An Improved Penalty-SA Based Resource Allocation Algorithm for OFDMA Cellular Systems
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Abstract—In a multi-cell OFDMA system, the total power is minimized by adopting a centralized approach to adjust the subcarriers and power allocation with the interference from other cells into consideration. In this paper, aiming at simplifying the model and reducing the computational complexity, we improved the Penalty-Simulation Annealing (PSA) resource allocation algorithm proposed by [4]. From the theoretical analysis and simulation results, we observe that the new method can dramatically reduce computational complexity compared with [4] without degrading the performance of the whole system. Compared with the multi-allocation algorithm in [2], our method can greatly improve the unit power throughput.

Index Terms—Resource Allocation; OFDM; cellular system; Penalty function; Simulation Annealing.

I. INTRODUCTION

The development of wireless communication technologies has dramatically changed people’s life. Since frequency resources are insufficient, it’s challenging to provide high speed data transmission and serve more users. The Orthogonal Frequency Division Multiplexing (OFDM) technology offers a solution to improve spectrum utilization due to its feature of combating frequency selective fading and inter symbol interference (ISI). Because of this, it is widely used in the lastest communication systems, such as wireless local area network and high speed ADSL access.

In a multi-user system, multi carrier technologies need to be combined with other multiple access technologies to achieve multi-user multiplexing. An Orthogonal Frequency Division Multiplexing Access (OFDMA) system assigns a different sub-carrier set to each user within a cell. It does not require the guard band between users and each sub-carrier used by a user does not necessarily continuous, but can be arbitrarily distributed as a unit[1]. Therefore it is more flexible than FDMA. Existing researches on resource allocation for an OFDM system have mostly been focused on single cell models. Abrardo has studied for multi cells [2], but the initial distribution is still based on the single cell model. The central controller serving multiple base stations carries out centralized resource allocation according to the feedback and allocates resources for multiple cells by removing the carrier that does not satisfy the multi-cell constraints from the carrier set through multiple allocation algorithms and then adjusting power to meet the requirement of throughput. Although this scheme reduces the complexity of the algorithm, the carrier resource can not be fully utilized. In [3] the allocation problem has been transformed into two sub-problems of allocating subcarrier and distributing power, which are solved separately using the traditional dichotomy. By doing this, a complex system is divided into two relatively simple models. However, this approach can only achieve a sub-optimal solution. It has been proved in [4-6] that in the model for rate maximization, with fixed total power using discrete power can greatly reduce the searching complexity without influencing the performance. Targeting at the resource allocation problem for a multi-cell OFDM system, Ji and Chen proposed a Penalty-Simulated Annealing (PSA) algorithm [7], which allocates sub-carrier and power resources at the same time for multiple cells and multiple users, distributes power with fixed rate, and carries out centralized resource allocation to achieve the optimal solution. Based on the scheme in [7], we propose an improved PSA (IPSA) algorithm in this paper, which dramatically reduces complexity without degrading performance. We also carry out a thorough investigation in the following two aspects: 1) whether the whole system will become stable with the increase of the number of users; 2) whether the choice of the number of discrete power values influence the performance of the whole system.

II. SYSTEM MODEL

The OFDMA system in this paper has a central controller, which analyzes the gain between different terminals and base stations and carries out resource allocation for multiple cells [8-9]. Consider N cells and the cell set is denoted as \( C = \{1, \ldots, K\} \). Cell \( k \) has \( U_k \) users and the user set is denoted as \( U_k = \{1, \ldots, U_k\} \). Let the carrier set be \( M = \{1, \ldots, x\} \) and the discrete power set be...
Also define variable \( x_{i,j,m} \) as follows:

\[
\begin{align*}
1, & \text{ The } J\text{-th carrier assigned to the } \ i\text{-th user and the power is } q_m, \\
0, & \text{ others}
\end{align*}
\]

Denote the set containing all users of the whole system by \( U = \bigcup_{k=1}^{K} U_k \) and let \( b(i) \) be the cell that user \( i \) belongs to.

