Angular Momentum

$$L = I\omega$$

If no NET external Torques act on a system then

Angular Momentum is Conserved.

$$L_{initial} = \mathrm{I}\omega \;=\; L_{final} = I\omega$$

Angular Momentum

$$L = I\omega$$

Angular Momentum

$$L = I\omega$$

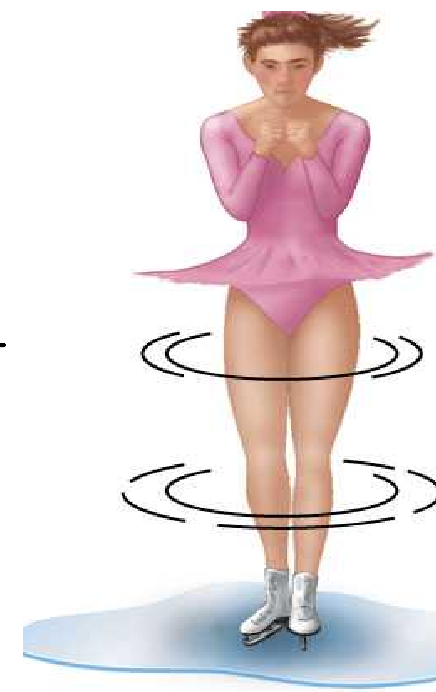
A Skater spins with an angular speed of 2 rev/s. If she brings her arms in and decreases her rotational inertia by a factor of 5, what is her new angular speed in rev/s?

$$L_0 = I_0\omega_0 = I_f\omega_f = L_f$$

$$\omega_f = \omega_0 \frac{I_0}{I_f}$$

$$= 2rev/s \frac{I}{I/5}$$

$$= 10rev/s$$

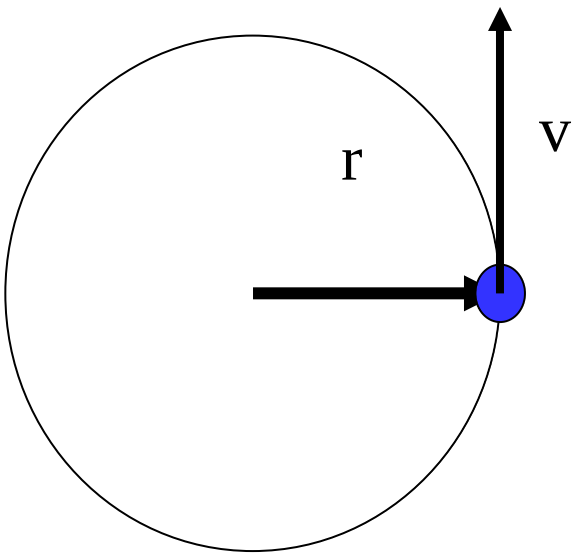


$$L_0 = I_0\omega_0$$

$$L_f = I_f\omega_f$$

Angular Momentum for a Point Particle

Single mass, a distance r from the axis of rotation.

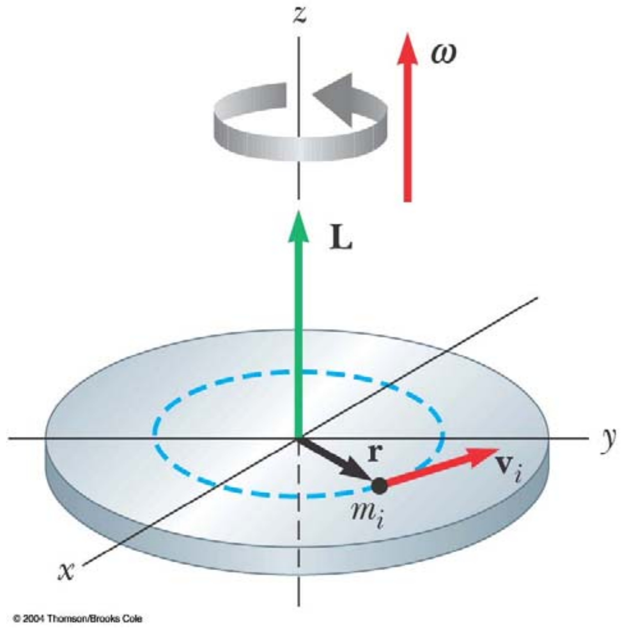


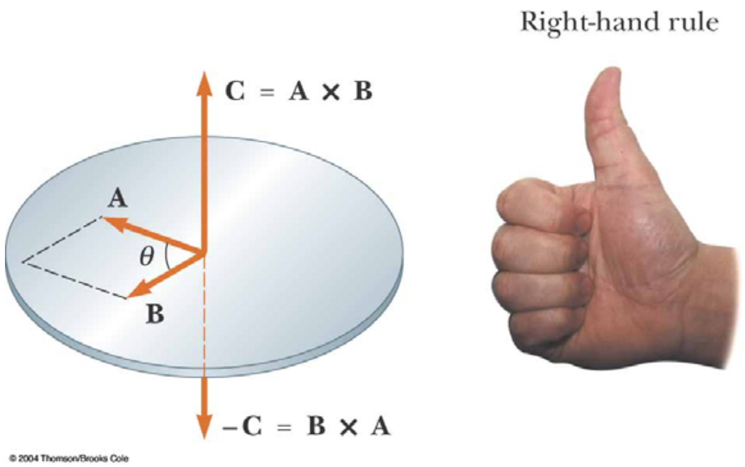
$$L = I\omega$$

$$= mr^2 \frac{v}{r} = mvr$$

$$\boxed{L = mvr}$$

Angular Momentum is a Vector
Angular Speed ω is a Vector

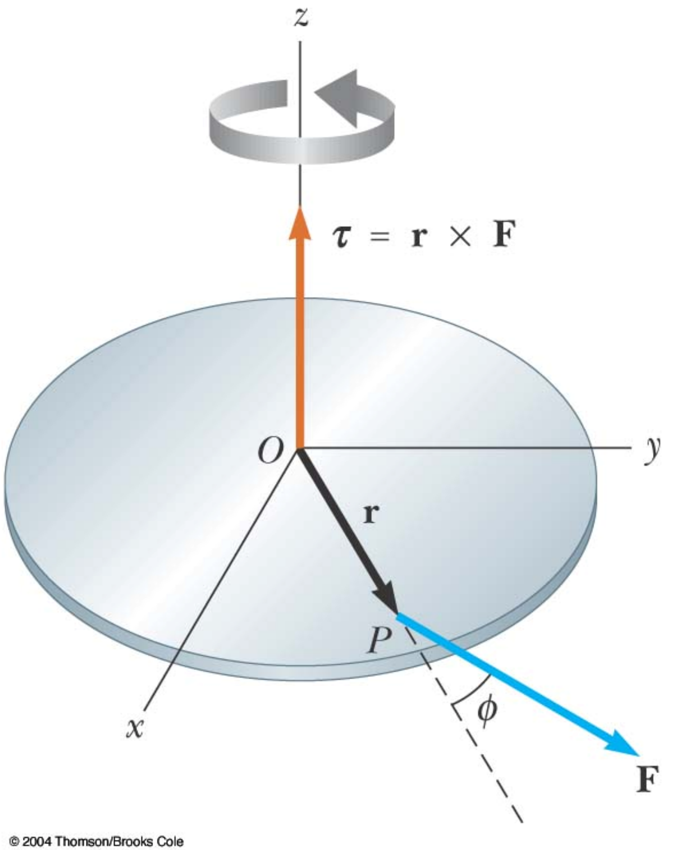




When a rigid object rotates about an axis, the angular momentum **L** is in the same direction as the angular velocity $\boldsymbol{\omega}$, according to the expression $\mathbf{L} = I\boldsymbol{\omega}$, both directions given by the RIGHT HAND RULE.

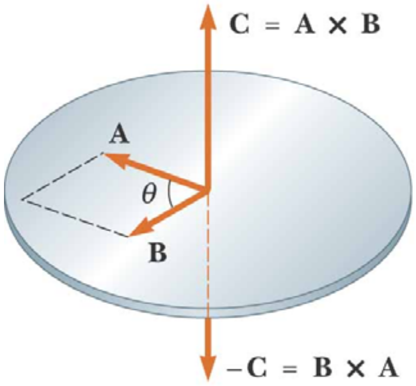
The Vector Product and Torque

- The torque vector lies in a direction perpendicular to the plane formed by the position vector and the force vector
- $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$
- The torque is the vector (or cross) product of the position vector and the force vector



More About the Vector Product

- The magnitude of C is $AB \sin\theta$ and is equal to the area of the parallelogram formed by **A** and **B**
- The direction of **C** is perpendicular to the plane formed by **A** and **B**
- The best way to determine this direction is to use the right-hand rule



Right-hand rule

$$\mathbf{A}\times\mathbf{B} = \left(A_yB_z - A_zB_y\right)\hat{\mathbf{i}} - \left(A_xB_z - A_zB_x\right)\hat{\mathbf{j}} + \left(A_xB_y - A_yB_x\right)\hat{\mathbf{k}}$$

