure, the oxygen level, and the drum water level. The fourth and fifth are the Ratios of Integrated Time multiplied Absolute Error for the steam pressure and the water level. The sixth and seventh are the Ratios of Integrated Absolute Variation of Control signals for the fuel flow and the water flow (see Morilla [2011]). A ratio less than one implies that the corresponding performance index is decreased by the tuning.

| Table 1. Evaluation indexes to $r_1(t)$ (test 1) |
| RIAE$_1$ | RIAE$_2$ | RIAE$_3$ | RITAE$_1$ |
| 0.1276 | 1.1454 | 0.3565 | 0.0656 |
| RIAE$_3$ | RIAU$^1_1$ | RIAU$^2_2$ | J |
| 2.4585 | 6.5161 | 0.8749 |

In the second test, we evaluate the responses to the disturbance of the load level change. The load level $d(t)$ is changed by 20% from 46.36% at $t = 100s$. The results are shown in Fig. 7. Fig. 7(b) shows that the response of $y_1$ to the load level change has been much improved. Fig. 7(d) shows that the response of $y_2$ to the load level change becomes faster. The evaluation indexes are listed in Table 3.

| Table 2. Evaluation indexes to $r_2(t)$ (test 1) |
| RIAE$_1$ | RIAE$_2$ | RIAE$_3$ | RITAE$_1$ |
| 0.4218 | 0.9998 | 0.4013 | 0 |
| RIAE$_3$ | RIAU$^1_1$ | RIAU$^2_2$ | J |
| 0.7238 | 6.4966 | 5.1980 | 1.2156 |

In the third test, we evaluate the responses to a load level change shown in Fig. 11 of the benchmark Morilla [2011]. The results are shown in Fig. 8, and the evaluation indexes are listed in Table 4.
Table 3. Evaluation indexes (test 2)

<table>
<thead>
<tr>
<th>RIAE1</th>
<th>RIAE2</th>
<th>RIAE3</th>
<th>RITAE1</th>
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<tbody>
<tr>
<td>0.1070</td>
<td>1.0011</td>
<td>0.1580</td>
<td>0.7170</td>
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Table 4. Evaluation indexes (test 3)

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<th>RIAE1</th>
<th>RIAE2</th>
<th>RIAE3</th>
<th>RITAE1</th>
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<td>1.0002</td>
<td>0.1384</td>
<td>0.1146</td>
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</tbody>
</table>

Fig. 8. Responses to a disturbance input (test 3)

Fig. 9. Responses to a staircase reference input $r_1$

4. CONCLUSION

In this paper, a multi-loop PID controller that consists of PID and PID-PD controllers have been designed for the TITO boiler plant by our data-driven method. Our method is applicable to SISO stable plants, and the integral gain is maximized subject to the stability margin constraint. The control performance was considerably improved compared with the initial PID gain that was conservatively tuned. We have shown the following. A two-step design procedure for a PID-PD controller was proposed so that our method can be applied to integrating plants. A design procedure for multi-loop systems was explained for the boiler TITO model.

It was shown that our method is easy to use and the design procedure is not iterative. Namely, the non-minimum phase property of the boiler plant was coped with successfully without any care to it, the design parameter could be easily set, and the same values of the design parameters were used for all the three designs. Further, only one shot response data of the plant excited by a step external input in the closed-loop operating was used for each design of the three PID controllers.

REFERENCES