In this paper, we have adopted a system model having seven cells, as shown in Figure 1. In each cell, users are randomly generated. The relationship between the interference and the signal is determined according to the distance between the user and the base station and the distribution of the carriers. For multiple cells, we can define the signal to noise ratio of each user as follows:

\[
SIR_i = \frac{p_i G_i(j)}{\sigma^2 + \sum_{h\neq i} p_h G_h(j)}
\]

Where \( p_i \) is the transmission power of the \( i \)-th user and \( G_i(j) \) is the channel gain if the \( j \)-th carrier is assigned to the \( i \)-th user. If users in other cells are assigned the same subcarrier as user \( i \), they will be regarded as the interference to user \( i \). Usually we only consider the interference from adjacent cells.

In OFDMA systems, the transmission rate is usually guaranteed by allocating a certain number of subcarriers and certain amount of power. Since the throughput is the summation of the transmission rates of all real-time services, optimization for the real-time service should minimize the transmission power under the premise that the required transmission rate is satisfied. Then the objective function can be defined as follows:

\[
f(x) = \sum_{i \in U} \sum_{j \in M} p_{i,m} x_{i,j,m}
\]

In order to reduce intra-cell interference, each carrier can only be assigned to one user in a cell, which means:

\[
\sum_{m \in iU_k} x_{i,j,m} \leq 1 \quad \forall j \in M
\]

Denote the number of carriers user \( i \) has used by \( R_i \), then

\[
r_i = R_i / \eta_i
\]

where \( R_i \) is the transmission rate and \( \eta_i \) is the spectrum efficiency, i.e.,

\[
\eta_i = \log_2 (1 + \text{SIR}_i)
\]

Therefore, we can simplify the model by transforming it into a single cell problem. Then the objective function becomes:

\[
p(x, M_k) = f(x) + M_k \{ \min(0, g(x)) \}^2
\]
where

\[ g(x) = G \sum_{m} p_{i,m} x_{i,m} - \sum_{k,b(k)\neq(b)} SIR^{-h(b)} \sum_{m} p_{i,m} x_{i,m} \]

(15)

\[ + SIR BN_{0} (Q-1) - SIR BN_{0} \sum_{m} x_{i,j,m} \]

and \( M_{k} \) is a relatively large number changing with a certain step size.

IV. IMPROVED PSA (IPSA) ALGORITHM

In this paper, we propose an improved PSA algorithm, where we require the total power in each cell to be less than a certain value and unify the Gaussian white noise and inter-cell interference into one ideal interference. Therefore,

\[ g(x) = g_{1}(x) + g_{2}(x) \]

(16)

where,

\[ g_{1}(x) = G \sum_{m} p_{i,m} x_{i,m} - \sum_{k,b(k)\neq(b)} SIR^{-h(b)} \sum_{m} p_{i,m} x_{i,m} \]

(17)

\[ + SIR BN_{0} (Q-1) - SIR BN_{0} \sum_{m} x_{i,j,m} \]

(18)

Our modification simplifies the penalty function. Therefore, the simulation latency is dramatically reduced. After simplification, the problem can be solved using the improved simulated annealing algorithm, which is described as follows.

Step 1: Initial value selection.

In our simulation we have used the randperm function to randomly allocate carrier and power resources and determine the initial values. Random numbers without repetition are generated as follows: since the ordering for generating any random number must be a sequence of integers. After generating \( n \) numbers among integers from 1 to \( n \) (allowing repetition), we assign a subscript to each of them, which represents their generation ordering. We can always order these numbers from small to large according to their values. After we reorder them, their subscripts are reordered as well. Then these reordered subscripts will form a sequence of random numbers without repetition.

Step 2: Determination of the solution region.

Any solution in the solution region must satisfy all the constraints; otherwise, the objective value will become very large due to the penalty function and the solution will be discarded. By setting the function \( e^{(-k_{i}/k_{b})} > \text{rand}(1) \), the algorithm can accept new solutions probability.

Step 3: Inner loop criteria.

Use the new state generation function to generate a new state \( x_{j} \). If the new state automatic function satisfies \( \Delta f_{j} = f(x_{j}) - f(x_{j}) > 0 \), then accept the new state. In a fixed temperature, if reached the maximum allowed times without improvement which defined by the objective function, and also achieved the lowest temperature of this loop, then stop the inner loop.