1. Two vectors lying in the *xy* plane are given by the equations **A** = 5**i** + 2**j** and **B** = 2**i** – 3**j**. The value of **A**x**B** is

a. 19**k**

b. –11**k**

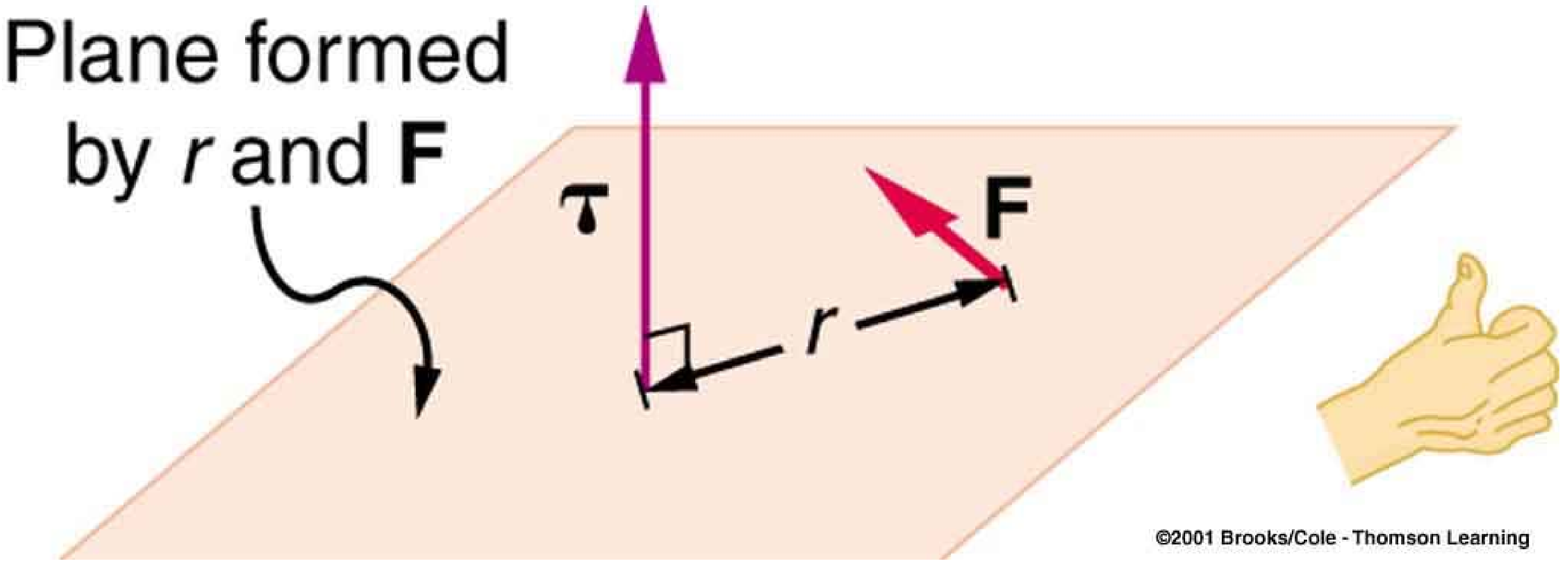
c. –19**k**

d. 11**k**

e. 10**i** – **j**

Torque is also a Vector!

$$\boxed{\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}}$$



The direction is given by the right hand rule where the fingers extend along r and fold into F. The Thumb gives the direction of τ.

COMPARE!

$$\boxed{\begin{aligned}\tau &= \boldsymbol{f} \times \boldsymbol{F}\\ \tau &= \boldsymbol{F}\boldsymbol{r}\sin\phi = \boldsymbol{F}\boldsymbol{d}\end{aligned}}$$

<u>CROSS PRODUCT</u>

F and d must be mutually perpendicular!

$$\boxed{\begin{aligned}\boldsymbol{L} &= \boldsymbol{r} \times \boldsymbol{p}\\ \boldsymbol{L} &= \boldsymbol{m}\boldsymbol{v}\boldsymbol{r}\sin\phi\end{aligned}}$$

<u>CROSS PRODUCT</u>

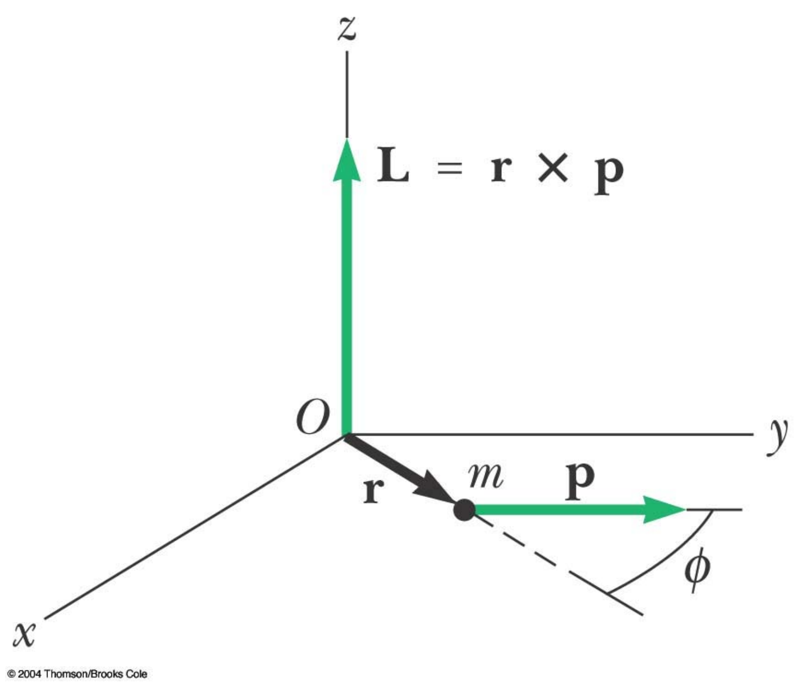
L and p must be mutually perpendicular!

$$\boxed{\boldsymbol{W} = \boldsymbol{F}\cdot\boldsymbol{d} = \boldsymbol{F}\boldsymbol{d}\cos\theta}$$

<u>DOT PRODUCT</u>

F and d must be mutually PARALLEL!

Angular Momentum is a Vector



Magnitude of L:

$$\boldsymbol{L} = \boldsymbol{mvr}\sin\phi$$

Only the perpendicular component of p contributes to L.

The angular momentum **L** of a particle of mass *m* and linear momentum **p** located at the vector position **r** is a vector given by **L** = **r** × **p.** The value of **L** depends on the origin about which it is measured and is a vector perpendicular to both **r** and **p**.

2. A particle whose mass is 2 kg moves in the *xy* plane with a constant speed of 3 m/s along the direction **r** = **i** + **j**. What is its angular momentum (in kg · m^2/s) relative to the origin?

$$L = mvr \sin\phi$$

a. 0 **k**

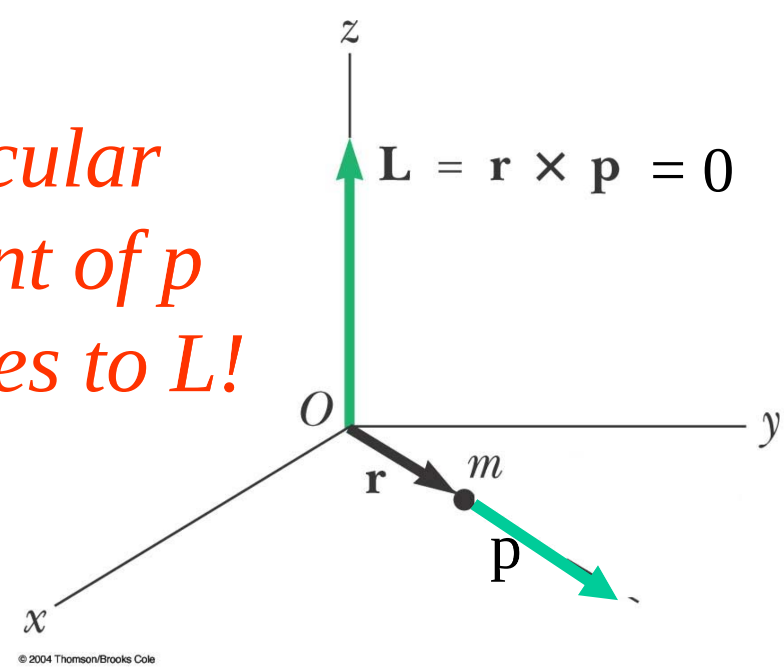
b. 6**k**

c. –6**k**

d. 6 **k**

e. –6 **k**

Only the perpendicular component of p contributes to L!



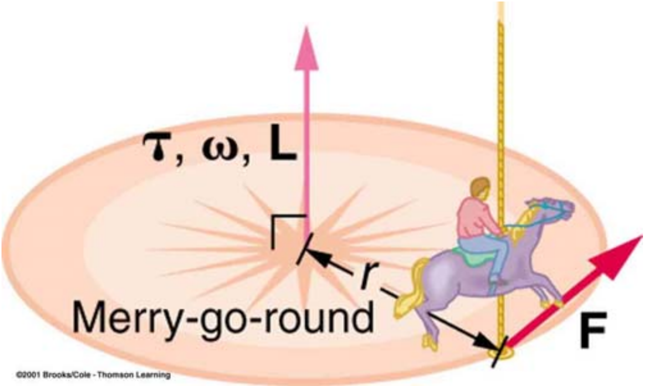
Torque Changes Angular Momentum

$$\sum \tau_{ext} = \frac{\Delta L}{\Delta t}$$

$$\boxed{\Sigma\boldsymbol{\tau}_{ext} = d\mathbf{L}/dt}$$

- $\Sigma\tau$ and **L** must be measured about the same origin
- This is valid for any origin fixed in an inertial frame

$$\boxed{\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = d\mathbf{L}/dt}$$



The torque is perpendicular to both the applied force and the lever arm, but parallel to the angular speed, angular acceleration and angular momentum.

3. YOU TRY: A puck on a frictionless air hockey table has a mass of 5.0 kg and is attached to a cord passing through a hole in the surface as in the figure. The puck is revolving at a distance 2.0 m from the hole with an angular velocity of 3.0 rad/s. The angular momentum of the puck (in kg · m2/s) is

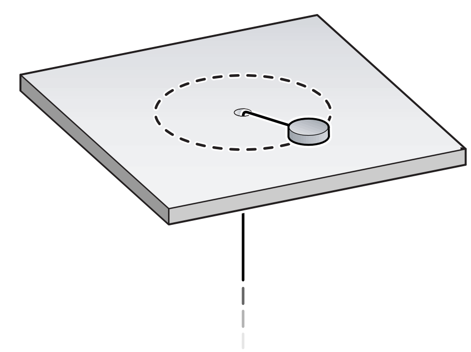
a. 80

b. 20

c. 30

d. 60

e. 50



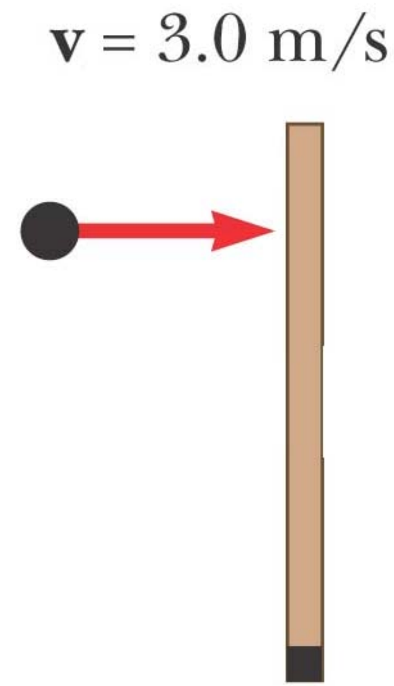
A meter stick is attached at one end (the zero mark) and is free to rotate on a horizontal, frictionless table. A particle of mass 0.400 kg is shot at the meter stick with initial speed 3.00 m/s, as shown. The particle strikes and sticks to the meter stick at the 75.0-cm mark. The meter stick has a mass 0.100 kg.

a) Calculate the rotational inertia and the center of mass of the stick/particle system.

