

# Electronic color charts for dielectric films on silicon

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**Abstract:** This paper presents the calculation of the perceived color of dielectric films on silicon. A procedure is shown for computing the perceived color for an arbitrary light source, light incident angle, and film thickness. The calculated color is converted into RGB parameters that can be displayed on a color monitor, resulting in the generation of electronic color charts for dielectric films. This paper shows generated electronic color charts for both silicon dioxide and silicon nitride films on silicon.

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**OCIS codes:** (330.1690) Color, (330.1710) Color measurement, (310.6860) Thin films, optical properties

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## Reference and Links

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## 1. Introduction

Silicon dioxide and silicon nitride are among the most ubiquitous of all materials used in the fabrication of integrated electronic and optical devices. It has long been common knowledge that slight changes in the thickness of these films create major shifts in their perceived color [1]. This color change is caused by variations in the reflectance of the thin dielectric film with wavelength [2]. Experienced technicians and researchers are accustomed to estimating a film's thickness simply by observing its color. However, the colors of dielectric films are typically either described in tables without showing the actual color, or a few silicon samples with different dielectric thicknesses and colors are kept for comparison purposes.

In this article, we calculate the perceived color of a dielectric film for any thickness and viewing angle. The perceived color is converted into RGB parameters such that the corresponding color can be displayed on a color computer monitor. These calculations allow the generation of complete electronic color charts over an arbitrary range specified by a user. These generated charts can show transitions between colors very accurately and can be displayed on monitors that are prevalent in most laboratory settings. This paper is meant to be a resource engineers and researchers can use for determining film thickness as well as a reference containing the physical parameters and methods for calculating film colors.

## 2. Calculating RGB parameters

The basic process used to calculate the RGB parameters for an arbitrary film thickness and viewing angle can be broken up into two steps. First, the reflected power as a function of wavelength is calculated. Second, the response of the eye is taken into account to calculate the CIE XYZ parameters from the wavelength spectrum. The XYZ parameters can then be converted into RGB parameters that are used by a typical computer monitor.

### 2.1 Calculating the wavelength spectrum

The spectrum of light reflected off a dielectric layer is the spectrum of the incident light source  $P_i(\lambda)$  multiplied by the total power  $P_o$  and the film reflectance  $R(\lambda)$ . Figure 1 shows the spectrum of a standard fluorescent light that is used in this work [3]. A fluorescent light spectrum is used because this type of lighting is typical in most laboratories where dielectric films are inspected.

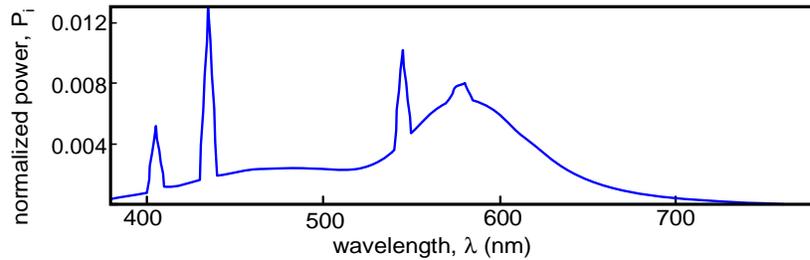


Fig. 1. Standard fluorescent light wavelength spectrum normalized by total power.

Figure 2 illustrates the reflection of an incident beam off of a dielectric layer deposited on a silicon substrate. The reflection of the transmitted wave  $P_t$  off of the back surface of the silicon is ignored since the silicon is absorptive at the visible wavelengths of interest. The reflectance  $R(\lambda)$  is calculated as a function of incident wavelength using a standard thin film matrix approach [4].

