Adaptive Fuzzy Model Predictive Control for Non-minimum Phase and Uncertain Dynamical Nonlinear Systems

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Abstract—this paper introduces a method to design a robust adaptive predictive control based on Fuzzy model. The plant to be used as predictive model is simulated by Takagi-Sugeno Fuzzy Model, and the optimization problem is solved by a Genetic Algorithms or Branch and Bound. The method to tune parameters of the model predictive controller based on Lyapunov stability theorem is presented in this paper to bring higher control performance and guaranty Global Asymptotical Stable (GAS) for the closed-loop system. This method is used for nonlinear systems with non-minimum phase (CSTR), uncertain dynamical systems and nonlinear DC motor. The simulation results for the Continuous Stirrer Tank Reactor (CSTR), nonlinear uncertain dynamical system and nonlinear DC motor are used for verifying the proposal method.

Index Terms - Model Predictive Control (MPC); Takagi Sugeno Fuzzy Model (TS); Genetic Algorithms (GAs); Branch and Bound (B&B), Multiple Inputs-Multiple Output (MIMO); Single Input-Single Output (SISO); Adaptive Fuzzy Model Predictive Control (AFMPC); Global Asymptotical Stable (GAS); Continuous Stirrer Tank Reactor (CSTR).

I. INTRODUCTION

Model Predictive Control (MPC) [1] has been widely and successfully applied in industrial process, especially the multi-input, multi-output (MIMO) nonlinear process. Several recent publications have provided a good introduction to theoretical and practical issues associated with MPC technology. In 1999, Allgower, Badgwell, Qin, Rawlings, and Wright [13] presented a more comprehensive overview of nonlinear MPC and moving horizon estimation, including a summary of recent theoretical developments and numerical solution techniques, Rawlings (2000)[25] provided an excellent introductory tutorial aimed at control practitioners. A comprehensive review of theoretical results on the closed-loop behavior of MPC algorithms was provided by Rawlings, Rao, and Scokaert (2000). Notable past reviews of MPC theory include those of Garsia, Prett, and Morari (1989)[18]; Ricker (1991)[21]; Morari and Lee (1991)[40][41]; Muske and Rawlings (1993)[43]; Rawlings, Meadows, and Muske (1994)[30]; Mayne (1997)[39], and Lee and Cooley (1997)[22]. Several books on MPC have recently been published by Allgower & Zheng (2000)[45]; Kouvaritakis & Cannon, 2001[46]; Maciejowski, 2002[47]. A survey of industrial MPC technology based on linear models at the 1996 Chemical Process Control V Conference was presented by the authors: Qin & Badgwell, 1997[48], Qin and Badgwell, 2000[49]. Froisy (1994)[50] and Kulhavy, Lu, and Samad (2001)[51] describe industrial MPC practice and future developments from the vendor’s viewpoint. Young, Bartusiak, and Fontaine (2001)[52], Downs (2001)[53], and Hillestad and Andersen (1994)[54] reported development of MPC technology within operating companies. A survey of MPC technology in Japan provides a wealth of information on application issues from the point of view of MPC users was provided by Ohshima, Ohno, & Hashimoto[44].

The principle of MPC is showed in Figure 1. The detail and advantages of this method were described in [1]. The MPC has some advantages as described in [1]. However, it also has some disadvantages as following.

Predictive model is required exactly to predict a process state in predictive horizon. In fact, this is a difficult problem. In order to solve this problem, Huang Y. L., Lou H. H., Gong J. P, Edgar T. F [57]... have used fuzzy set theory in system modeling, Markus Lendl, Udo H. Schwarz, Hans-Joachim Romeiser, Rolf Unbehauen, Michael Georgieff and Gütz F. Geldner [59], Najávéř[60], J.Wang and G.Thomas[61], Kaouter Iaibidi and Faouzi Bouan[56], Tatjewski, P., Lawryncuk, M[16]... have used Neural Networks in system modeling, X.J. Lin, C.W.Chan [63], J. Shing and R. Jang [64]... have combined the fuzzy system and Neural Networks in system modeling, the simulation results for the plants is rather good.
time for tuning the weight is also longer than usual, and even making the closed-loop system is unstable.

The second drawback is that do not have algorithm for adaptive tuning of the weight coefficients for global asymptotical stable (GAS) of the closed-loop system [76].