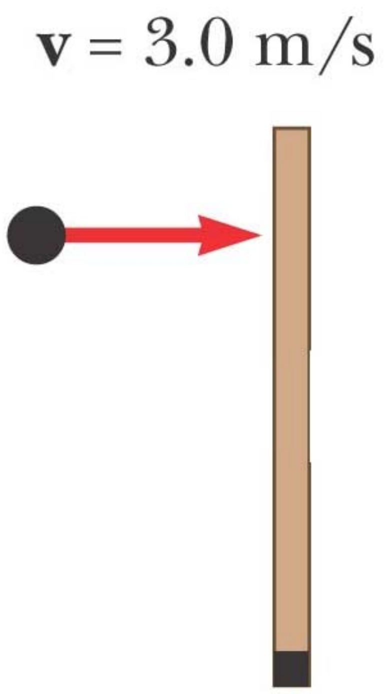
b) Calculate the angular speed of the system just after the particle hits and sticks to the meter stick.

c) Calculate the angular momentum of the system before and after the collision.

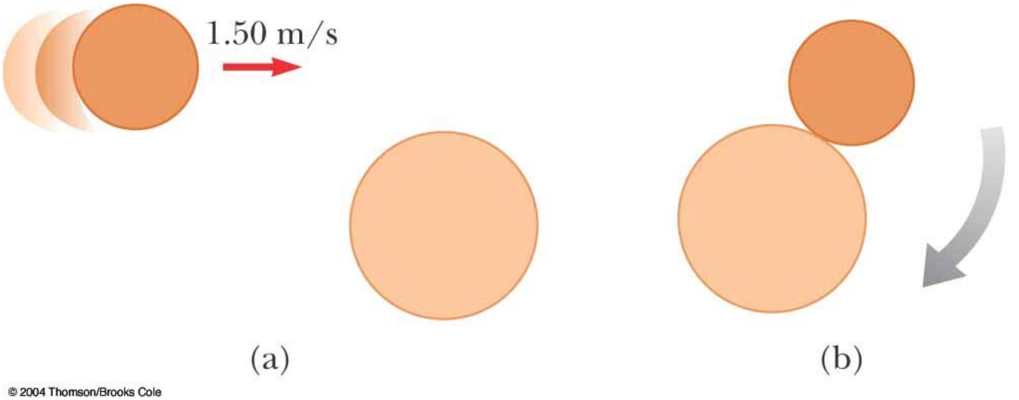
d) How long does it take for the system to complete one revolution after the collision?

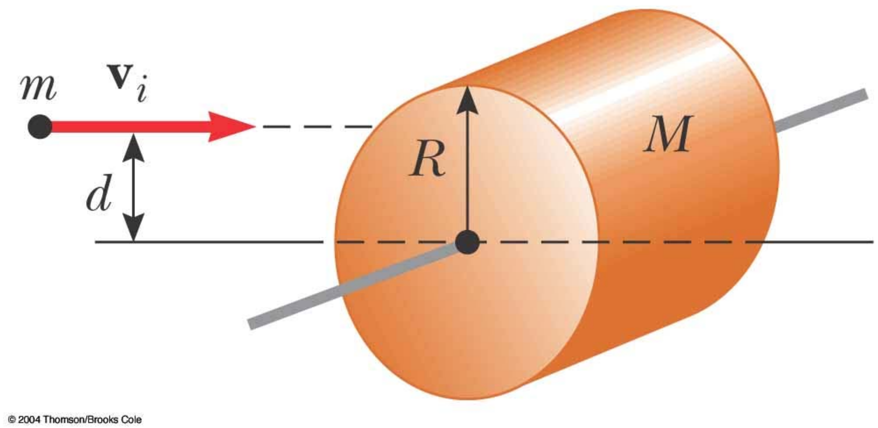


e) If instead the meter stick is NOT fixed at one end but was free to translate, explain and/or sketch the resultant motion. What would change? Calculate whatever would change and find the velocity of the center of mass.

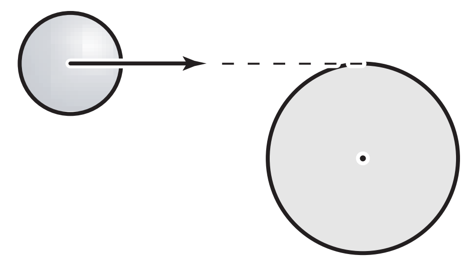


HW Problems I like!





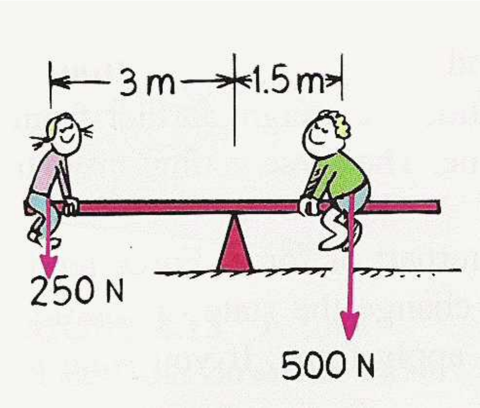
4. A particle of mass m = 0.10 kg and speed $v0$ = 5.0 m/s collides and sticks to the end of a uniform solid cylinder of mass M = 1.0 kg and radius R = 20 cm. If the cylinder is initially at rest and is pivoted about a frictionless axle through its center, what is the final angular velocity (in rad/s) of the system after the collision?



a. 8.1
b. 2.0
c. 6.1
d. 4.2
e. 10

Newton's 1st Law: Conditions for Equilibrium

If the sum of the net external torques is zero, the system is in rotational equilibrium.



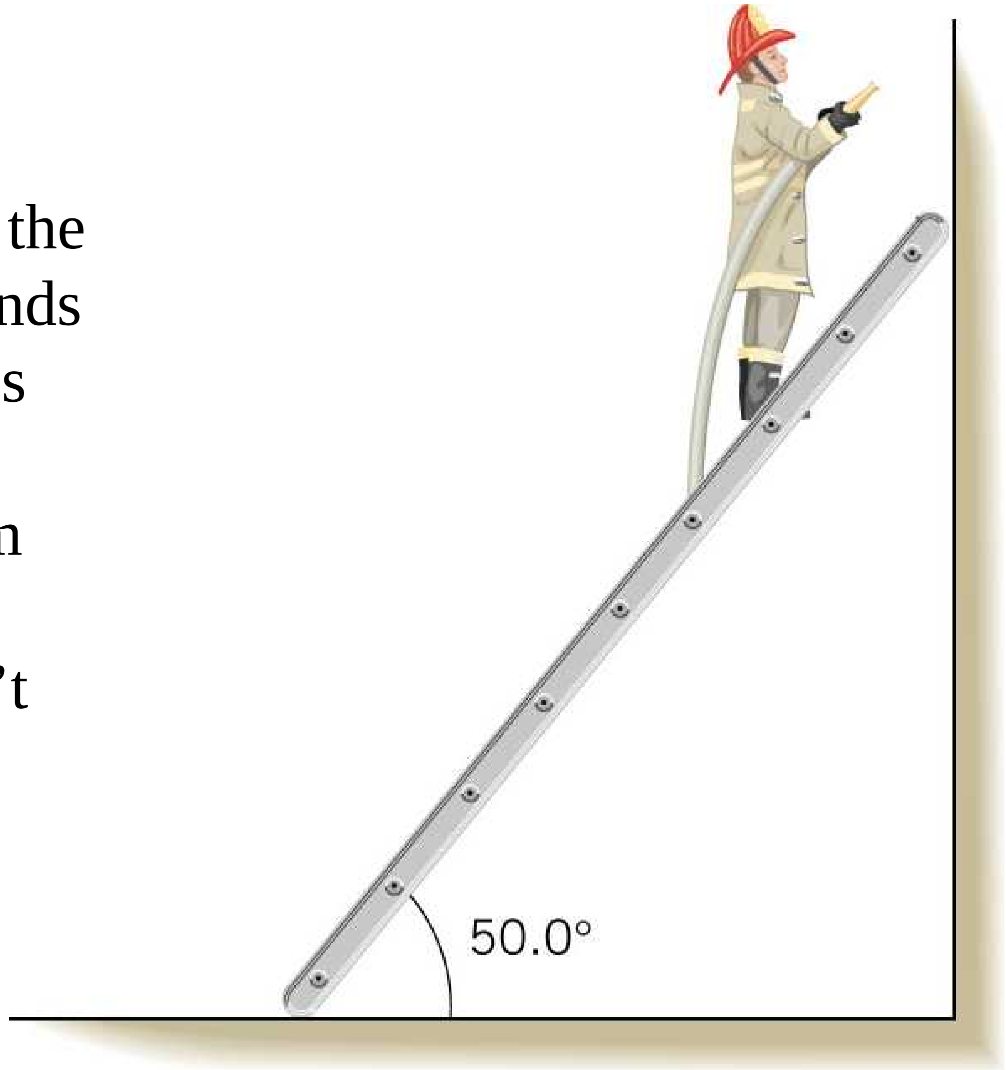
$$\boxed{\sum \tau = 0}$$

If the sum of the net external forces is zero, the system is in translational equilibrium.

