

Quantitative Approaches to Fiscal Sustainability Analysis: A Case Study of Turkey since the Crisis of 2001

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This case study of fiscal sustainability in Turkey after the crisis in 2001 reviews and extends quantitative approaches to fiscal sustainability analysis and brings them together in a user-friendly tool applicable in a data-sparse environment. It combines a dynamic simulations approach with a steady-state consistency approach. It also incorporates user-defined stress tests and stochastic simulations to deal with uncertainty. And it derives the future distribution of debt-output ratios, evaluating the fiscal adjustment required to stabilize them. Value at Risk analysis shows that considerable risks remain unless explicit feedback rules from debt surprises to the primary surplus are implemented. JEL codes: E61, E62, F34, C15

Long-run sustainability has moved to center stage in the analysis of fiscal policy. This reorientation has been part of rethinking the role of government, with less emphasis on active involvement and more on providing a stable environment for the private sector. Unsustainable policies will change and will be expected to do so; they are a natural cause of instability. Fiscal sustainability analysis has thus become a key element of macroeconomic analysis. This study reviews and extends various recent approaches to fiscal sustainability analysis and combines them into one model for a study of fiscal sustainability in Turkey since the 2001 crisis. The analysis is of interest in its own right, but it also demonstrates that the model is useful for widespread application in low-income and semi-industrialized countries.

Fiscal sustainability analysis has, for example, become an important part of the design and evaluation of anti-inflation programs. Many elements of the analysis can be found in the International Monetary Fund's (IMF) workhorse for standby programs, the Polak monetary programming model. And a long

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series of balance of payments crises has been linked to a lack of fiscal sustainability, particularly the series of failed stabilization efforts in Latin America in the late 1970s and 1980s. There, unsustainable fiscal policies, or the anticipation of policy changes because of a perceived lack of sustainability, triggered the balance of payments crises that brought several stabilization programs to a crashing halt. Even the Asia crisis in 1998, where the main protagonists had no major visible fiscal imbalances, has been linked by some to off-budget fiscal liabilities related to implicit or explicit bailout guarantees in the financial sector (Burnside 2005; Valderrama 2005). There is little doubt that countercyclical fiscal policy (allowing deficits to increase in “bad” times) could backfire seriously if fears of fiscal sustainability were to lead to expectations of runaway deficits in response to a deficit increase.

There are thus many reasons to assess the consistency of the various policies and measures brought together under the heading of fiscal policy. Yet in practice the approaches are often ad hoc and qualitative. A standardized framework is not yet available, though there has been substantial progress in various directions compared, for example, with the IMF’s early monetary programming model, which can be interpreted as an attempt at fiscal sustainability analysis, with a short-run focus.¹ A major objective of this study and associated software development is to combine and extend the various strands. It uses a modern tool for fiscal sustainability analysis that reflects progress on this topic, links easily with existing data sources, and is user-friendly for the practitioner and country economist.²

I. RECENT APPROACHES TO FISCAL SUSTAINABILITY ANALYSIS

So, what are these “various directions” of substantial progress that will be brought together in the model presented here? An early approach to fiscal sustainability analysis is presented in Anand and van Wijnbergen (1988, 1989) and van Wijnbergen (1989). Anand and van Wijnbergen, like the Polak model, tightly link inflation, monetary policy and fiscal deficits, an approach that requires consolidating the central bank into the public sector accounts (Anand and van Wijnbergen 1988). But the focus is more on long-run consistency of inflation targets (and their fiscal impact if achieved), fiscal deficits, and debt management policies. Anand and van Wijnbergen pay a great deal of attention to the structure of the financial system as a key determinant of the link between inflation and inflation tax “revenues.” But there is no serious attention to out-of-steady state (debt) dynamics, and no attention at all to uncertainty.

The IMF’s efforts (IMF 2002, 2003) have taken a different direction. The Polak model’s integration of the central bank and the public sector, more

1. See Anand and van Wijnbergen (1988) for a discussion of the link between the Polak model and fiscal sustainability analysis.

2. The model and associated documentation are available from the authors.

traditionally defined, has apparently been abandoned, and the focus has shifted toward public debt dynamics. Without the integration of the central bank, seigniorage income and the inflation tax have to be dropped from the analysis, so the link with anti-inflation programs cannot be made within the new IMF approach.

There have recently been several extensions to the IMF approach. Celasun, Debrun, and Ostry (2006) introduce uncertainty using simulation methods. They use stochastic properties of key variables determining fiscal deficits to simulate debt dynamics using Monte Carlo simulation techniques and to derive the probability distribution of debt stocks at given moments in the future. The authors also introduce fiscal reaction functions, an important shortcut to modeling the dynamic properties of fiscal policymaking (see Bohn 1998). At the World Bank, the IMF's emphasis on debt dynamics has been combined with Anand and van Wijnbergen's integration of public sector and central bank accounts to reintroduce seigniorage to the analysis, but without the long-run consistency approach and detailed financial sector modeling characterizing the Anand and van Wijnbergen models (see Burnside 2005).

The academic literature has focused mainly on techniques to establish whether historical debt and deficit processes are characterized by unit roots (see Hamilton and Flavin 1986 for an early example). A disadvantage of this line of research is that it is of necessity backward looking, limiting its usefulness after policy reform. More recent work has extended the analysis by resorting to full-fledged dynamic general equilibrium models, in practice at the expense of empirical verification (dynamic, stochastic general equilibrium models are typically "calibrated" rather than estimated). Mendoza and Oviedo (2006) present an interesting variant, deriving maximum debt levels below which governments can be expected to both maintain acceptable expenditures and service their debts. Other research has focused more on detailing the distribution of shocks using modern techniques such as bootstrapping to explore the existence of fat tails and asymmetries. Perotti (2007) provides an overview of the academic research on sustainability.

The approach here combines all these strands into a simple template usable for country economists yet sufficiently rich to incorporate the results of research. The model follows Anand and van Wijnbergen (1988) by tightly integrating the central bank into the public sector, which reintroduces seigniorage income and the inflation tax into the analysis. This allows analysis of the consistency and sustainability of inflation targets within a given set of fiscal plans, a crucial point since structural reform packages are often embedded in a macroeconomic stabilization program. But the study goes beyond the steady-state analysis of Anand and van Wijnbergen by also introducing debt dynamics, as in IMF (2002, 2003) and Burnside (2005). This permits analyzing how serious the deviations from consistency are, and what the tradeoff is between adjusting now and adjusting later, buying time at the expense of a larger required adjustment.

