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ELECTROMAGNETIC RESONANT BEHAVIOR OF A CONFOCAL SPHEROIDAL CAVITY SYSTEM IN THE MICROWAVE REGION

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Abstract

The resonance of a small spheroidal object in a large spheroidal cavity is investigated. This approximates the case of a thin needle-like antenna in a large cavity. It is shown that this needle, if thin enough, shows marked resonant properties, in that when the cavity is tuned to a resonant frequency defined by the needle, the magnetic field on the surface of the needle is greatly enhanced. This property can be used in a practical way, in measuring surface impedance of the material of which the needle is composed: at resonance most of the loss in the cavity is located at the surface of the needle and depends on the material of which it is composed.

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A thin spheroidal antenna in empty space is known (1) to show resonance effects when its wavelength is approximately a whole number of half-wavelengths; the thinner the antenna and the more closely it approaches a line, the closer the resonant lengths are to whole numbers of half-wavelengths. Also the Q of the resonance becomes infinite as the antenna becomes infinitely thin. These facts suggest that a thin spheroidal antenna in a resonant cavity might show similar results. If so, it would have a practical value for certain microwave measurements. It is often desired to investigate the surface impedance of a sample of material; that is, its contribution to the Q of a cavity containing it. If the whole cavity is made of the material in question, the calculation of the relation between Q and the surface impedance is simple. If, however, it is only convenient to introduce a sample of one material into a cavity of another, both materials will make contributions to the Q of the composite cavity. It is then obviously desirable to concentrate the tangential magnetic field, and related surface current, on the surface of the sample as far as possible, so that it will contribute most of the losses, and a comparatively small part will come from the remaining wall of the cavity. The facts regarding high-Q antennas in empty space suggest that if the sample were made in the form of such an antenna, the larger part of the loss would be concentrated on the sample. Pippard (2) has used resonant samples in his studies of the surface impedance of conductors and semiconductors at very low temperatures and microwave frequencies.

To investigate this problem, a natural method is to solve the case of resonance of a confocal spheroidal cavity, the inner spheroid representing the sample antenna, the outer spheroid the cavity. This case can be handled analytically, and it is possible to compute both the resonant frequencies as functions of the various parameters, and the ratio of the contributions of inner and outer conductors to Q. Solutions of this problem have been obtained for a number of values of the parameters. We shall not present the detailed results in the present note. The solutions elucidate the qualitative nature of the result, however, and that is simple enough so that it seems worth while to describe it briefly.

Let us start by fixing the distance between the foci of the spheroids. The infinitely thin spheroid will then be a line or needle, whose length is the interfocal distance. Special resonance properties will be shown for those frequencies for which this infinitely thin antenna is $\ell+1$ half-wavelengths long (so that ℓ measures the number of nodes of the magnetic field in the length of the antenna). Let us now set up a standing electromagnetic wave of one of these resonant frequencies, with the appropriate number of nodes along the length of the needle, and satisfying the proper boundary conditions on the surface of the antenna. This standing wave will have an infinite number of confocal spheroidal surfaces surrounding the antenna, distant approximately a half-wavelength from each other, on which the tangential electric field will be zero, so that any one of these surfaces could be replaced by a metallic cavity wall without interfering with the radiation pattern. The larger spheroids, with many wavelengths inside them, will of course approach spheres. Such a cavity, then, is so chosen that the frequency of the resonant modes corresponding to a given ℓ value within it will equal the frequency of the corresponding mode of the free antenna. We may call such a cavity a standard cavity.

The solution we have just set up is not the only type consistent with the resonant frequencies. At any such resonant frequency, we have not only the solution just described, which satisfies boundary conditions on an infinitely thin spheroid; we have also an independent solution, related to the first solution as the sine is to the cosine at large distances, and having a singularity at the origin of such variety that it cannot satisfy suitable boundary conditions at a needle-shaped antenna. We can then make any linear combination of these two solutions, and each such linear combination will have a family of confocal spheroids on which the tangential component of electric field is zero. We could make two such spheroids, of which one may be small, the other large, into conductors, and thus set up a resonant solution of this type, and a standard cavity, for an inner spheroid of any arbitrary eccentricity.

We may now take such a standard cavity and perturb it by either increasing or decreasing the size of the outer wall, keeping it always a confocal spheroid and leaving the inner spheroid unchanged. As we do this, we can follow the behavior of each of the resonant modes of the cavity, and each such mode will have its frequency tuned, but without change of its fundamental nodal structure. As the cavity is shrunk the frequency will increase, since we still have the same number of nodes between the antenna and the wall, and this distance decreases, bringing a decrease of wavelength, or increase of frequency; similarly, increase of the size of the cavity decreases the frequency. The effect of these changes on the wave pattern may be decribed qualitatively. As the cavity shrinks and the frequency increases, the field tends to decrease near the antenna, and at the same time the region of high field near the antenna shrinks down toward the center of the antenna, so as to preserve the correct wavelength. If the cavity is greater than the standard size the variation is of the opposite sort, the field being stronger nearer the antenna.

The essential point for our purpose, however, is not this variation of field strength, but one particular quantity to be derived from it, the integral of H^2, where H is the magnetic field, over the inner spheroid, divided by the corresponding integral over the outer spheroid; that is, the ratio of losses on inner and outer spheroid, assuming them to be made of the same material. When we make the calculations we find that the loss on the inner spheroid is small for frequencies lower than the resonant frequency, increases as the frequency increases, goes through a maximum, and then decreases again at still higher frequencies, the losses on inner and outer spheroids becoming equal at infinite frequency. The thinner the inner spheroid, the smaller the frequency range over which the loss of the inner spheroid is greater than half its maximum value, or the greater the Q as measured by the breadth of the resonance peak; also the thinner the spheroid, the higher the maximum value of loss on the inner spheroid. For very thin spheroids the maximum loss on the antenna comes almost exactly at the resonance frequency, corresponding to the standard cavity. For thicker spheroidal antennas, however, the maximum shifts to lower frequencies. Both these features of the behavior, the dependence of resonant frequency and of the breadth of the resonant peak on the thickness of the antenna, are qualitatively similar to the situation found for antennas in free space.

The calculations which have been carried out have been only exploratory, directed toward elucidating the qualitative nature of the results, as described in the present note; in particular, the methods used have not allowed the evaluation of results for frequencies much greater than resonance and are not adapted to very thin antennas and high-Q. More work would be necessary to get quantitative values of the Q of the resonance (as measured by maximum loss of the inner spheroid, or half-width of the resonance) and the resonant frequency as functions of the eccentricity or thickness of the inner spheroid. The results clearly confirm, however, the supposition that by using a very thin antenna it is possible to concentrate most of the field on the antenna; to do this, the outer cavity must be tuned to a frequency closely agreeing with the free-space resonant frequency of the antenna. Since this result is of such a simple qualitative nature it is to be anticipated that a similar resonant antenna put inside some other shape of external cavity will show similar results as the cavity is tuned to the resonant frequency of the antenna. We may plausibly generalize somewhat further, and suppose that other shapes of resonant objects, as for instance objects shaped like small Hertz oscillators, put inside of ordinary cavities, would show similar properties: as the cavity is tuned to a frequency which would be a resonant frequency of the oscillator if it were in free space the field near the oscillator will build up greatly, so that most of the loss in the cavity will be found on the surface of the oscillating antenna, very little on the surface of the cavity.

References

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