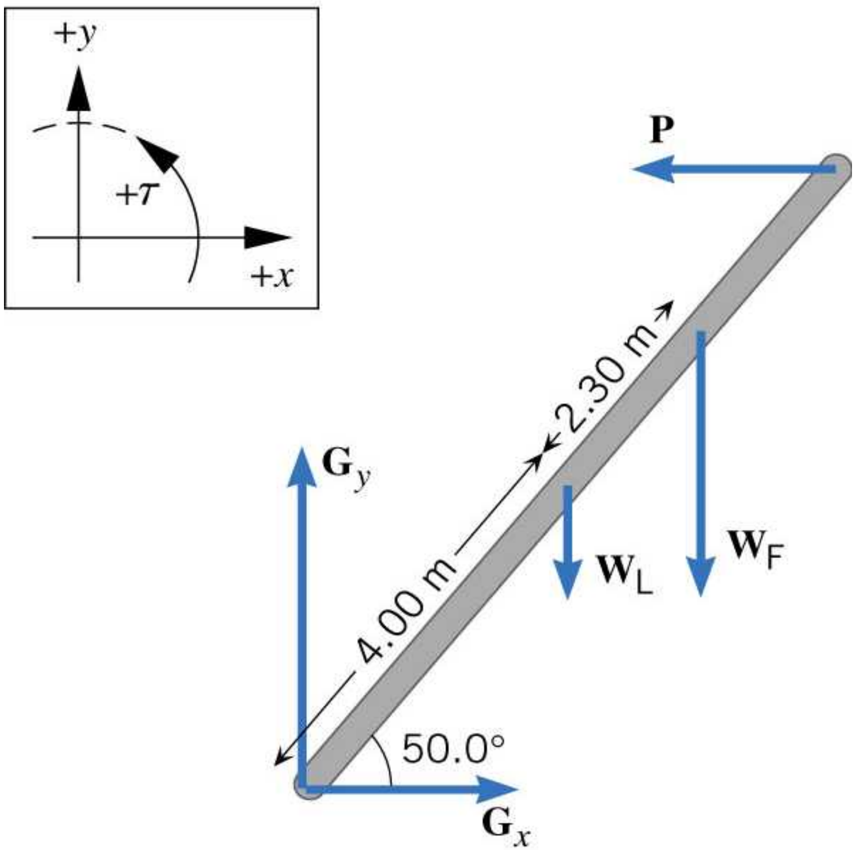
$$\boxed{\Sigma F = 0}$$

The Ladder Problem

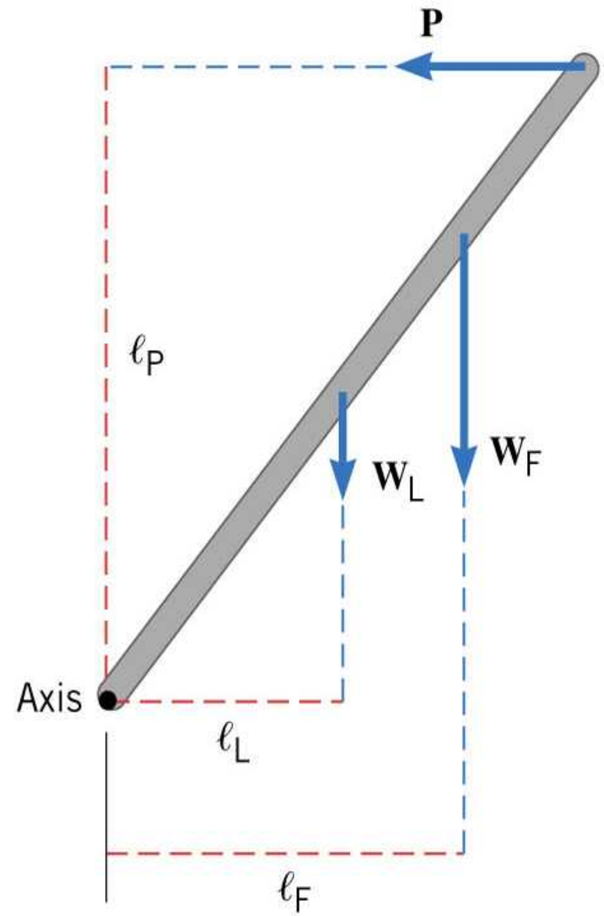
The ladder is 8m long and weighs 355 N. The weight of the firefighter is 875N and he stands 2.30m from the center of mass of the ladder. If the wall is frictionless, find the minimum coefficient of friction of the floor so that the ladder doesn't slip.



$$\sum F = 0 \qquad \sum \tau = 0$$



(*b*) Free-body diagram of the ladder



6. A uniform ladder 15 ft long is leaning against a frictionless wall at an angle of 53° above the horizontal. The weight of the ladder is 30 pounds. A 75-lb boy climbs 6.0-ft up the ladder. What is the magnitude of the friction force exerted on the ladder by the floor?

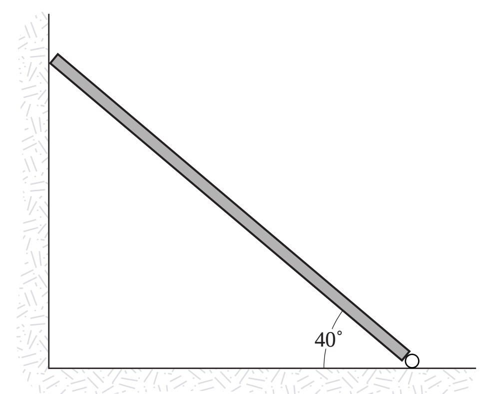
a. 43 lb

b. 34 lb

c. 38 lb

d. 47 lb

e. 24 lb



7. A uniform beam having a mass of 60 kg and a length of 2.8 m is held in place at its lower end by a pin. Its upper end leans against a vertical frictionless wall as shown in the figure. What is the magnitude of the force the pin exerts on the beam? The angle is 40 degrees.