Possibly even more important is that explicitly introducing debt dynamics into the analysis also permits introducing debt structure and the attendant exposure to risk. Vulnerability to sudden stops of external financing is often related to debt structure, especially if there are high peaks in refinancing needs. This can occur when a substantial part of the debt is issued at short maturity or indexed to foreign exchange. Debt issued in or linked to foreign exchange leads to large capital losses after devaluations of the exchange rate. Analysis of these issues requires explicitly introducing the structure and composition of public sector debt.

Moreover, the vulnerability of market-access countries to sudden stops and reversals of capital inflows makes it critical to incorporate uncertainty in the analysis of fiscal sustainability. Uncertainty surrounding public debt dynamics is often related to uncertainty about medium-term projections of the economic growth rate, the primary balance, the cost of public sector borrowing, and the existence of either explicit or implicit guarantees of debt or bank deposits.

This study uses two approaches to introduce uncertainty and risk to the analysis. To deal with vulnerability to specific shocks, the template provides a variety of single-factor stress tests, as used for example in IMF (2002, 2003). To get a broader view on the riskiness of the basic projections, the study also introduces the possibility of a full set of stochastic simulations using the stochastic properties of key variables in debt dynamics.

Stress tests provide valuable insights into the robustness of the projections to specific shocks to individual exogenous variables and allow explicit analysis of the consequences of extreme events. The stochastic simulations approach has the advantage of deriving the entire distribution of future debt stocks, based on stochastic realizations of key debt determinants, and accounting for their variances and covariance structure. The tool can incorporate any number of distributional assumptions on fat tails and asymmetries, though the default assumption is a multivariate normal. The simulations allow using a Value at Risk approach, or calculating the likelihood that in a given period maximum “safe” debt levels, like the ones derived in Mendoza and Oviedo (2006), will be exceeded with more than a certain threshold probability. Fan charts represent the results, as in Celasun, Debrun, and Ostry (2006). The fiscal sustainability tool goes beyond Celasun, Debrun, and Ostry (2006) in presenting not just the distribution of debt-output ratios at various moments in the future, but also the distribution of the fiscal adjustment necessary to restore consistency and stability. This measure could be more useful to policymakers.

Stochastic simulations can be misleading if they assume unchanged primary deficits after stochastic shocks while the government has a record of responding to debt shocks by tightening its belt. So, the analysis here also incorporates fiscal policy reaction functions and endogenous debt feedback rules for the primary surplus as an additional option for stochastic simulations and other stress tests, as suggested in Bohn (1998) and Celasun, Debrun, and Ostry

(2006). Bohn (1998) shows that fiscal reaction functions are a key requirement for the stability of fiscal programs.

Section II outlines the analytical framework, and sections III and IV demonstrate the application of the fiscal sustainability tool for the case of Turkey.

II. ANALYTICAL FRAMEWORK

Important to stress from the outset is that the fiscal sustainability analysis presented here is not based on a fully specified model. Rather, an accounting framework is the basis of fiscal sustainability analysis, with the exception of the fiscal policy reaction function parameter and the parameters of the stochastic processes used in the simulations. The fiscal sustainability analysis template thus does not provide a tool to set policy variables optimally, such as maximizing a particular welfare function. The more modest goal is to assess the sustainability of whatever policy package is chosen: What is the likelihood that solvency limits will be breached at unchanged policies?

Solvency and Debt Dynamics

Solvency is not much more than an intertemporal extension of staying within one's means: a government is solvent if it does not intend to spend more than its income and initial wealth (or minus net debt). The intertemporal equivalent of staying within one's means implies that the discounted value of current and future income plus initial wealth should at least be equal to the discounted value of all current and future noninterest expenditure. Interest expenditure and income are incorporated through the discounting procedure. Formally, this comes down to:

$$(1) \quad b_0 + \sum_1^{\infty} \frac{g_i}{(1+r)^i} = \sum_1^{\infty} \frac{t_i + s_i}{(1+r)^i}$$

$$b_0 = \sum_1^{\infty} \frac{ps_i + s_i}{(1+r)^i}.$$

Equation (1) states that initial debt, b_0 , plus the discounted value of all non-interest expenditure, g_i , should (at most, but equality is assumed in what follows) equal the discounted value of all public sector noninterest income, here summarized as the sum of tax revenues, t_i , and seigniorage revenues, s_i . Seigniorage is the net income the public sector derives from issuing money. This can be rewritten as the second line in equation (1): initial (net) debt should at most equal the discounted value of the primary or noninterest government surplus, ps_i , plus seigniorage revenues, s_i .

To understand the implications and structure of equation (1), it helps to write a simpler construct, the so-called flow budget constraint:

$$(2) \quad b_t = b_{t-1}(1+r) - (ps_t + s_t)$$

or, equivalently:

$$b_{t-1} = \frac{ps_t + s_t}{1+r} + \frac{b_t}{1+r}$$

Equation (2) states that initial debt plus interest payments plus the primary deficit (or rather minus the primary surplus), and, finally, minus seigniorage revenues, equal the new level of debt. This can be rewritten as an expression for initial debt, as is done in the second line of equation (2), where initial debt equals the discounted sum of the primary surplus plus seigniorage and the end of period debt. This way of writing the flow budget constraint should also show how discounting takes care of interest payments. Including them in the deficit definition would essentially count them twice. Substituting equation (2) repeatedly into itself, for t starting at 0, yields:

$$(3) \quad \begin{aligned} b_0 &= \frac{ps_1 + s_1}{1+r} + \frac{b_1}{(1+r)^1} \\ &= \frac{ps_1 + s_1}{1+r} + \frac{ps_2 + s_2}{(1+r)^2} + \frac{b_2}{(1+r)^2} . \\ &= \lim_{t \rightarrow \infty} \sum_1^t \frac{ps_i + s_i}{(1+r)^i} + \lim_{t \rightarrow \infty} \frac{b_t}{(1+r)^t} \end{aligned}$$

Combining equation (3) with equation (1) shows that solvency requires:

$$(4) \quad \lim_{t \rightarrow \infty} \frac{b_t}{(1+r)^t} = 0.$$

In words, the debt should ultimately not grow faster than the rate of interest.

