A New Decentralized Fuzzy Model Reference Adaptive Controller for a Class of Large-scale Nonaffine Nonlinear Systems

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This paper proposes a new method to design a decentralized fuzzy adaptive controller (FAC) for a class of large-scale nonaffine nonlinear systems in which functions of the systems and interactions are unknown. Comparing to previous paper which mainly concentrates on affine large-scale system (LSS), the proposed method is on nonaffine nonlinear LSS. The stability of the closed-loop system is guaranteed based on Lyapunov theory. The proposed controller is robust against uncertainty and external disturbance. To show the effectiveness of the proposed method, an illustrative example is given. The simulation results are very promising.

Keywords: Lyapunov Stability, Robust Adaptive Control, Non-affine Nonlinear Systems, Fuzzy Systems, Large Scale System

1. Introduction

In the recent years, control design for large-scale systems (LSSs) and effort to extend it have attracted much attention. Research in a control of LSSs is motivated by many emerging applications that employ novel actuation devices for active control of industrial automation, cooperating robotic systems, power systems, and aerospace processes. Centralized control for the LSSs is usually impractical due to the requirement of a large amount of information exchanges between subsystems and the lack of computing capacity [14].

Nowadays, fuzzy adaptive controller (FAC) has attracted many researchers to develop appropriate controllers for nonlinear systems especially for LSSs because of the following reasons.

1. Due to its tunable structure, the performance of the FAC is superior to that of the fuzzy controller.
2. Instead of using adaptive controller, FAC can use the knowledge of the experts in the controller.

In the recent years, FAC has been fully studied as follows:

1. The Takagi–Sugeno (TS) fuzzy systems have been used to model nonlinear systems and then TS-based controllers have been designed with guaranteed stability [5, 6]. In ref. [9], modeling of affine nonlinear system is discussed and subsequently a stable TS-based controller is developed. Designing of the sliding mode FAC for a class of multivariable TS fuzzy systems are presented in ref. [2]. In refs. [7, 20], the nonaffine nonlinear function are first approximated by the TS fuzzy systems, and then stable TS fuzzy controller and observer are designed for the obtained model. In these papers, modeling and controller have been designed simply, but the systems must be linearizable around some operating points.
2. The linguistic fuzzy systems have been used to design controllers for nonlinear systems.
Ref. [12, 24, 26, 29] have considered linguistic fuzzy systems to design stable adaptive controller for affine systems based on feedback linearization and furthermore in [26] the zero dynamic has been considered to be stable. Stable FAC based on sliding mode is designed for affine systems in ref. [16]. Designing of the FAC for affine chaotic systems is presented in refs. [1, 23]. How to design stable FAC and linear observer for class of affine nonlinear systems is presented in refs. [8, 21, 25, 33]. Fuzzy adaptive sliding mode controller is presented for class of affine nonlinear time delay systems in refs. [3, 13, 32]. The output feedback FAC for class of affine nonlinear MIMO systems is suggested in ref. [30]. The main incompetency of these papers are those restricted conditions on their functions.

References [17, 18] are involved in stable FAC for class of nonaffine nonlinear systems. The inefficiency of these papers is bad performance of the controller when the controller has not been adjusted. Stable adaptive controller for class of linear LSS is proposed in refs. [10, 19, 22, 31]. Ref. [4] deals with designing FAC based on sliding mode for class of large-scale affine nonlinear systems. Ref. [34] presents decentralized sliding mode fuzzy adaptive tracking for a class of affine nonlinear systems in LSSs. Ref. [28] designed FAC for a class of affine nonlinear time delay systems. These papers have many restricted conditions.

Compared to previous paper, which mainly concentrates on affine systems LSS, the proposed method is a nonaffine nonlinear LSS. In this paper, we propose a new method to design a decentralized robust adaptive controller based on fuzzy systems for a class of large-scale nonaffine nonlinear systems with guaranteed stability. The capability of stabilizing closed-loop system, robustness against external disturbance and uncertainty and also convergence of the tracking error and boundedness of the estimation errors are advantages of the proposed controller.