a. 0.68 kN

b. 0.57 kN

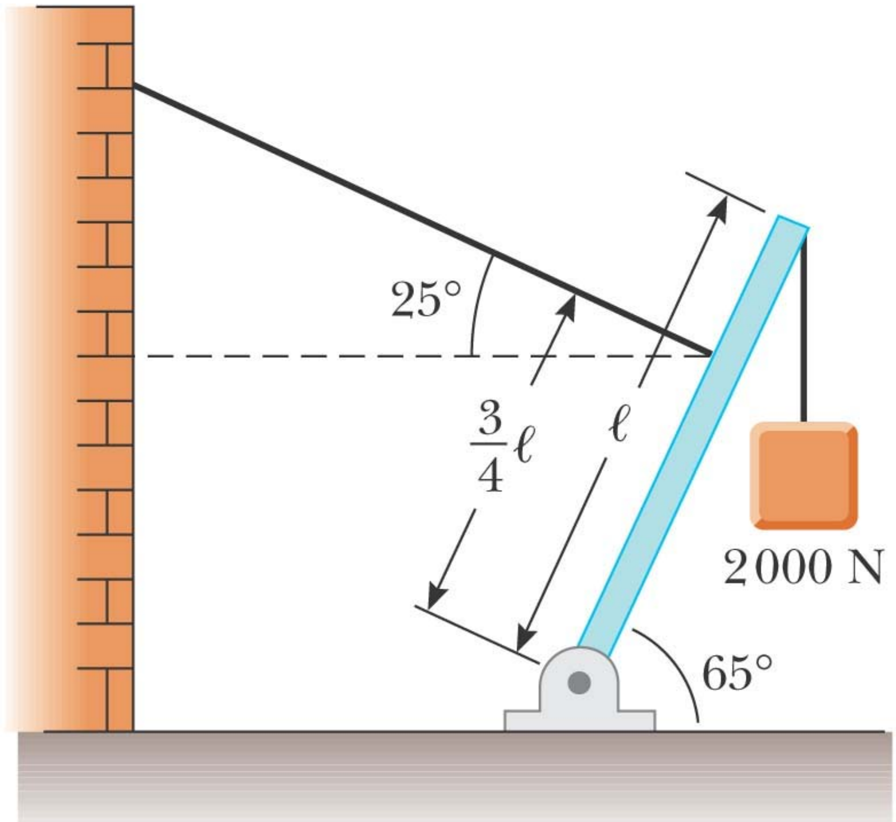
c. 0.74 kN

d. 0.63 kN

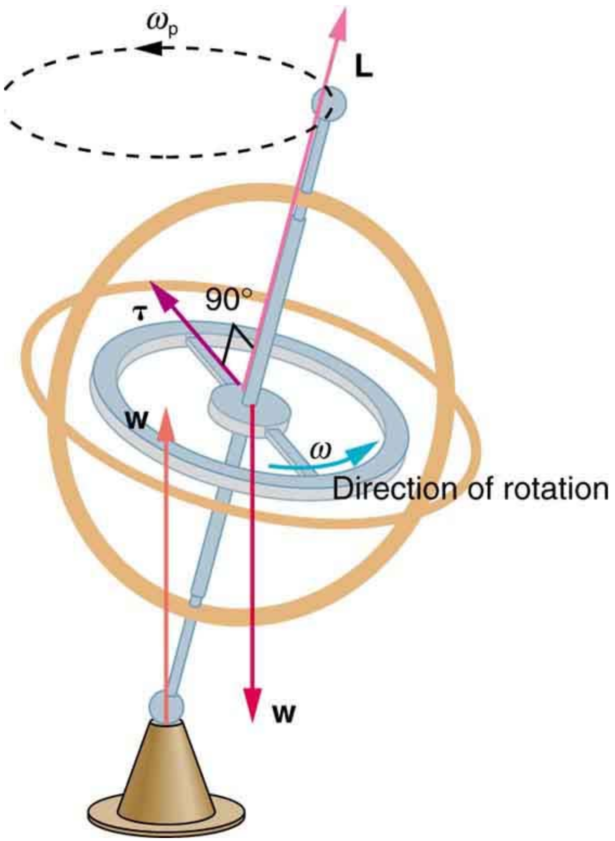
e. 0.35 kN

HW Problem

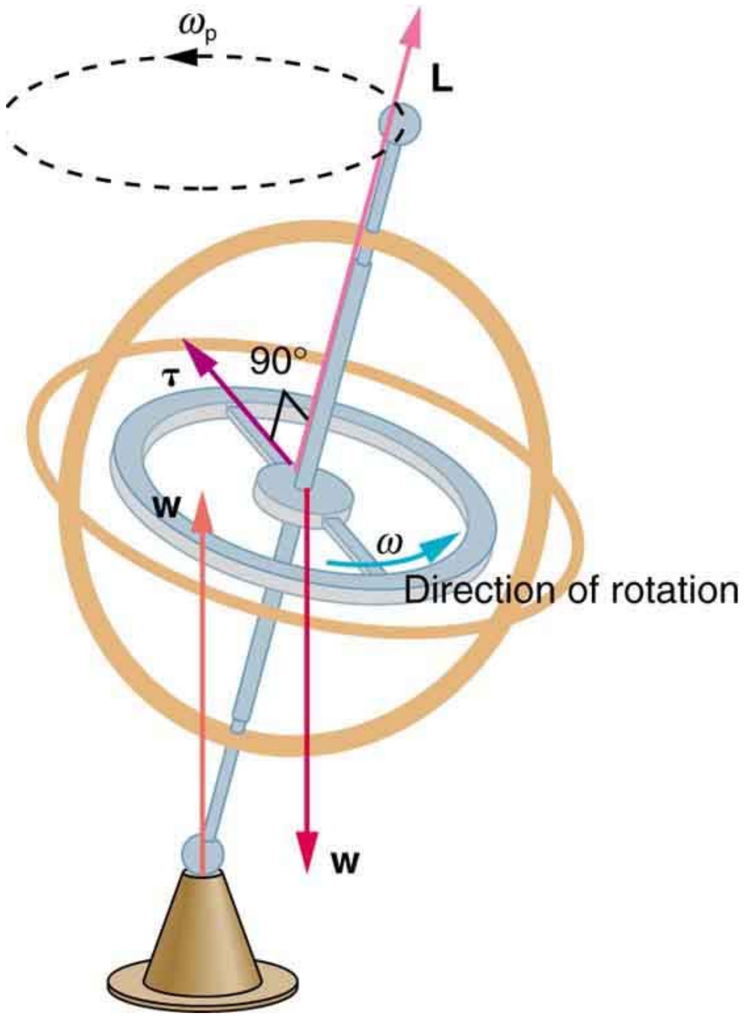
A 1 200-N uniform boom is supported by a cable as shown. The boom is pivoted at the bottom, and a 2 000-N object hangs from its top. Find the tension in the cable and the components of the reaction force by the floor on the boom.



What Keeps a Spinning Gyroscope from Falling Down?

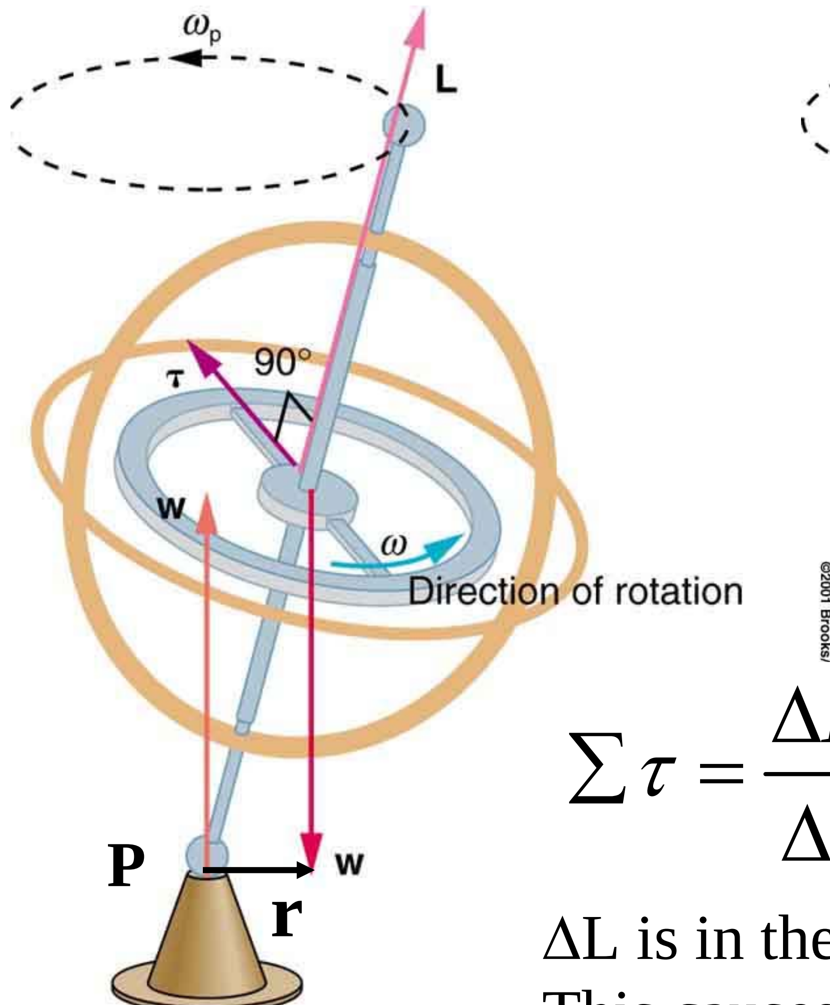


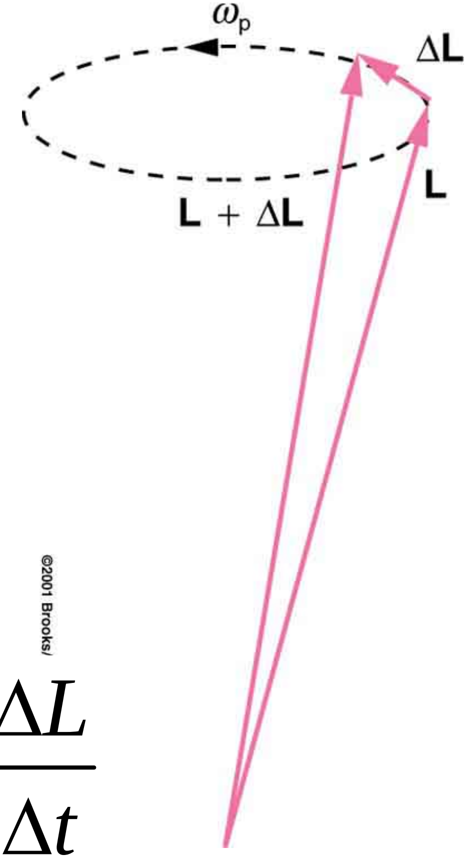
What Keeps a Spinning Gyroscope from Falling Down?



Precession Keeps it from Falling!

The “Couple” are equal and opposite forces so
$\Sigma F = 0$.
Since the normal force is through the axis of rotation P (the support) it produces no torque.
Only the weight of the CM produces a torque with lever arm **r** that changes the angular momentum, **L**.





$$\sum \tau = \frac{\Delta L}{\Delta t}$$

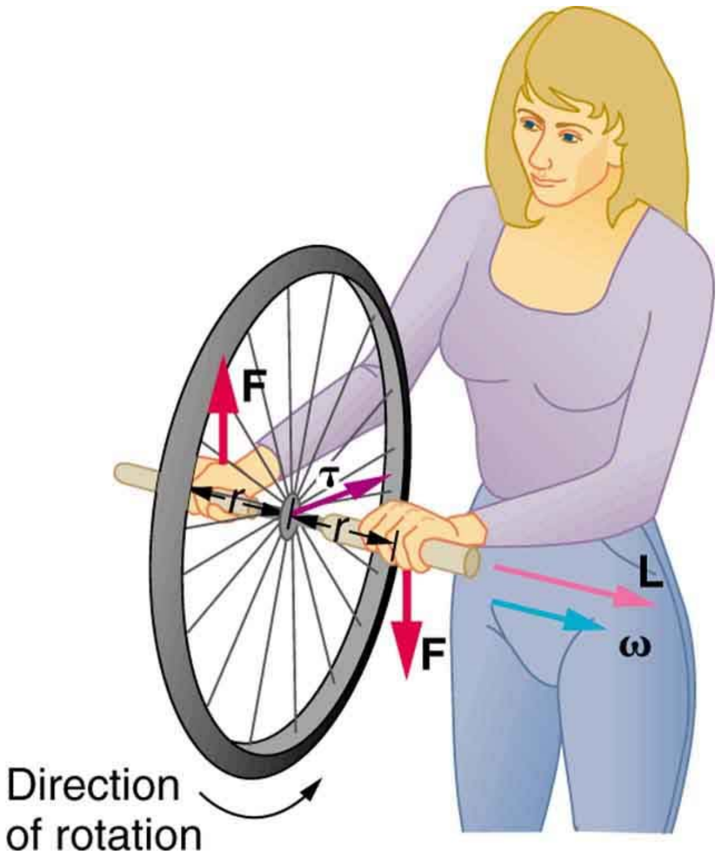
ΔL is in the direction of τ!
This causes the *Precession*!

Spinning Wheel

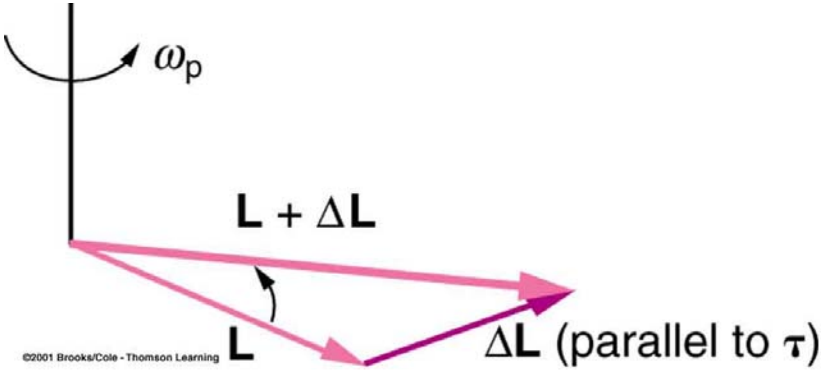
What happens if you rotate the wheel while sitting in a spin stool?

The stool will spin in the direction of the torque.