This result is behind the econometric approaches to testing for solvency mentioned in the introduction. Hamilton and Flavin (1986) use equation (4) as a starting point for a series of unit root tests to establish the compliance of a given time series of debt stocks with equation (4). Passing the unit root test (first for the budget surplus, then for the debt stocks) means that the process, if it continues to conform to the econometrically recovered structure, will be within solvency limits. Failure does not necessarily mean insolvency, however, since the test will also fail when b_t converges to a positive number, but at a rate that is positive but lower than r . A variant on the Hamilton–Flavin approach uses regression analysis to see whether coefficients on terms

proportional to $(1 + r)^t$ are significant in an equation linking debt to past deficits and to past debt stocks (see Burnside 2004 for a survey of research in this area). This second econometric approach also has weak power against near-alternatives.

But there is a bigger problem with econometric approaches: they are not really useful for policy analysts looking at the stability of reformed budget processes. The reason is that both approaches are backward looking and can by construction say nothing about recently reformed budgetary policies and the resulting debt stocks, since the new processes are without a track record.

Forward-looking Approaches: Dynamic Simulations

Because of the problems with backward looking approaches, the approach here is to take a forward-looking, dynamic simulations approach. The first step is to create a baseline scenario of the likely future time path of the public debt to GDP ratio. The baseline is derived using the flow budget equation (2) to update future debt stocks based on macroeconomic projections of key determinants of public debt dynamics, such as growth, inflation, projected primary surpluses, and interest rates.

To ensure consistency among debt stocks, deficits, and revenues from seigniorage, it is necessary to consolidate the general government accounts with the central bank's profit and loss account (see Anand and van Wijnbergen 1988). Otherwise seigniorage, an important source of revenue in many developing countries, will not show up in the budget dynamics, and debt may be mismeasured by failing to count assets held by the central bank.³ Public foreign debt should be measured net of the (net) foreign asset holdings of the central bank, and public domestic debt, net of holdings of such debt by the same central bank.

Seigniorage $(g + \pi)m$ equals the real value of the nominal increase in base money, $\Delta M/(PY)$, where π equals the target inflation rate. The first term, gm , equals the increase in real balances that people are willing to hold simply to keep the money-output ratio constant in the face of growth, g . The second term, $\tilde{\pi}m$, equals the increase in nominal money balances necessary to keep the real value of money constant given inflation, $\tilde{\pi}$. The relation between demand for base money and inflation depends, among other things, on the financial structure and regulation (such as reserve requirements). So, it is likely to change after financial sector reform (see Anand and van Wijnbergen 1988 for an extensive discussion).

But given the lack of long time series for many developing countries, and the requirement to make a generally usable template for fiscal sustainability analysis, the fiscal sustainability tool simplifies estimation of the revenue from

3. In Turkey the seigniorage revenues were about 2 percent of GDP in 1991–93, then rose to 2.5–3.5 percent of GDP in the high inflation years 1994–99, and dropped to around 1 percent in the post-crisis period after 2001.

seigniorage by using a simple Cagan money demand function. The only additional requirement then is to estimate the coefficient of the elasticity of money demand to the nominal interest rate (or inflation), which represents the opportunity cost of holding money. The estimated coefficient will determine the amount of seigniorage to be expected given the assumed inflation targets. Of course tracing the budgetary impact of inflation through seigniorage does not constitute an argument to raise inflation. Inflation has high costs, which are not the focus of this model. So, outlining the fiscal consequences of inflation through seigniorage should not be construed as an argument to raise inflation in a response to debt concerns.

Steady-State Consistency and the Required Deficit Reduction Measure

A major disadvantage of the debt simulation approach is that it does not tell policymakers how much adjustment is required to keep the debt-output ratio stable. So, a second indicator is introduced, the required deficit reduction measure rdr that gives precisely that information.⁴ The rdr indicator equals the deficit reduction necessary to stabilize the debt-output ratio at its current value—that is, the value it has in the year for which the rdr is calculated. Reducing the primary deficit by rdr restores consistency between projected growth rates, interest rates, and inflation targets on the one hand, and the requirement of a stable debt/output ratio on the other.

Incorporating Uncertainty

So far, deterministic paths have been assumed for the underlying variables. But there is little doubt that all input projections are surrounded by a great deal of uncertainty and so are the results of any deterministic analysis. Uncertainty leads to two separate questions, requiring separate approaches for their answer. First, with uncertainty attached to projections of such variables as interest and growth rates and exchange rate developments, how sensitive are the results to a given shock in any of the variables used as input in the exercise? Second, again given the uncertainty surrounding almost all variables, how much confidence can there be in the outcome of the base run? Or, phrased differently and more in line with the Value at Risk approach now commonly used by commercial banks, what is the probability that certain thresholds will not be exceeded in a given period? For the first question stress tests are introduced dealing with specific risks (in the next section). For the second question stochastic simulation methods are resorted to, using empirical information about the distribution of the input variables.

The sensitivity of the results to specific shocks can be tested by conducting sensitivity tests to the baseline scenario, assuming for example that the underlying variables swing away from their means by one or two standard deviations. Examples are stress tests for real interest rates, real output growth,

4. For extensive discussion of this measure see Anand and van Wijnbergen (1988, 1989).

primary balance, changes in the real exchange rate, and unanticipated realizations of contingent liabilities.

Stress tests can also be used to assess robustness to extreme events, possibly involving adverse changes in a variety of input variables (“crisis scenario”). Varieties of such scenarios are included and can be run at the discretion of the user. Under a full Monte Carlo simulation, extreme events are just one of many realizations and will largely be averaged out. Demonstrating robustness under extreme events may contribute importantly to credibility, so stress tests focusing on extreme events are a useful tool.

An alternative to stress tests is a full-scale Monte Carlo simulation (IMF 2003; Celasun, Debrun and Ostry 2006). Using estimated parameters of the joint distribution of all input variables, the distribution of these variables can be jointly simulated using Monte Carlo methods. This implies that for n input variables and a horizon of T years, $n \times T$ random numbers are generated repeatedly until the generated and empirical distribution are sufficiently close (5,000 runs are generated by default). And for each run, the model is applied to derive the full path of debt stocks and values of the required deficit reduction measure rdr . In this way, the full probability distribution of debt-output ratios and the rdr measure at each future point in time is derived. The probability density of the outcomes of the debt ratio and of the rdr measure of necessary fiscal adjustment in each year can be plotted from the stochastic simulations, generating a “fan chart” for the debt to GDP ratio and the rdr variable.