This paper proposes a method for tuning of the weight coefficients. The method is Model Predictive Control based on Fuzzy Logic, Genetic Algorithms or B&B, and Adaptive Control (AFMPC). First, the plant is modeled by using TS model. Then GAs or B&B is used for solving an optimal problem in objective function. Finally, the weight coefficients are tuned using adaptive technique based on the error between the reference and the output. The simulation results on the Continuous Stirred Tank Reactor (CSTR), uncertain dynamical nonlinear systems and nonlinear DC motor verified that the control performance is good, and the closed-loop system is global asymptotical stable.

The remainder of this paper is organized as follows. Section 2 is a brief description of model predictive control based on fuzzy logic. In the same section we also describe briefly the objective function and Genetic Algorithms (GAs) and B&B method. In section 3, we propose an adaptive tuning for weight coefficients. In section 4 the proposed AFMPC methodology is applied to a SISO chemical reactor (CSTR), nonlinear uncertain dynamical systems and nonlinear DC motor. The results are discussed and compared with conventional MPC method. The paper ends with concluding remarks.

II. MODEL PREDICTIVE CONTROL BASED ON FUZZY LOGIC AND GENETIC ALGORITHMS AND B&B METHOD

A. Fuzzy Model Prediction

Fuzzy set theory is used in system modeling. Modeling is implemented by fuzzy inference system (FIS). FIS are unit process which change digital information into language variable by fuzzy process, i.e. process change physical values into fuzzy values through fuzzy sets.

FIS is all-powerful approximate tool. So FIS can approximate any continuous function in domain with high accuracy. However, the all-powerful approximation of fuzzy model is not the most important problem, but the information can be given by the model. This information depicts the system response which is modeling in language [66].

In order to ensure the accurate of Predictive Model, the Takagi-Sugeno fuzzy model [9] is used broadly. The advantage of TS model is that the observable output-input data is easy receiving by grouping engineering. Furthermore, this model has simple structure and high accuracy. The computing of TS fuzzy model is also faster than Mamdani model [10]. The output of TS as following [11]

\[ \hat{y}(k+d) = \frac{\sum_{i=1}^{L} \theta_i \mu_i(\varphi(k+d)) \mu_i(\varphi(k+d))}{\sum_{i=1}^{L} \mu_i(\varphi(k+d))} \] (1)
The rapid increase of control signal is not only making the fuzzy system as given in following table I [11].

\[ q(k+d) = \left[ y(k+d-1), \ldots, y(k+d-N_y), u(k-l), \ldots, u(k-N_u) \right] \]

\( d \) is the time-delay of the plant

There are many methods to define the parameters of the fuzzy system as given in following table I [11].

**TABLE I. METHODS OF FUZZY MODELLING**

<table>
<thead>
<tr>
<th>Method</th>
<th>Type of MFs</th>
<th>Number of MFs</th>
<th>Location of the MFs</th>
<th>Consequences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mosaic scheme</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Adjusted</td>
</tr>
<tr>
<td>Gradient descent</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Adjusted</td>
<td>Adjusted</td>
</tr>
<tr>
<td>Clustering + gradient descent</td>
<td>Adjusted</td>
<td>Adjusted</td>
<td>Adjusted</td>
<td>Adjusted</td>
</tr>
<tr>
<td>Evolution strategies</td>
<td>Adjusted</td>
<td>Adjusted</td>
<td>Adjusted</td>
<td>Adjusted</td>
</tr>
</tbody>
</table>

For the uncertain dynamical nonlinear systems, the system modeling shall be more complex than the certain dynamical nonlinear system. In order to ensure the accurate of Predictive Model, we have to understand the plant and put this information in the approximation algorithm by Takagi-Sugeno model. The steps for identifying plant are as following:

From the mathematical model of plant is described, we can divide plant into Model 1, Model 2, ..., Model N and using TS fuzzy model to approximate. Mechanisms of this problem is to use “fuzzy key” (using the command *if ... else ...end*) according to the advanced stipulation. So, predictive model is organized by TS 1, TS 2...TS N and a fuzzy key.

**B. Objective Function and general solution**

The difference of MPC algorithms are the difference for objective function considerations for obtaining the control rules. The final purpose is the future signal \( \hat{y} \) (in the predictive horizon) to track reference signal (set-point) \( r \), and simultaneously, defining the optimal value of the control signal \( \Delta u \) of objective function \( J \).