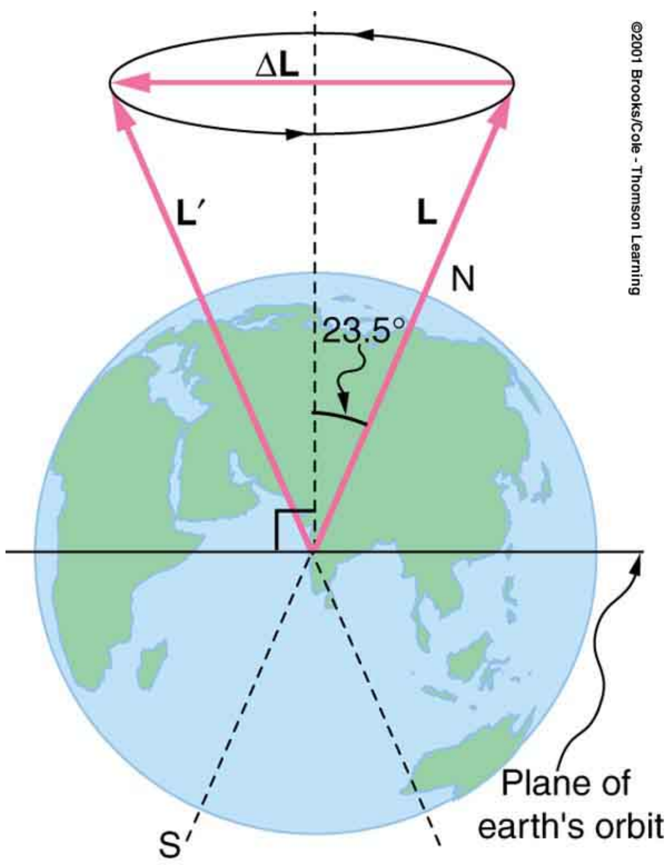
Why?



Conservation of Angular Momentum!

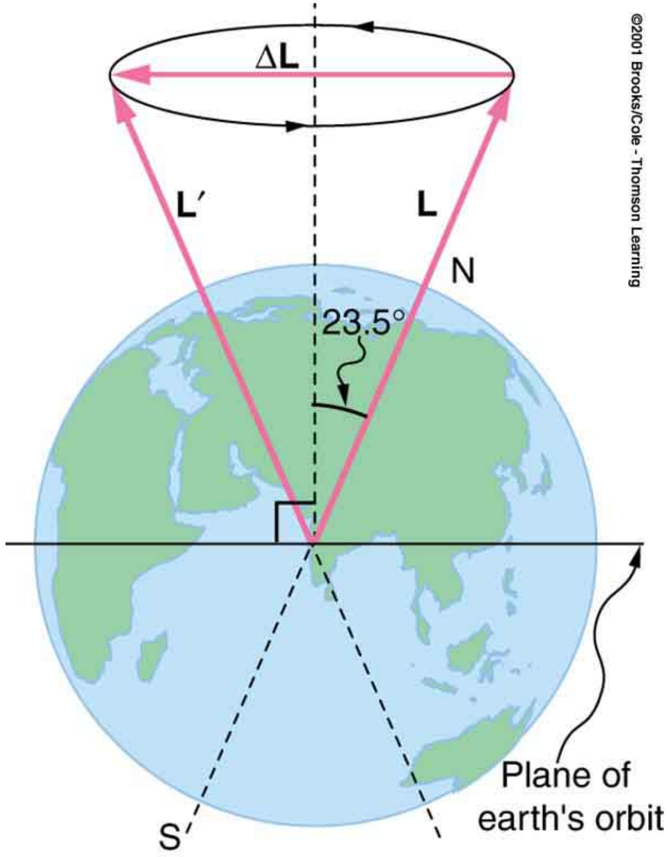


What is the best gyroscope on Earth?



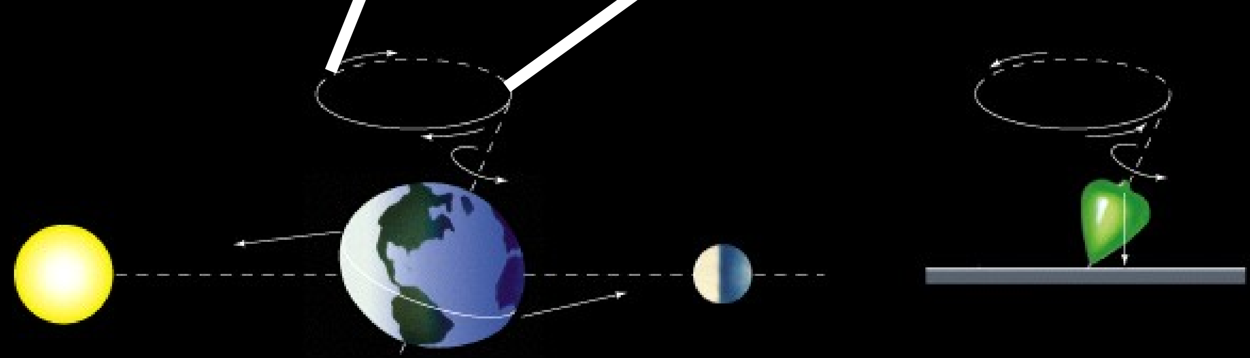
The Earth!

Earth’s Precession



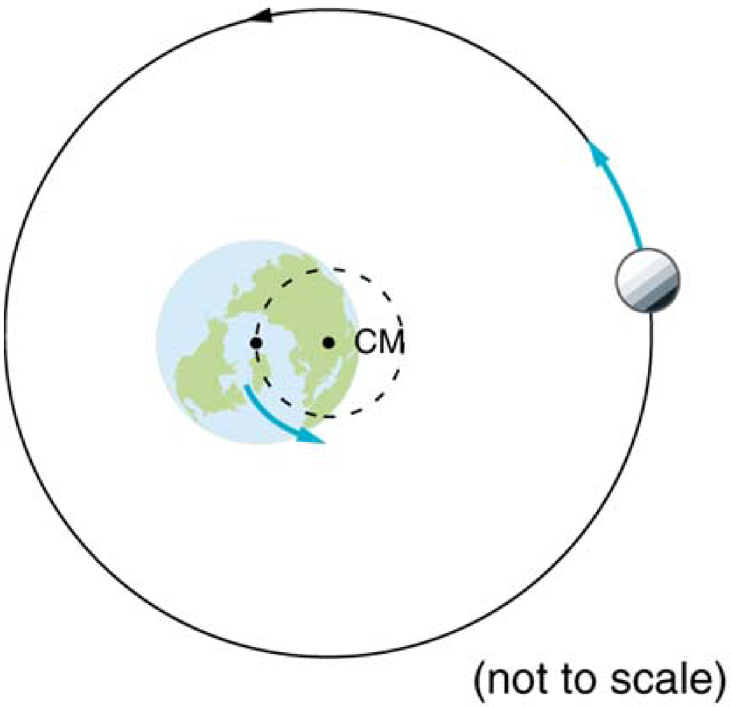
The Earth’s precession is due to the gravitational tidal forces of the Moon and Sun applying torque as they attempt to pull the Equatorial Bulge into the plane of Earth’s orbit.

The Earth's Precession causes the position of the North Pole to change over a period of 26,000 years.



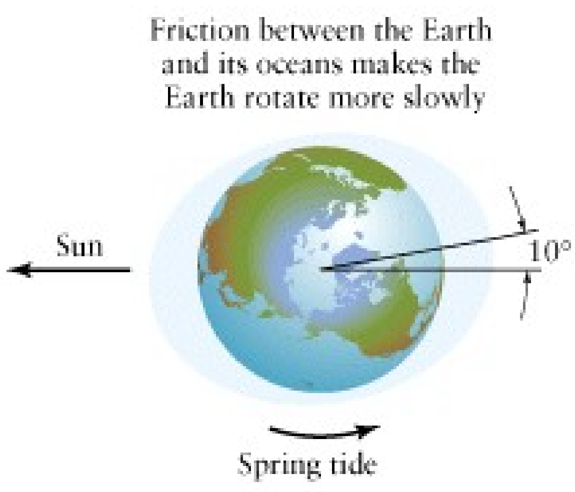
TOTAL Angular Momentum
Orbital and Spin Angular Momentum

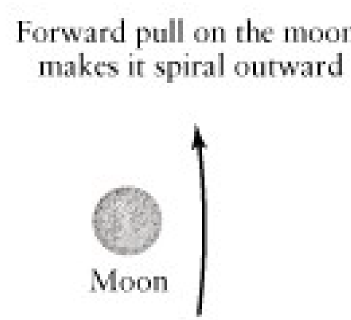
The TOTAL angular momentum includes both the angular momentum due to revolutions (orbits) and rotations (spin). It is the TOTAL angular momentum that is conserved.



Earth- Moon System:
Total Angular Momentum is Conserved!

- Earth Rotation Slowing due to friction of ocean on bottom
- .0023 s per century: 900 Million yrs ago, Earth day was 18 hrs!
- Decrease of Earth’s spin angular momentum, increases the orbital angular momentum of the Moon by increasing the distance, r, in order to keep L conserved!
- Earth is slowing down and Moon is moving further away!