One way of obtaining the relevant variance–covariance information is to run a Value at Risk on historical variables and transform the generated random numbers in such a way that the resulting distribution conforms to the Value at Risk estimates of the true distribution of the input variables.⁵ For a multivariate normal, a transformation using the Cholesky decomposition of the empirical covariance matrix can transform independent and identically distributed generated random variates into variates corresponding to the empirical distribution (Bandiera and others 2007). Alternative distributional assumptions can also be incorporated.

Both stress tests and Monte Carlo simulations may overestimate the impact of shocks, since the government may take deliberate corrective actions as its debt stock rises. Bohn (1998) shows that, if all other determinants of fiscal policy are stationary, a positive correlation between the primary surplus and the past level of the public debt to GDP ratio is sufficient to guarantee fiscal sustainability. The fiscal sustainability template therefore also provides the option to simulate with a fiscal policy reaction function, introducing feedback from deviations from base-level debt stocks to deviations from base-level primary surpluses.

5. These days, most middle-income countries have data series long enough do to meaningful simple Value at Risk regressions, particularly if quarterly data can be used.

A cautionary note may be in order. The Monte Carlo analysis outlined here presumes that the stochastic structure prevalent in the past will persist in the future. That may not be the case, particularly after reform periods, although the Value at Risk is not used for the budgetary processes and debt stocks are most likely changing. Moreover, simulating empirical distributions based on point predictions to get the mean and using estimated covariance matrices to generate the random terms around those predictions, do not incorporate uncertainty about the empirically obtained estimates for the parameters of that distribution. So, the distributions are simulated conditional on obtained prediction means. Another way of introducing uncertainty in modeling would be to explicitly model risk premia and their impact on asset returns (see Budina and van Wijnbergen 2007, where that is done, but it is also explained that the data requirements of doing so with sufficient empirical content would preclude application to most developing countries of interest).

III. SUSTAINABILITY OF PUBLIC DEBT IN TURKEY: STRESS TESTS

Throughout the 1980s and 1990s there have been many attempts to stabilize inflation in Turkey, but all of them failed until recent successes changed the pattern. After each failed attempt, inflation jumped to a higher rate. Similarly, the debt to GNP ratio increased steadily from 1990 onwards, doubling through 1999, and tripling through 2001 (from 30 percent in 1990 to 90 percent).

The latest financial crisis seems to have triggered a break in the cycle of high inflation, failed stabilization attempts, rising debt burdens, and higher inflation. Following the crisis, Turkey adopted economic reforms to address the underlying economic vulnerabilities and regain macroeconomic stability. These reforms led to high primary surpluses (about 6 percent of GDP for the past five years), which reduced the overall fiscal deficit substantially, as improved credibility led to lower interest rates. The large fiscal adjustment, with robust economic growth and a strong real appreciation of the currency, led to a sizable reduction in the public debt ratio. The composition of public debt also improved considerably, as the share of foreign-currency-denominated debt fell. And the average annual inflation rate fell to single digits in 2004 and declined further in 2005.

An important question is whether these positive developments are likely to continue. Can Turkey grow out of its debt problem? Is the current fiscal policy sustainable and consistent with single-digit inflation? To answer these questions, some stress tests are presented in this section, and the results of a full Monte Carlo simulation in the next.

Public debt dynamics are derived using a variant of equation (2), after first consolidating the central bank and distinguishing foreign and domestic debt. Then increases in net public debt (measured net of the net foreign asset and public debt holdings of the central bank) can be decomposed in various contributing factors. Switching to ratios to GDP and indicating them by a tilde above

the relevant variables, public debt dynamics can be broken down into several components: the primary fiscal deficit net of seigniorage revenues, growth-adjusted real interest payments on domestic debt, and growth-adjusted real interest payments on external debt, including capital gains and losses on net external debt due to changes in the real exchange rate:

$$(5) \quad \dot{\tilde{d}} = (\tilde{pd} - \tilde{s}) + (r - g)\tilde{b} + (r^* + \hat{e} - g)(\tilde{b}^* - n\tilde{fa}^*).e + OF$$

where \tilde{d}_t is the net public debt to GDP ratio, \tilde{pd}_t is the primary deficit as a share of GDP, r is the real interest rate on domestic debt, g is the real GDP growth rate, r^* is the real interest rate on external debt, \hat{e} is the change in the real exchange rate (with $\hat{e} > 0$ denoting a real exchange rate depreciation), and e is the real exchange rate. A catch-all term, other factors, OF , collects residuals due to cross-product terms arising from the use of discrete time data (see annex A.1 in Bandiera and others 2007 for explicit discrete time formulas) and the impact of debt-increasing factors that in a perfect accounting world would be included in deficit measures but in the real world are not. Examples are contingent liabilities that actually materialize, such as the fiscal consequences of a bank bailout and one-off privatization revenues.⁶ Note that in this single-equation exercise, debt levels are generated, but all other variables are considered exogenous (feedbacks from shocks to debt levels are not incorporated).

To derive public debt dynamics for the consolidated public sector, the fiscal sustainability tool also provides an estimate of the revenue from seigniorage during the projection period. Key inputs in the calculations of seigniorage revenue are projections for average inflation rate, average price level, and real GDP. A simple Cagan money demand function is assumed for base money, which because of the presence of unit roots had to be estimated in an error correction model framework (see table 1, with obvious definitions of variables).

The implied long-run value of the semi-elasticity of money demand to inflation is $-(-0.21039/-0.669) = -0.31$. Seigniorage revenue is projected at about 0.6 percent of GDP on average during the projection period. The value of 0.31 gives a maximum seigniorage inflation rate (the inverse of the semi-elasticity of inflation) equal to 318 percent, very close to the value found with the detailed financial sector model estimates in Anand and van Wijnbergen (1988).