![Figure 1 Flow chart of the simulation annealing algorithm](image)

Step 4: Outer loop criteria.

Decrease the lowest temperature of the inner loop through the annealing temperature function \( T(k+1) = \alpha T_{k} \), where \( \alpha \) is between 0.8-0.99. Once the whole system is in the sample steady state, terminate the algorithm and return the lowest energy state found, which is the optimal solution.
The Improved PSA (IPSA) algorithm flow same as the original simulated annealing algorithm, it can be expressed in Figure 1, this paper improved the original algorithm in some way, as the new state generation methods and annealing temperature functions. In the process of simulation gradually adjust the parameters, including the Hamming distance, anneal function coefficients, so as to achieve the purpose of maximizing the system throughput.

V. SIMULATION RESULTS

The parameters in our simulation are set in the same way as in [9]: Let the total bandwidth of the system be 5MHz, the number of cells be 7, spectral efficiency $\eta = 4$, the power spectral density of the zero-mean thermal noise $N_0$ be $10^{-20}$, $G_{ij} = S_{ij}G_0A(\theta_{ij})/L(d_{ij})$ and S is for the shadow fading value. We have used log-normal distribution random numbers to represent shadow fading values; $G_0$ is comprised of a series of parameters, including the transmitter antenna gain, the receiver antenna gain, noise level, cable damage and penetration damage parameters, etc. Here we set $G_0 = 0 dB$, $A(\theta) = 25.11 dB$ and large scale fading $L(d) = 128.1 + 37.6 \log_{10}(d)$, where $L$ is the fading value in dB, and d is the distance between the user and the base station in km.

To reduce latency, we set the maximum number of the inner loop iterations to 500 and that of the outer loop iterations to 1000. When the number of iterations exceeds the maximum value, the loop will be terminated.

Figure 2 shows the convergence of the proposed IPSA algorithm under different spectral efficiency. We can see the number of iterations needed for different spectral efficiency is different.

When searching for the optimal solution, besides accepting the optimal solution, the SA algorithm can also accept some deteriorated solutions using a random acceptance criteria (Metropolis), and make the probability of accepting deteriorated solutions approach zero. This makes it possible for the algorithm to escape the local optimal solution and reach the global optimal and it also guarantees the convergence of the algorithm. This algorithm can described in math by the ergodic theory of the Markoff chain. It has been proved that if the initial temperature is high enough and the annealing process is slow enough, then the SA algorithm converges to the global optimal solution with probability 1.

In [2], it has been shown that better performance can be achieved by using equal interval discrete power. In this paper, we will also discuss this in detail. The speed of computation is doubled for the improved model. Figure 3 and Figure 4 depict the impact of different number of discrete power on the overall system performance. By repeating the experiment and taking the average value, we found that due to the randomness of the discrete power allocation, the power curve and throughput curve is mainly stable, but with some randomness. For discrete power with four, eight, sixteen, thirty-two, and sixty-four equal divisions, the total power and throughput of the system do not have certain trend. However, with the increase of the number of users, they will increase proportionally.

![Figure 2 Number of iterations under different spectral efficiency](image2)

![Figure 3 total power figure](image3)
VI. CONCLUSION

The radio resources such as the carrier power, etc., must be fully utilized since they are scarce. The model of this paper is to allocate subcarrier and power resources under a certain rate so as to minimize the total power of the system. We simplified the existing model and improved the computation speed dramatically. When allocating power and carrier for multi-cells at the same time, the model is much more complex than a single cell case. In this paper, we proposed an improved penalty function simulated annealing algorithm. The average throughput per user is improved by 30% compared with multi-cell allocation. The advantages of discretting power are more apparent.

Performance of a multi-cell system is one of the focuses in wireless communications, so our future research will be: 1) Change the problem model and study with fixed power how to allocate carriers, bandwidth, etc. to maximize the speed of the system. 2) Introduce MIMO technology and study the overall performance of the system.

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