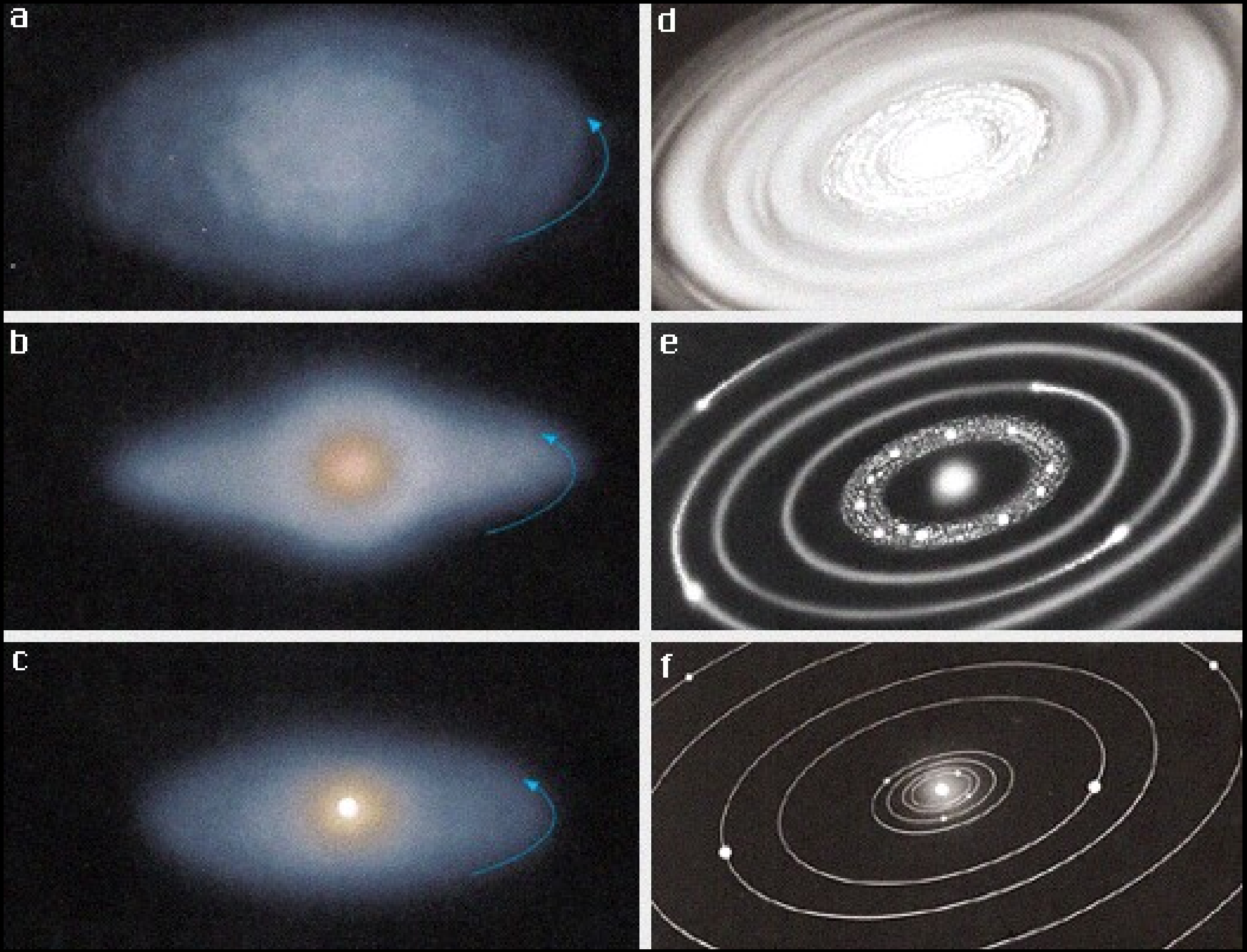


Bouncing laser beams off the Moon demonstrates that it slowly moving away from the Earth ~.25 cm/month

Angular Momentum as a Fundamental Quantity

- The concept of angular momentum is also valid on a submicroscopic scale
- Angular momentum has been used in the development of modern theories of atomic, molecular and nuclear physics
- In these systems, the angular momentum has been found to be a fundamental quantity
  - Fundamental here means that it is an intrinsic property of these objects
  - It is a part of their nature

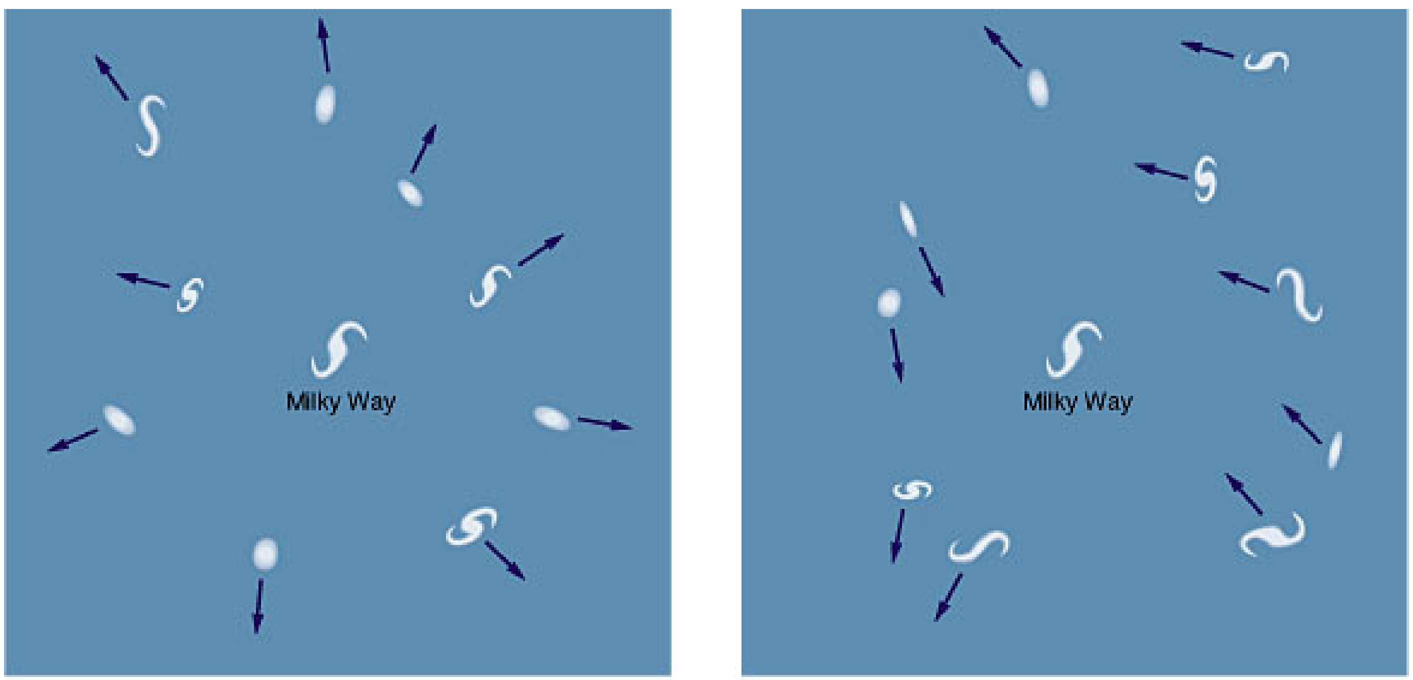
Conservation of Angular Momentum



Does the Universe have NET Angular Momentum?

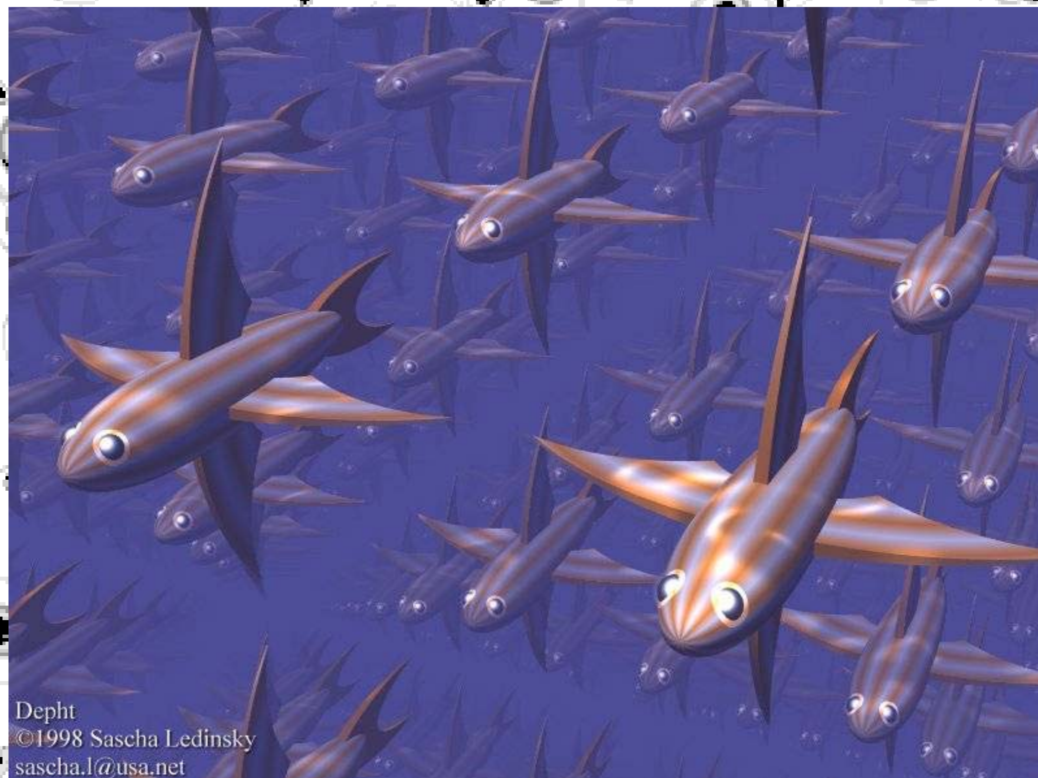
NO!

According the General Relativity,
The Universe is ISOTROPIC – it looks the same in every direction. Net Angular Momentum would define a direction.



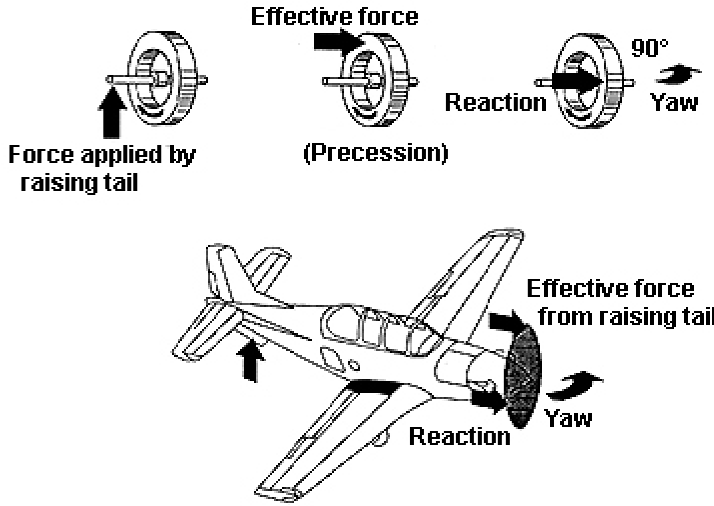
a b

Isotropic Universe



The Gyroscopic Effect

Once you spin a gyroscope, its axle wants to keep pointing in the same direction. If you mount the gyroscope in a set of **gimbals** so that it can continue pointing in the same direction, it will. This is the basis of the **gyro-compass** which is used in navigation systems.