The baseline scenario for public debt dynamics assumes continued commitment to the high primary surpluses of 2000–05 over the projection period (6.5 percent of GDP). Consumer price index inflation is expected to decline further, consistent with the recently adopted inflation targeting rule. Real interest rates on domestic public debt are projected to stabilize at about 10 percent a year and those on external public debt at about 8 percent. The real exchange rate is

6. See Bixi and Schick (2002) and IMF (2005) for a more detailed discussion on contingent liabilities.

TABLE 1. (Base) Money Demand Estimates

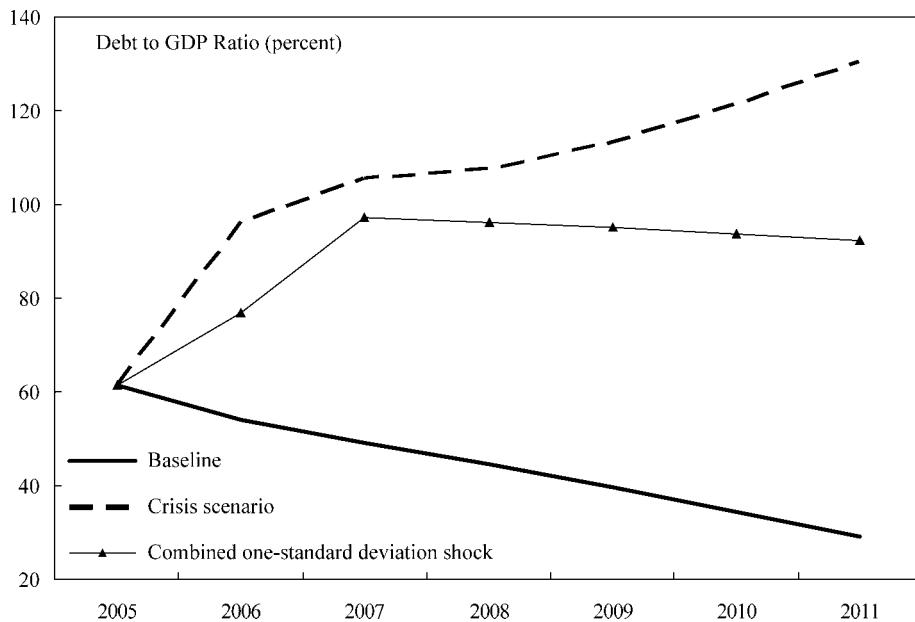
Variable	Coefficient	Standard Error	t-Statistic	Probability
C	-0.787	0.149	-5.284	0
INF(-1)	-0.210	0.138	-1.523	0.132
LM0Y(-1)	-0.669	0.126	-5.301	0
D(INF)	-0.315	0.097	-3.253	0.002
T	-0.005	0.002	-2.556	0.013
R-squared	0.381			
Adjusted R-squared	0.347			
Standard error of regression	0.231			
Sum squared residual	3.849			
Log likelihood	6.087			
Durbin-Watson statistic	1.862			
Mean dependent variable	-0.005			
Standard deviation dependent variable	0.286			
Akaike info criterion	-0.028			
Schwarz criterion	0.124			
F-statistic	11.076			
Prob(F-statistic)	0			

Source: Authors' analysis based on data from International Monetary Fund, International Financial Statistics database.

also assumed to remain broadly stable. Following the strong performance in 2002–04, growth rates are expected to stabilize at 5 percent, the potential growth rate. As a result, in this baseline estimation, the net debt to GDP level continues to decline, falling below 60 percent in 2006 and below 50 percent in 2008 and reaching 29 percent in 2011 (figure 1). The projected trajectories for the exogenous variables are arbitrary in that they are not derived from any underlying model (the Value at Risk estimations were not used to generate them).

The baseline scenario may be rather optimistic, particularly on maintaining the very high primary surplus. To assess the robustness of the base case, various stress tests were performed. One scenario is a structured “extreme event” scenario corresponding to a recurrence of the 1999–2001 crisis (labeled “crisis scenario” in figure 1). This scenario was formulated to illustrate risks related to a possible loss of market confidence. The key assumption is that the political will to continue the fiscal program falters, so the projected primary surplus does not exceed 3 percent of GDP. Weaker fiscal efforts will again trigger a loss of market confidence, exchange rate instability, and thus a worsening debt dynamics. A loss of market confidence is simulated through a sudden stop in the availability of foreign financing, assuming no new disbursements in 2006. This means that all the interest and principal falling due on external debt has to be repaid, so the government has to issue new domestic debt not just to finance its current deficit but also to pay back external debt falling due.

FIGURE 1. Baseline Projections for Net Public Debt to Gross Domestic Product Ratio



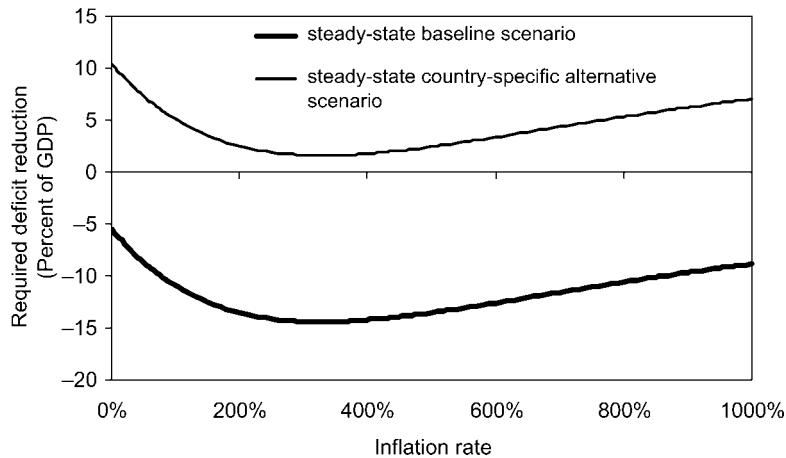
Source: Authors' analysis based on data from the International Monetary Fund (IMF) *World Economic Outlook* (various years) and IMF staff estimates.

In the crisis scenario, exchange rates overshoot initially, as they did in 1999–2001, leading to a substantial real depreciation (30 percent in 2006 and 15 percent in 2007). The depreciation puts pressure on domestic real interest rates, which increase to 40 percent in 2006 and 20 percent in 2007, again following the recent crisis pattern. Similarly, the loss of market confidence is reflected in a higher default premium, peaking at 12 percent in 2006 and 2007. As a result, the economy is assumed to contract by 6 percent in 2006 and then recover, but at a much slower pace. The sizable depreciation is also assumed to create pressures in the banking system, adding up to some 15 percent of GDP with the recognition of contingent liabilities in public debt. As a result, the public debt ratio is projected to increase to 130 percent of GDP by 2011.

An alternative scenario is more mechanical: four key variables deteriorate by one standard deviation for two years (GDP growth, the two real interest rates, and the primary surplus). In those two years the debt-output ratio deteriorates rapidly, to stabilize afterward at a high 100 percent of GDP (combined one standard deviation shock in figure 1).

The stress tests show that the favorable baseline scenario is not robust. Adverse shocks that are not all that different from the recent shocks could rapidly reverse the favorable debt trends of the baseline scenario.