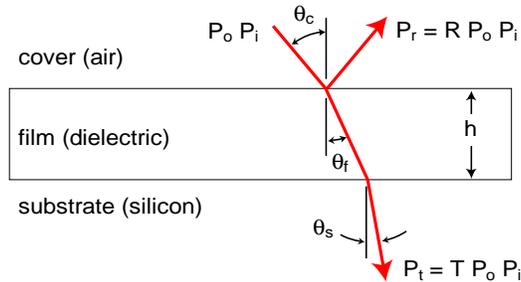


Fig. 2. Dielectric film.

The reflectance for the single dielectric layer is given by

$$R(\lambda) = \frac{\left| Y_f (Y_c - Y_s) \cos \phi + i (Y_c Y_s - Y_f^2) \sin \phi \right|^2}{\left| Y_f (Y_c + Y_s) \cos \phi + i (Y_c Y_s + Y_f^2) \sin \phi \right|^2}. \quad (1)$$

The parameters  $Y_c$ ,  $Y_f$ , and  $Y_s$  are respectively the impedance of the cover, film, and substrate that are given by

$$Y_m = \eta_o n_m \cos \theta_m, \quad (2)$$

for transverse electric (TE) polarization and

$$Y_m = \eta_o n_m / \cos \theta_m, \quad (3)$$

for transverse magnetic (TM) polarization, where  $m = c, f,$  or  $s$  for the various regions,  $\eta_o = \sqrt{\epsilon_o / \mu_o}$  is the impedance of free-space,  $\epsilon_o$  is the electrical permittivity, and  $\mu_o$  is the magnetic permeability. The absorption loss of the silicon is taken into account by using the complex index of refraction  $n_s = n - jk$ . The phase delay caused by propagation through the dielectric slab is given by

$$\phi = k_o n_f h \cos \theta_f, \quad (4)$$

where  $k_o = 2\pi/\lambda$  is the wave number of free-space,  $h$  is the thickness of the dielectric layer, and  $\theta_f$  is the propagation angle in the dielectric film.

The wavelength dependence of Eq. (1) is caused by two factors. First, Eq. (4) shows that the phase delay changes with wavelength. Second, the material dispersion causes the indices of refraction for the film and substrate to change with wavelength. Figure 3 shows the index of refraction for silicon dioxide  $\text{SiO}_2$ , silicon nitride  $\text{Si}_3\text{N}_4$ , and silicon, where the complex index of refraction of silicon is  $n_{si} = n - jk$  [5].

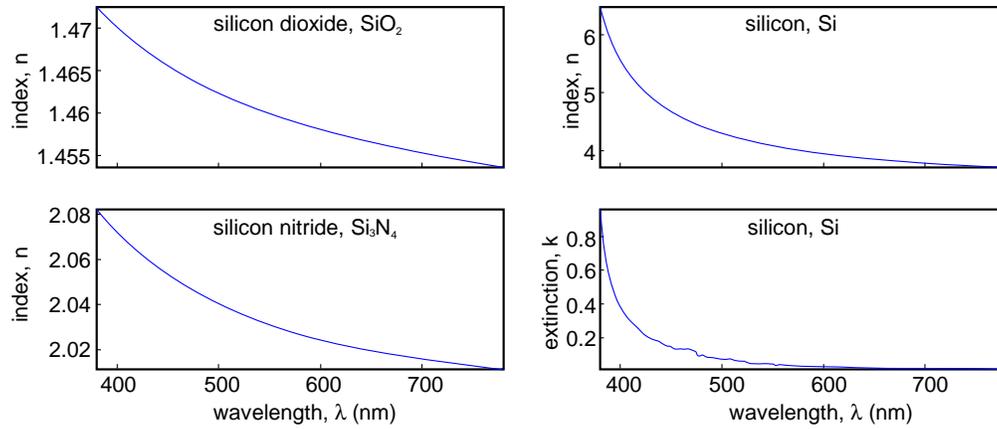


Fig. 3. Material parameters for silicon dioxide, silicon nitride, and silicon.