Assume that \( J \) is chosen as following [1].

\[ J = \frac{1}{2} \delta(k)[e(k+d)]^2 + \frac{1}{2} \lambda(k)\Delta u(k)^2 \]  \hspace{1cm} (3)

Then, the general solution is defined by the approximate method:

\[ \frac{\Delta J}{\Delta u(k)} = \delta(k) e(k+d) \frac{\Delta e(k+d)}{\Delta u(k)} + \lambda(k) \Delta u(k) = 0 \]  \hspace{1cm} (4)

From (3), we obtain

\[ u(k) = u(k-1) - \frac{\delta(k)}{\lambda(k)} e(k+d) \frac{\Delta e(k+d)}{\Delta u(k)} \]

\[ = u(k-1) - \frac{\delta(k)}{\lambda(k)} e(k+d) \xi \]  \hspace{1cm} (5)

We can see that when the error \( e(k+d) \) is larger then the control signal is also increased. But, sometimes the rapid increase of control signal is not only making the decrease system performance but also may lead to unstable of the closed-loop system.

Therefore, in this paper, the method for tuning the weights of objective function is proposed to ensure the global asymptotical stable of the closed-loop system based on the sensitive function:

\[ \xi = \Delta \hat{y}(k+d) \]  \hspace{1cm} (6)

**C. Genetic Algorithms**

Genetic Algorithms (GAs) are adaptive heuristic search algorithm premised on the evolutionary ideas of natural selection and genetic. The basic principle of GAs is introduced by John Holland in 1962 [12]. In the optimal area, the GAs is used widely in different fields as optimization, image processing, system identification and control.

Using the GAs for optimal problem will search the global neighborhood extreme [14] [31], this avoid the local extreme as previous studies. The steps of using GAs for solving the optimal problem are introduced detail in [14] [31].

**D. Branch and Bound method [8] [68] [69].**

The Branch and Bound (B&B) method is to search for a discrete optimal solution of the optimization and especially found in the nonlinear predictive control problem. The main idea of this method is dividing problem into small sub-problems using a tree structure. The technique is based on the fact that, in general, only a small number of possible solutions actually need to be enumerated, so the remaining solutions are eliminated through the application of bound, i.e., upper and lower bounds of the objective function are used to decide whether a branch is examined or not. The B&B algorithm applied to MPC has three major advantages:

- The global optimum is always found in the discrete space of control alternatives, guaranteeing a good control performance when the number of control actions is sufficiently large.
- The algorithm does not need any initial guess.
- It implicitly deals with the constraints.

In fact, the present of constrains improve the efficiency of bounding, restricting the search space by eliminating non-feasible sub-problems. Two serious drawbacks of B&B method are the exponential increase of computational time with the control horizon and the number of alternatives, and the discretization of the possible control actions. This discretization can cause chattering, overshoots and slow step-responses. A solution to deal with these problems is using dynamic grid size method.

**III. ADAPTIVE TUNING THE WEIGHTS OF THE OBJECTIVE FUNCTION**

On the recent studies of MPC based on Fuzzy Logic, the weight coefficients \( \delta(k) \) and \( \lambda(k) \) are chosen as fix coefficients, and that choice depends on the experience. Therefore, it spends a lot of time for searching and difficulty for choosing suitable parameters. This paper proposes a method for defining the weight coefficients...
based on the relationship between these coefficients to ensure the stable of the closed-loop system.

Define the Lyapunov function of closed-loop system as following:

\[ V = \frac{1}{2} e(k + d)^2 + \frac{1}{2} \Delta e(k + d)^2 \]  

(7)

Where

\[ e(k + d) = r(k + d) - \dot{y}(k + d) \]  

(8)

\[ \Delta e(k + d) = e(k + d) - e(k + d - 1) \]  

(9)

In case of the set-point is constant, then

\[ \Delta e(k + d) = \Delta \dot{y}(k + d) \]  

(10)

The condition for closed-loop stability is

\[ \Delta V \leq 0 \]  

(11)

From (5), we obtain

\[ \Delta V = e(k + d) \frac{\Delta e(k + d)}{\Delta u(k)} + \Delta e(k + d) \frac{\Delta (\Delta e(k + d))}{\Delta u(k)} \]  

(12)

We can be demonstrated

\[ \Delta e(k + d) = \frac{\Delta e(k + d)}{\Delta u(k)} \Delta u(k) \]  