FIGURE 2. Required Deficit Reduction for Different Inflation Rates



Source: Authors' analysis based on data from the International Monetary Fund (IMF) *World Economic Outlook* (various years) and IMF staff estimates.

Finally the rdr , the fiscal consistency measure, is calculated for the baseline and the crisis scenarios. The baseline assumptions of continued large fiscal primary surpluses and strong growth create substantial headroom for seigniorage and debt carrying capacity, given stable debt-output targets (figure 2). If the favorable macro data persists as assumed in the baseline scenario, there is substantial overadjustment: more than 5 percent of GDP, an overshooting policy that seems wise given the continuing problems of external credibility.

There should be concern about the robustness of the favorable baseline scenario. The rdr measure in the crisis scenario turns positive for all inflation rates and reaches 10 percent of GDP at the currently (2007) targeted inflation rate of 3.5 percent.

IV. SUSTAINABILITY OF PUBLIC DEBT IN TURKEY: STOCHASTIC SIMULATIONS

Stochastic simulations are used to account for the high volatility of key variables, such as real interest rates, real growth rates, and changes in the real exchange rate. A Value at Risk model is estimated, using the time series for 1990q1–2004q4: the log-difference of the real effective exchange rate ($dlog(reer)$), real GDP growth ($dlog(gdpr)$), and real domestic interest rates ($rdom$) from the International Finance Statistics database and IMF staff estimates. For foreign interest rates, the US interest rate on treasury paper ($rtreas$) is used.

Unit roots of individual time series are checked to make sure that they are stationary, performing augmented Dickey–Fuller tests on each series. Unit roots at the 5 percent level are rejected for all series (table 2). With all variables

TABLE 2. Unit Root Tests, Quarterly Data, 1990q1–2004q4

Variable from Value at Risk	With Intercept Only	With Intercept and Trend
dlog(reer)	-6.591**	-6.555**
dlog(gdpr)	-8.227**	8.167**
rdom	-8.821**	-9.212**
rtreas	-3.198*	-5.558**

*Reject unit root at 5 percent level; **reject unit root at 1 percent level.

Source: Authors' analysis based on data from International Monetary Fund (IMF), International Financial Statistics database and IMF staff estimates.

TABLE 3. The Appropriate Lag Structure

Lag	Schwarz's Bayesian Information Criterion
0	23.9938
1	19.9415*
2	20.5332
3	21.204
4	21.485

*Indicates lag order selected by the criterion at 5 percent level of significance.

Source: Authors' analysis based on data from International Monetary Fund (IMF), International Financial Statistics database and IMF staff estimates.

stationary, ordinary least squares can be used without concern about spurious regression results.

Since the tool performs stochastic simulations based on annual data for the public debt-updating equation, these variables had to be annualized before estimating a Value at Risk model, using an appropriate lag structure on the basis of Schwarz's Bayesian information criterion, which selected just one lag (table 3).

Table 4 shows the estimated coefficients of the Value at Risk model. The normality of the error structure was tested by performing a Cholesky transformation and testing the resulting four orthogonal empirical distributions for skewness and kurtosis. The Jarque-Bera test was also performed on each. The tests in the four dimensions separately rejected skewness and kurtosis for all (transformed) variables, and the Jarque-Bera test also accepted normality. On the four together the skewness and Jarque-Bera test accepted normality at a 5 percent level, and the kurtosis test at a 2.5 percent level (but on the four transformed variables individually at the 5 percent level). So, normality is assumed in what follows.

The estimated Value at Risk model corresponds to the variance–covariance matrix in table 5.

TABLE 4. Estimated Value at Risk Model for Turkey for 1990–2004

	Depreciation of RER	Domestic Interest Rate	Foreign Interest Rate	Growth Rate
Depreciation of RER (-1)	0.6164132*** (0.1214663)	-0.0764374 (0.0745745)	0.0071392*** (0.0032883)	-0.0163632 (0.0578173)
Domestic interest rate (-1)	0.5183742** (0.1300432)	0.7703966*** (0.0798403)	-0.0034098 (0.0035205)	0.2292575*** (0.0618999)
Foreign interest rate (-1)	2.613194** (1.310771)	0.1130593 (0.8047509)	0.9929505*** (0.0354851)	0.8727535 (0.6239205)
Growth rate (-1)	0.0078362 (0.2706376)	-0.0719824 (0.1661585)	-0.0050722 (0.0073267)	0.6440931** (0.1288221)
Constant	-17.6484** (6.057141)	3.862517 (3.718795)	0.0658478 (0.1639787)	-5.609101* (2.883169)

*Significance at the 10 percent level; **significant at the 5 percent level; ***significant at the 1 percent level.

Note: Numbers in parentheses are standard errors.

Source: Authors' analysis based on data from the International Monetary Fund (IMF), International Financial Statistics database and IMF staff estimates.

TABLE 5. Estimated Variance–Covariance Matrix ($\hat{\Sigma}$)

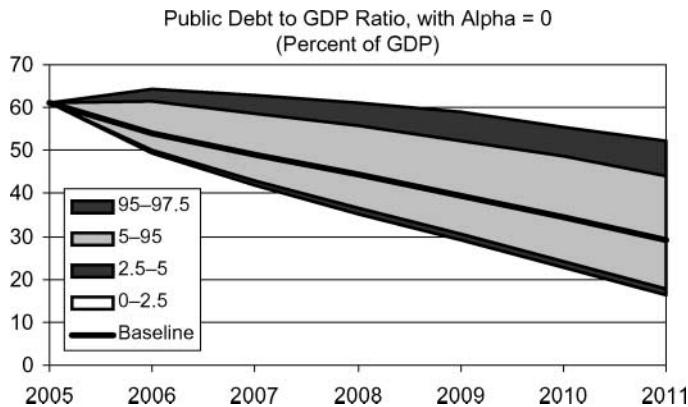
	Depreciation of RER	Domestic Interest Rate	Foreign Interest Rate	Growth Rate
Depreciation of RER	81.1591	-6.30625	-0.165489	23.1178
Domestic interest rate	-6.30625	30.5919	-0.164606	7.36655
Foreign interest rate	-0.165489	-0.164606	0.059481	0.112955
Growth rate	23.1178	7.36655	0.112955	18.3883

Source: Authors' analysis based on data from the International Monetary Fund (IMF), International Financial Statistics database and IMF staff estimates.