## 2.2 Calculating the RGB parameters

There are two steps required to convert the reflected wavelength spectrum  $P_r(\lambda)$  into RGB parameters that can be displayed on a standard computer monitor. These two steps are calculating the XYZ parameters from the wavelength spectrum, and converting the XYZ parameters into RGB parameters.

The CIE color space is a color standard that is modeled on the color sensitivity and perception of the human eye. The gamut of colors that it describes is used as a high fidelity device-independent color system that closely approximates the range and scope of colors that are perceivable. Any given spectrum can be decomposed into X, Y, and Z tristimulus components by applying the CIE color-matching equations [6] to the spectrum.

Figure 4 shows the color matching functions for the three tristimulus components. The X, Y, and Z components are calculated by integrating the wavelength spectrum of the reflected light  $P_r(\lambda)$  times the color matching functions as given by

$$X = \int_0^{\infty} R(\lambda) P_o P_i(\lambda) X_\lambda(\lambda) d\lambda, \quad (5)$$

$$Y = \int_0^{\infty} R(\lambda) P_o P_i(\lambda) Y_\lambda(\lambda) d\lambda, \quad (6)$$

$$Z = \int_0^{\infty} R(\lambda) P_o P_r(\lambda) Z_\lambda(\lambda) d\lambda, \quad (7)$$

where  $P_o$  is the total source power. A change in the source luminance or total power will cause a scaling of the X, Y, and Z parameters.

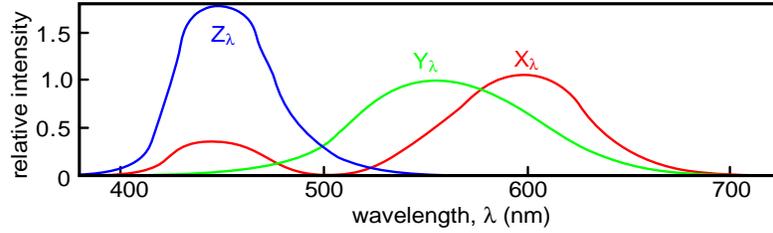


Fig. 4. CIE color-matching functions (CIE 1931 2 degree observer).

The XYZ color parameters need to be converted to a device-dependent color scheme (such as RGB or NTSC) before they can be displayed or printed. The standard IBM-compatible monitor uses sRGB. The conversion to sRGB [7, 8] is accomplished by multiplying the XYZ vector by a transformation matrix.

$$\begin{bmatrix} R & G & B \end{bmatrix} = \begin{bmatrix} X & Y & Z \end{bmatrix} [M]^{-1} \quad (8)$$

The transformation matrix  $M$  is generated using the monitor's phosphor chromaticity coordinates  $(x_r, y_r)$ ,  $(x_g, y_g)$ , and  $(x_b, y_b)$  and the reference white of the source  $(X_w, Y_w, Z_w)$ . In this work, the sRGB standard phosphor values are used [9, 10] as given by  $(x_r=0.64, y_r=0.33)$ ,  $(x_g=0.30, y_g=0.06)$ , and  $(x_b=0.15, y_b=0.06)$ . The reference white is obtained by applying the color-matching functions Eqs. (5)-(7) to the source spectrum. The white point of the standard fluorescent light used in this work is calculated to be  $(X_w, Y_w, Z_w) = P_o (0.10, 0.10, 0.06)$ . The transformation matrix is given by [8]

$$[M] = \begin{bmatrix} S_r X_r & S_r Y_r & S_r Z_r \\ S_g X_g & S_g Y_g & S_g Z_g \\ S_b X_b & S_b Y_b & S_b Z_b \end{bmatrix}, \quad (9)$$

where  $X_\ell = x_\ell / y_\ell$ ,  $Y_\ell = 1$ , and  $Z_\ell = (1 - x_\ell - y_\ell) / y_\ell$  with  $\ell = r, g, \text{ or } b$  and

$$\begin{bmatrix} S_r & S_g & S_b \end{bmatrix} = \begin{bmatrix} X_w & Y_w & Z_w \end{bmatrix} \begin{bmatrix} X_g & Y_r & Z_r \\ X_g & Y_g & Z_g \\ X_b & Y_b & Z_b \end{bmatrix}. \quad (10)$$