(13)

From (5), \( u(k) \) can be rewritten as

\[ u(k) = u(k - 1) + \Delta u(k) \]  

(14)

Replacing (13) and (14) into (12), we obtain

\[ \Delta V = \frac{\lambda(k)}{\delta(k)} \Delta u(k)^2 + \left[ \frac{\Delta e(k + d)}{\Delta u(k)} \right]^2 \Delta u(k)^2 \]  

(15)

For satisfying (11), then

\[ \left[ \frac{\Delta e(k + d)}{\Delta u(t)} \right]^2 - \frac{\lambda(k)}{\delta(k)} \leq 0 \]  

(16)

(16) can be rewritten as

\[ \frac{\delta(k)}{\lambda(k)} \leq \frac{1}{\left[ \frac{\Delta e(k + d)}{\Delta u(k)} \right]^2} \]  

(17)

From (1), and (5), we obtain

\[ \delta(k) \leq \frac{1}{\left[ \frac{\Delta e(k + d)}{\Delta u(k)} \right]^2} \]  

\[ \lambda(k) \leq \frac{1}{\left[ \frac{\max \left( \Delta \dot{y}(k + d) \right)}{\Delta u(k)} \right]} \]  

(18)

Where

\[ 0 < \delta(k), \lambda(k) < 1 \]  

(19)

\[ \delta(k) + \lambda(k) = 1 \]  

(20)

The inequality (18) is the relationship between \( \delta(k) \) and \( \lambda(k) \) in the objective function (3). The system is also Lyapunov stable.

The weight coefficients \( \delta(k) \) and \( \lambda(k) \) are adaptive tuned according to (18). The system performance is better, and the closed-loop system is GAS.

From the proposal algorithm as above, the block diagram of model predictive control based on Fuzzy Model as Figure 2.
is reaction temperature
is heat transfer factor.
The parameters of CSTR are chosen as \( D_o = 0.072, \quad \phi = 20, \quad B = 8, \quad \beta = 0.3 \). For these parameters, the CSTR is unstable (Chen, and Peng, 1997).

The structure of MPC based on TS is chosen as following

There are 2 fuzzy sets in each of inputs of linguistic variable, and the membership function form is chosen as trapezoid form. Predictive output has the following form

\[
\hat{y}(k+d) = f[y(k+d-1), y(k+d-2), u(k-1)]
\]

(22)

Control horizon \( H_c \) is 2, output horizon \( H_p \) is 6, and sample time is 0.5 second.

The parameters of GAs are chosen as:
- Bit numbers: 10
- Chromosome numbers: 50
- Generation numbers: 6

The parameters \( \delta(k) \) and \( \lambda(k) \) are tuned as (18).

B. Simulation and results

We use Matlab for simulation of the system. The simulations results as shown in Figures 3-8. Figures 5 is the simulation results using AFMPC, the weight coefficients are tuned adaptive. Figure 3 describe the input/output of the plant with family signal (10,000 samples). Figure 4 describe the offline training results.

As the simulation results in Figures 5-8, the simulation results using AFMPC are better than simulation results using conventional MPC, mean that the output signal track to reference signal precisely, transient period (settling time and rise time) is shorter and system performance is better than conventional MPC (figures 6,8). This can be clearly when the system is effected by external disturbance (figures 7, 8).
4.2. Using AFMPC and fuzzy key for uncertain dynamical nonlinear systems

C. Mathematical Model

The mathematical model of nonlinear uncertain dynamical systems is described as [56].

\[
y(k) = \frac{ay(k-1) + cu(k-1)}{1 + by(k-1)}
\]  

(23)

The parameters a, b and c are taken as follows: (a, b, c)=(1, 0.1, 0.5) for the first 800 iterations and (a, b, c)=(0.5, 0.8, 1) for the remaining iterations. So Predictive model is organized by two models (TS 1 and TS 2) and a fuzzy key. The structure of MPC based on TS is chosen as following:

There are 2 fuzzy sets in each of inputs of linguistic variable, and the membership function form is chosen as trapezoid form. Predictive output has the following form

\[
\hat{y}(k) = f(y(k-1), u(k-1), u(k-2))
\]  

(24)

Control horizon \( H_C \) is 2, output horizon \( H_P \) is 6, sample time is 4 second and \( 0 \leq u \leq 0.7, -0.2 \leq \Delta u \leq 0.2 \). The parameters of GAs are chosen as:

- Bit numbers: 10
- Chromosome numbers: 50
- Generation numbers: 6

The parameters \( \delta(k) \) and \( \lambda(k) \) are tuned as (18).