The Cholesky decomposition of the covariance matrix of table 4 is used in the stochastic simulations. A series of orthogonal independent variables is generated on [0,1] and transformed into orthogonal normal variables with variance one by applying the inverse normal density function. These variates are then transformed into normal variates with the appropriate covariance matrix by multiplying each vector of drawings by the inverse of the Cholesky matrix. This procedure produces a distribution matching the empirical estimates of the Value at Risk results. The fiscal sustainability tool then simulates the model in response to the shocks generated that way and produces a “fan chart” of the resulting debt to GDP ratios (figures 3 and 4).

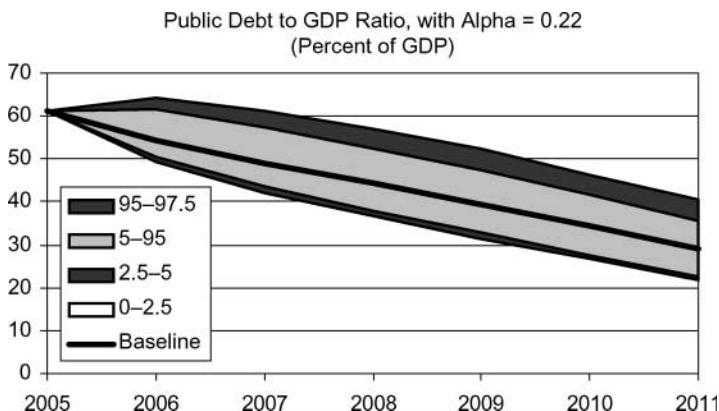
Figures 3 and 4 show the distribution of debt stocks as a function of time that comes out of the simulations. Figure 3 indicates a 50 percent chance that public debt will be below 29 percent of GDP at the end of the projection period: the chance that public debt will be below 50 percent at the end of the projection period is a high 95 percent. Although extreme events may throw the economy off track, the variance around the baseline due to uncertainty from

FIGURE 3. Fan Chart for Public Debt to GDP Ratio, with Alpha = 0



Source: Authors' analysis based on data from the International Monetary Fund (IMF) *World Economic Outlook* (various years) and IMF staff estimates.

FIGURE 4. Fan Chart for Public Debt to GDP Ratio, with Alpha = 0.22



Source: Authors' analysis based on data from the International Monetary Fund (IMF) *World Economic Outlook* (various years) and IMF staff estimates.

interest rates, exchange rates, and growth does not really change the basic optimistic note that comes out of the baseline scenario.

With debt staying as high as the upper end of the distribution shown in figure 3, the assumption of unchanging primary surpluses may not be realistic. There is strong evidence that the primary surplus in Turkey is influenced by total government debt: higher debt tends to lead to higher primary surpluses. A simple regression of the primary surplus on past debt and the output gap (and a dummy variable for the crash stabilization program adopted in 2001) confirms that result (table 6), showing a very substantial response coefficient of about 20 percent.

If the estimated value for feedback coefficient alpha is introduced to the stochastic simulations, the distribution of debt stocks narrows considerably

TABLE 6. Fiscal Reaction Function

Dependent Variable: Primary Balance

	Coefficient	t-statistic
Lagged debt	0.227***	4.94
Output gap	-0.005	-0.02
Dummy variable for 2001	3.247	0.85
Constant	-10.837***	-4.34
Number of observations	15	
F (3, 11)	8.95	
Prob > F	0.003	
R-squared	0.710	
Adjusted R-squared	0.630	
Durbin-Watson d-statistic	1.392	

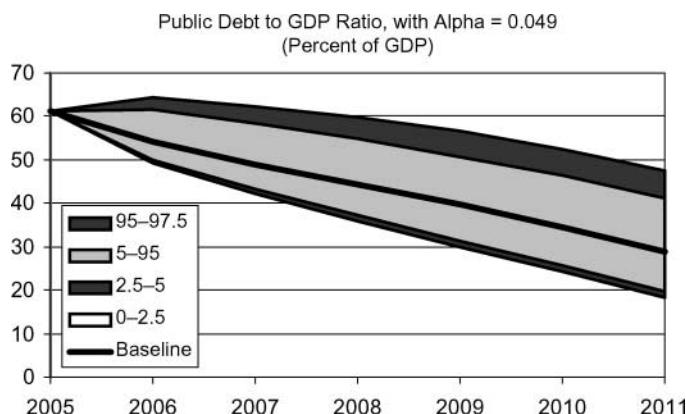
***Significant at the 1 percent level.

Source: Authors' analysis based on data from the International Monetary Fund (IMF), International Financial Statistics database and IMF staff estimates.

(figure 4). Instead of fanning out to a 95 percent range of almost 35 percentage points, as in figure 3, the same range narrows considerably once the empirically obtained feedback rule is incorporated. A Value at Risk analysis would thus come up with better numbers. Of course, a regime shift might also mean that the feedback coefficient obtained from past data might lose relevance for analysis of the future.

The high feedback coefficient reflects Turkey's strong adjustment effort after the crisis. Budina, van Wijnbergen, and Li (2008) find a lower coefficient of 4.9 percent in a sample of resource-rich countries, close to the 4.3 percent found by Celasun, Debrun, and Ostry (2006). Redoing the simulation with that lower value of alpha results, not surprisingly, in less variance reduction (figure 5).

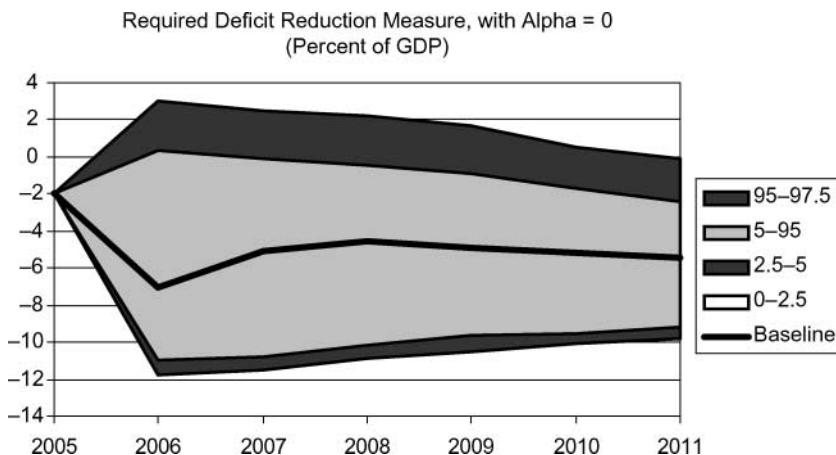
FIGURE 5. Fan Chart for Public Debt to GDP Ratio, with Alpha = 0.049



Source: Authors' analysis based on data from the International Monetary Fund (IMF) *World Economic Outlook* (various years) and IMF staff estimates.