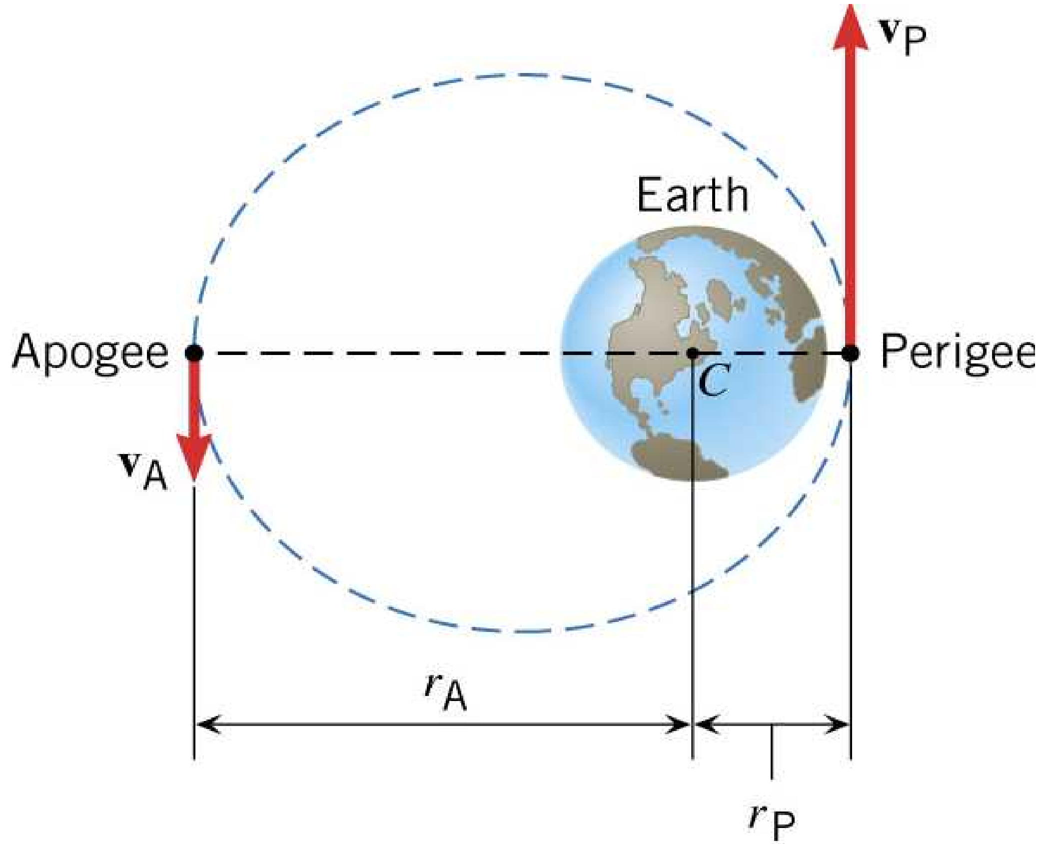


$$r_P = 8.37x10^6\ m$$

$$r_A = 25.1x10^6\ m$$

$$v_P = 8450 m/s$$

$$v_A = ?$$



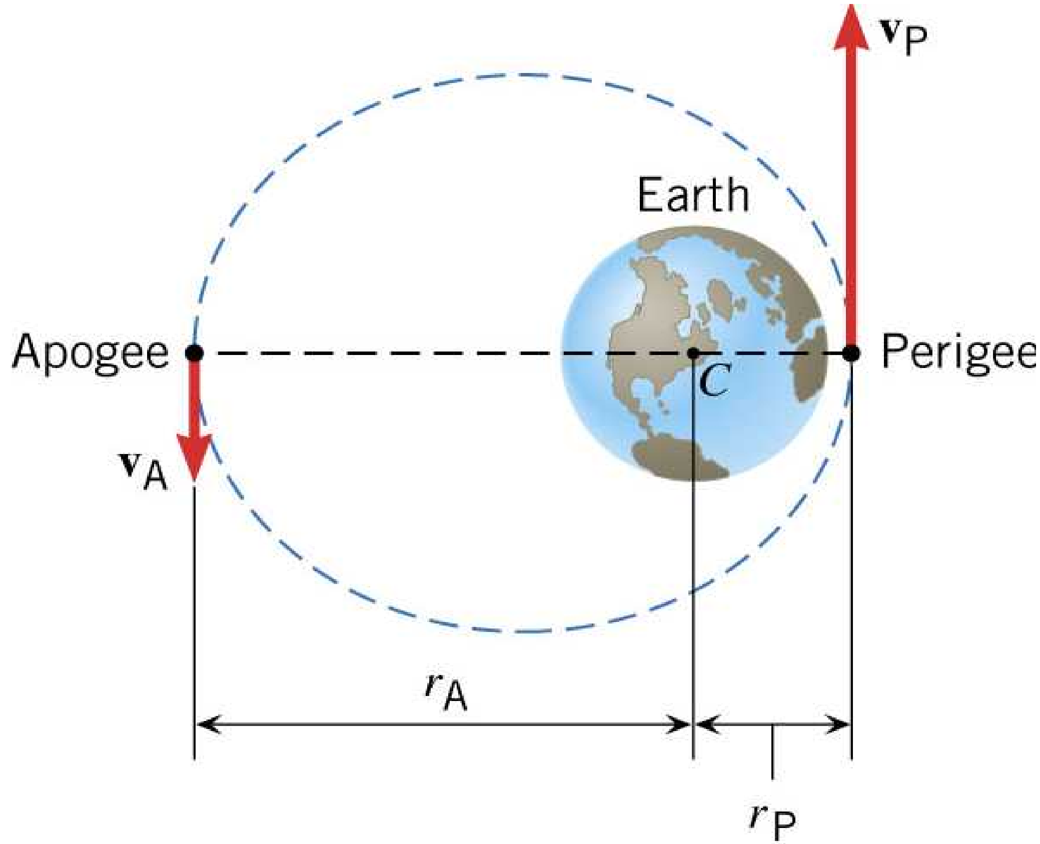
What is the velocity
of the satellite at apogee?

$$r_P = 8.37x10^6 m$$

$$r_A = 25.1x10^6 m$$

$$v_P = 8450 m/s$$

$$v_A = ?$$



Is the angular momentum of the system *CONSERVED?*

$$L_A = L_P$$

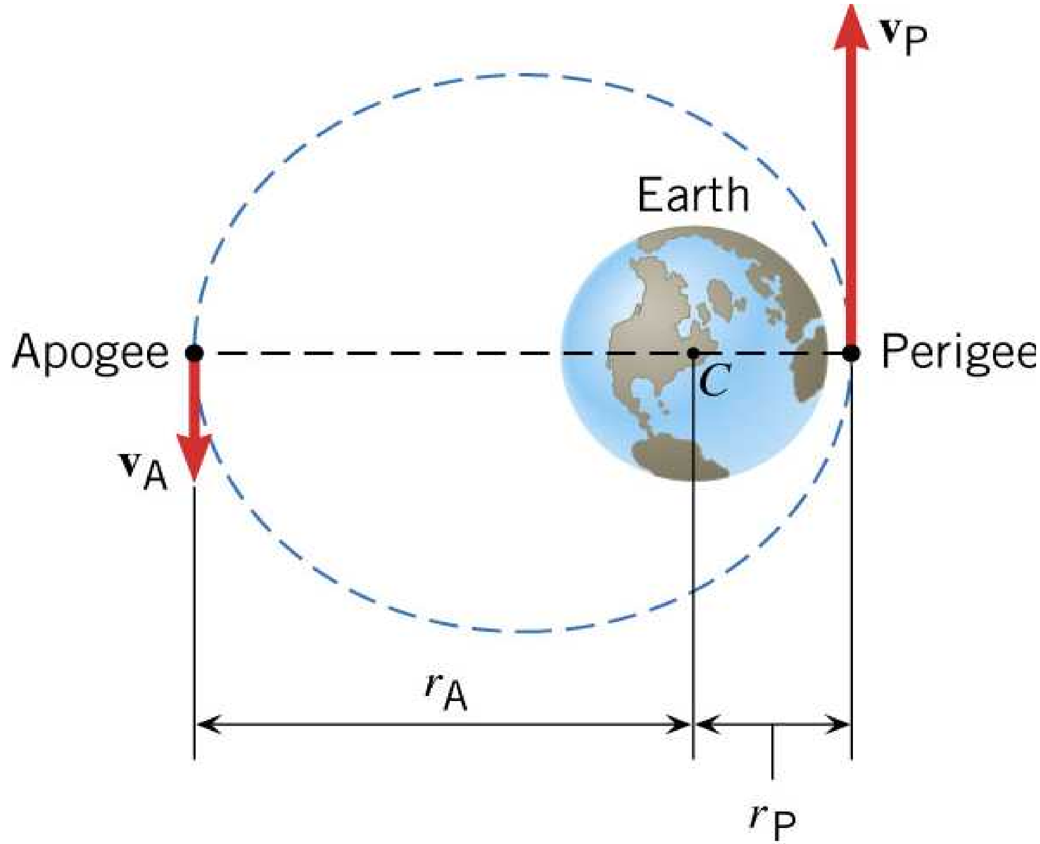
yes!

$$r_P = 8.37x10^6\ m$$

$$r_A = 25.1x10^6\ m$$

$$v_P = 8450 m/s$$

$$v_A = ?$$



$$mr_A v_A = mr_P v_P$$

$$v_A = \frac{r_P}{r_A} v_P = 2820 m/s$$

GRYO MOVIE!