D. Simulation and results

We use Matlab for simulation of the system. The simulations results as shown in Figures 9-14. Figures 11 and 13 are the simulation results using AFMPC, the weight coefficients are tuned adaptive. Figure 9 discribe the input/output of the plant with family signal (10,000 samples) using Fuzzy key. Figure 10 discribe the offline training results.

The reference signal \( r(t) \) is trapezium. As the simulation results in Figures 11-13, the simulation results using MPC-FGA are better than simulation results using conventional MPC, mean that the output signal track to reference signal precisely, transient period (settling time and rise time) is shorter and system performance is better than conventional MPC (figures 12, 14). This can be clearly when the system is effected by external disturbance (figures 13, 14).
4.3. Using AFMPC and fuzzy key for nonlinear DC motor

E. Mathematical Model

The mathematical model of nonlinear DC motor is described as [29] [64].

\[
\dot{\omega} = -k_1 \omega + k_2 i - k_3 \text{sgn}(\omega) - k_4 \exp(-k_5 |\omega|) \text{sgn}(\omega)
\]

\[
i = -k_6 \omega - k_7 i + k_8 u
\]

\[
y = \omega
\]

(25)

<table>
<thead>
<tr>
<th>Positive direction parameters</th>
<th>Negative direction parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1</td>
<td>0.0110</td>
</tr>
<tr>
<td>K2</td>
<td>16.1656</td>
</tr>
<tr>
<td>K3</td>
<td>20.6626</td>
</tr>
<tr>
<td>K4</td>
<td>1.3142</td>
</tr>
<tr>
<td>K5</td>
<td>20.8965</td>
</tr>
<tr>
<td>K6</td>
<td>19.2766</td>
</tr>
<tr>
<td>K7</td>
<td>16.2566</td>
</tr>
<tr>
<td>K8</td>
<td>0.0035</td>
</tr>
</tbody>
</table>

TABLE II. PARAMETERS OF NONLINEAR DC MOTOR

\(u\) is terminal voltage of armature circuit (V), \(\omega\) is the angular speed of the rotor (rad/s)

So Predictive Model is organized by two models: TS 1 for positive direction parameters, TS 2 for negative direction parameters, using TS 1 or TS 2 by a fuzzy key. The structure of MPC based on TS is chosen as following:

There are 2 fuzzy sets in each of inputs of linguistic variable, and the membership function form is chosen as trapezoid form. Predictive output has the following form

\[
\hat{y}(k) = f(y(k-1), u(k-2), u(k-3))
\]

(26)

Control horizon \(H_c\) is 1, output horizon \(H_p\) is 10, sample time is 0.01 second, and \(-10 \leq u \leq 10\), \(-0.2 \leq \Delta u \leq 0.2\)

B&B is used for solving an optimal problem in objective function. The parameters \(\delta(k)\) and \(\lambda(k)\) are tuned as (18).

F. Simulation and results

We use Matlab for simulation of the system. The simulations results as shown in Figures 17–20. Figures 17 and 19 are the simulation results using AFMPC, the weight coefficients are tuned adaptive. Figure 15 describe the input/output of the plant with family signal (10,000 samples) using Fuzzy key. Figure 16 describe the offline training results.

The reference signal \(r(t)\) is trapezium. As the simulation results in Figures 17–19, the simulation results using AFMPC are better than simulation results using conventional MPC, mean that the output signal track to reference signal precisely, transient period (settling time and rise time) is shorter and system performance is better than conventional MPC (figures 18, 20). This can be clearly when the system is effected by external disturbance (figures 19, 20).
V. CONCLUSIONS

The combination of MPC with TS fuzzy model and GAs or B&B with adaptive tuning of weight coefficients of objective function, the nonlinear predictive control problem is solving completely. This method is used for nonlinear systems with non-minimum phase (CSTR), uncertain dynamical systems and nonlinear DC motor.
The simulation results showed that the system performance when using AFMPC or AFMPC and fuzzy key are better than conventional MPC. The output \( y(t) \) track the reference \( r(t) \) precisely. The close-loop system is also GAS.

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