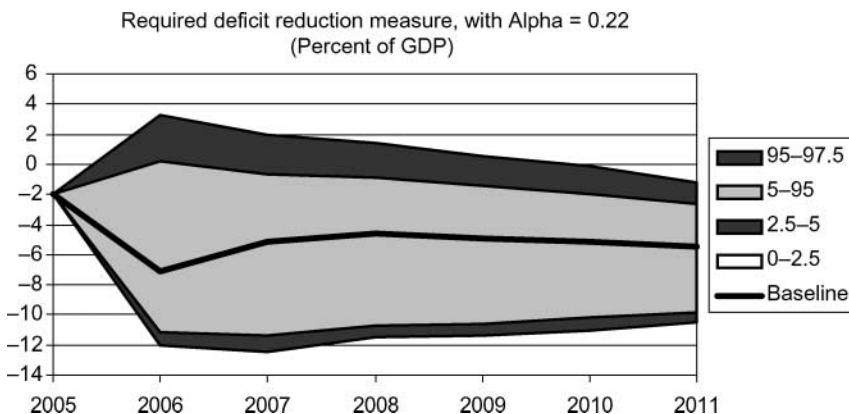
The same Monte Carlo runs can also show the probability distribution of the required deficit reduction measure—the additional fiscal effort needed to stabilize the public debt to GDP ratio in any given year at the value it has in that year with respect to GDP. Figures 6 and 7 show the results, again for a zero alpha (figure 6) and with a feedback rule (figure 7). Figure 6 indicates that if Turkey maintains its sizable primary surplus of 6.5 percent of GNP, there is

FIGURE 6. Fan Chart for Required Deficit Reduction Measure, with Alpha = 0



Source: Authors' analysis based on data from the International Monetary Fund (IMF) *World Economic Outlook* (various years) and IMF staff estimates.

FIGURE 7. Fan Chart for Required Deficit Reduction Measure, with Alpha = 0.22



Source: Authors' analysis based on data from the International Monetary Fund (IMF) *World Economic Outlook* (various years) and IMF staff estimates.

a 97.5 percent probability that fiscal policy will be at least consistent with stable debt ratios and targeted inflation rates in 2011. So there is only a 2.5 percent probability that additional fiscal adjustment may be needed in 2011 to ensure consistency with the adopted inflation targets and to stabilize the debt to GNP ratio at its 2011 level.

Introducing a positive alpha reduces rdr at the top of the distribution (now the 97.5 percent boundary is almost –2 percent, indicating overadjustment). Interestingly, no such narrowing occurs at the bottom end of the distribution; the lower end corresponds to higher debt stocks than in the alpha = 0 scenario, so rdr does not fall.

V. CONCLUSIONS

This article reviewed various quantitative approaches to fiscal sustainability analysis with the objective of combining them into a user-friendly tool that reflects modern developments. Fiscal sustainability is defined as reaching stable debt-output ratios at unchanged policies. This is in most cases ($r > n$) a sufficient though not necessary condition for solvency.

The analysis built on a simplified version of the steady-state consistency approach introduced by Anand and van Wijnbergen (1988), combining it with dynamic simulations (as in Burnside 2005) to be able to analyze debt dynamics. Two methods were incorporated to deal with uncertainty: user-defined stress tests, to assess vulnerability to specific shocks in individual variables and analyze the robustness of strategies under extreme events, and stochastic (Monte Carlo) simulations to derive the complete probability distribution of future debt stocks (though they cannot focus on extreme events). This makes it possible to take a Value at Risk approach to fiscal sustainability analysis. The tool is more policy oriented than most approaches by going beyond distributions of debt stocks to the evaluation of the full future distribution of the fiscal adjustment required to stabilize debt-output ratios (rdr). The fiscal sustainability tool incorporates an endogenous debt feedback rule for the primary surplus, a fiscal policy reaction function.

The authors have not taken the step toward full-fledged macroeconomic modeling as the underlying basis for the stochastic simulations, instead relying on a reduced form Value at Risk approach to extract distribution parameters. This enables wide applicability to many countries in data-sparse environments. Alternative, possibly more sophisticated econometric techniques and non-normal distributional assumptions (“fat tails”) can easily be incorporated.

The second part of the article presented a fiscal sustainability analysis for Turkey using the new tool. Applying the tool involved:

- Using public debt dynamics to create a baseline projection of future trends in public debt to GDP ratio, with existing macroeconomic projections.

- Conducting sensitivity tests to the baseline scenario, to debt dynamics and to the consistency measure rdr .
- Performing Monte Carlo simulations using distributional assumptions obtained from a Value at Risk estimation to derive the distribution over time of debt-output ratios and the required deficit reduction measure rdr .
- Checking the sensitivity of the public debt and rdr probability distributions to the adoption of an empirically verified fiscal policy reaction function.

The results suggest that if the current fiscal adjustment persists, with primary surpluses of about 6 percent of GDP, there will be a rapid decline in public debt (as a share of GDP) over the projection period. Moreover, stochastic simulations suggest that if the current fiscal strategy is to be maintained, there is still considerable leeway: a 95 percent chance that the public debt ratio will be below 50 percent at the end of the projection period and a 50 percent chance that the public debt ratio will be below 29 percent of GDP at the end of the projection period.

But risks to fiscal sustainability remain. There is concern that the quality and durability of the fiscal adjustment may not be sustainable. In particular, while the overall primary surplus is impressive by international standards, substantial pension deficits crowd out the rest of public expenditure and make it harder for Turkey to sustain this primary surplus. Pension reform may be necessary to allay this fear. A crisis scenario, conducted as a deterministic stress test, shows that the sustainability conclusion is not robust to faltering fiscal reforms, rising interest rates, and a resulting sudden stop of capital inflows. But the Monte Carlo analysis does show that if the fiscal adjustment stays on track, debt stocks are unlikely to derail.

The application to Turkey demonstrates the flexibility and easy applicability of the fiscal sustainability analysis tool presented here. Be aware, however, that all conclusions remain probability statements, with substantial judgmental elements. Despite increasing sophistication of the tools, fiscal sustainability analysis remains as much art as science.

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