The transformation equation for the standard fluorescent lighting and sRGB phosphors is calculated to be

$$\begin{bmatrix} R & G & B \end{bmatrix} = \frac{1}{P_o} \begin{bmatrix} X & Y & Z \end{bmatrix} \begin{bmatrix} 23.42 & -9.83 & 1.16 \\ -11.11 & 19.02 & -4.24 \\ -3.60 & 0.42 & 21.96 \end{bmatrix}^{-1}. \quad (11)$$

Notice that the source power in the transformation matrix cancels out the source power in Eqs. (5)-(7). The RGB parameters are then scaled to compensate for the display gamma as given by

$$\ell = \begin{cases} 12.92 \ell & \ell \leq 0.00313 \\ 1.055 \ell^{1/2.4} - 0.055 & \ell \geq 0.00313 \end{cases}, \quad (12)$$

where  $\ell$  is the  $R$ ,  $G$ , or  $B$  parameter. The  $R$ ,  $G$ , and  $B$  parameters in this formulation should vary from 0 to 1. However, the XYZ color space contains colors that do not lie within the RGB subspace; this can cause the  $R$ ,  $G$ , and  $B$  parameters to be outside of that range. Therefore, any value less than 0 or greater to 1 are respectively set to 0 and 1, which results in an approximation of the colors not represent by the chosen RGB system. In addition, a typical display uses an RGB color scheme that ranges from 0 to 255. To convert to such a scheme, the RGB parameters are multiplied by 255 and rounded to the nearest integer.

### 3. Sample calculation

The wavelength spectrum of the light reflected off of the dielectric layer  $P_r$  is the power of the incident light  $P_o * P_i$  multiplied by the reflectance of the film  $R$ . Figure 5 shows the calculated film reflectance  $R$ , the incident power spectrum  $P_i$ , and the corresponding reflected power  $P_r$  for two silicon dioxide films. The thicknesses of these films were chosen to illustrate the very different reflected spectrums that can be produced – leading to different apparent colors.

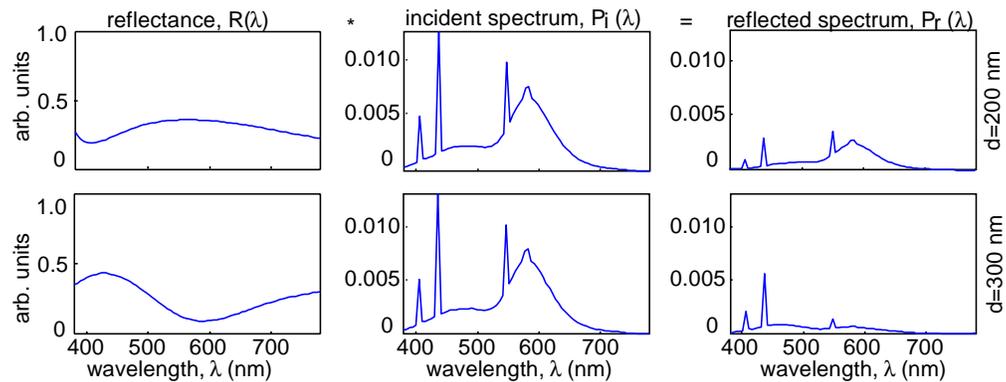


Fig. 5. Reflectance  $R(\lambda)$ , normalized spectrum  $P_i(\lambda)$ , and corresponding reflected spectrum  $P_r(\lambda)$  for silicon dioxide films of thickness  $d=200\text{nm}$ , and  $d=300\text{nm}$ .

The reflected power spectrum  $P_r$  is converted into CIE XYZ parameters using Eqs. (5)-(7). The sRGB parameters are then calculated from the XYZ parameters using Eq. (11). Table 1 shows the color parameters and the corresponding color.

Table 1. color parameters for two  $\text{SiO}_2$  films.

Thickness	X	Y	Z	R	G	B	Color
$d=200\text{nm}$	0.035	0.037	0.015	159	162	128	
$d=300\text{nm}$	0.014	0.014	0.024	87	100	186	

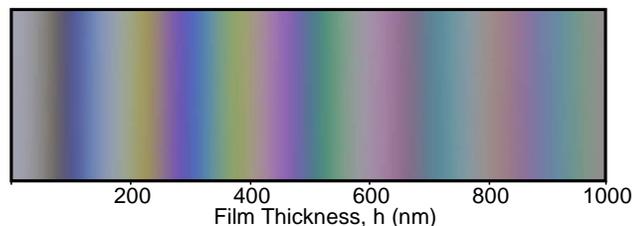


Fig. 6. Apparent color of a  $\text{SiO}_2$  film on silicon as a function of thickness.

### 4. Color charts

The color of the dielectric film on silicon depends on the incident angle of the light source, the luminance of the source, the thickness of the dielectric film, and the dielectric material. Figure 6 shows the color of a silicon dioxide ( $\text{SiO}_2$ ) film as a function of film thickness with a source

and viewing angle of  $\theta_i=0^\circ$ . Figure 7 shows a similar color chart for a silicon nitride ( $\text{Si}_3\text{N}_4$ ) film viewed at normal incidence ( $\theta_i=0^\circ$ ). The large variation in color across Figs. 6 and 7 makes it difficult to use these images to compare against actual dielectric films. Therefore, a more interactive electronic color chart has been designed and is available on an internet site [11]. This interactive color chart allows for a user specified range of film thickness, source angle, and variation in the source luminance.

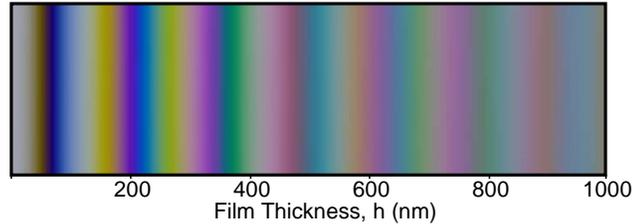


Fig. 7. Apparent color of a  $\text{Si}_3\text{N}_4$  film on silicon as a function of thickness.

Figure 8 is a Quicktime movie that provides the color of a silicon dioxide film as a function of film thickness. Each frame of this movie has a thickness variation of 50nm. The movie can be paused and the frame can be selected that corresponds to the desired film thickness. Thus, this movie provides a semi-interactive electronic color chart. Similarly, Fig. 9 shows the movie for a silicon nitride film. These two movies contain the same data shown in Figs. 6 and 7 along with the same values of incident angle and source luminance.

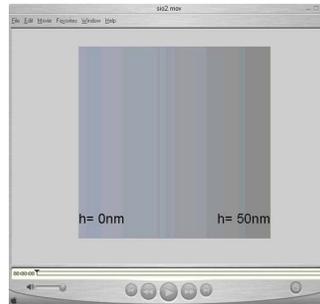


Fig. 8. (457 KB) Quicktime movie showing the color of a silicon dioxide ( $\text{SiO}_2$ ) film as a function of film thickness.

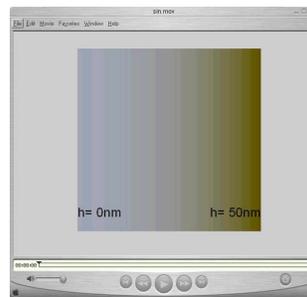


Fig. 9. (511 KB) Quicktime movie showing the color of a silicon nitride ( $\text{Si}_3\text{N}_4$ ) film as a function of film thickness.