The rest of the paper is organized as follows. Section 2 gives problem statement. General concept of the fuzzy systems is formulated in Section 3. The designing of FAC is proposed in Section 4. Section 5 shows simulation results of the proposed controller and Section 6 concludes the paper.

2. Problem Statement

Consider the following large-scale nonaffine nonlinear system.

\[
\begin{align*}
    \dot{x}_{ij} &= x_{ij,l+1} \quad \text{for } l = 1, 2, \ldots, n_i - 1 \\
    \dot{x}_{i,n_i} &= f_i(x_i, u_i) + m_i(x_1, x_2, \ldots, x_N) + d_i(t) \\
    y_i &= x_{i,1}
\end{align*}
\]

where \( x_i = [x_{i,1}, \ldots, x_{i,n_i}]^T \in \mathbb{R}^{n_i} \) is the state vector of the system which is assumed available for measurement, \( u_i \in \mathbb{R} \) is the control input, \( y_i \in \mathbb{R} \) is the system output, \( f_i(x_i, u_i) \) is an unknown smooth nonlinear function, \( m_i(x_1, x_2, \ldots, x_N) \) is an unknown nonlinear interconnection term, and \( d_i(t) \) is bounded disturbance.

The control objective is to design an adaptive fuzzy controller for system (1) such that the system output \( y_i(t) \) follows a desired trajectory \( y_d(t) \) while all signals in the closed-loop system remain bounded.

In this paper, we will make the following assumptions concerning the system (1) and the desired trajectory \( y_d(t) \).

**Assumption 1:** Without loss of generality, it is assumed that the nonzero function \( f_u(x_i, u_i) = \partial f_i(x_i, u_i) / \partial u_i \) satisfies the following condition:

\[
f_u(x_i, u_i) \geq f_{\text{min}} > 0 \quad \text{for all } (x_i, u_i) \in \mathbb{R}^{n_i} \times \mathbb{R}
\]

\[
    \frac{df_u(x_i, u_i)}{dt} \geq f_{\text{dm}}
\]

\( f_{\text{dm}} \in \mathbb{R} \) is known and constant.

**Assumption 2:** The desired trajectory \( y_d(t) \) and its time derivatives \( \dot{y}_d(j)(t) \), \( j = 1, 2, \ldots, n_i \), are all smooth and bounded.

**Assumption 3:** The interconnection term satisfies the following:

\[
    |m_i(x_1, x_2, \ldots, x_N)| \leq \zeta_i(||x_i||)
\]

where \( \zeta_i(||x_i||) \) is an unknown nonlinear function.

**Assumption 4:** The disturbance in the above equation is bounded by:

\[
    d_i(t) \leq d_{\text{max}}
\]

Define the tracking error vector as:

\[
    e_i = [e_{i,1}, e_{i,2}, \ldots, e_{i,n_i}]^T \in \mathbb{R}^{n_i}
\]

where

\[
    e_{i,1} = y_d - y_i
\]

Taking the \( n_i \)th derivative of both sides of the equation (6) we have
where \( \tanh \) is a small positive constant.

Using assumption (1), equation (11) and the signal \( v_i \), which is not explicitly dependent on the control input \( u_i \), the following inequality is satisfied:

\[
\frac{\partial (f_i(x_i, u_i) - v_i)}{\partial u_i} = \frac{\partial f_i(x_i, u_i)}{\partial u_i} > 0
\]

Invoking the implicit function theorem, it is obvious that the nonlinear algebraic equation \( f_i(x_i, u_i) - v_i = 0 \) is locally solvable for the input \( u_i \) for an arbitrary \( (x_i, v_i) \). Thus, there exists some ideal controller \( u_i^*(x_i, v_i) \) satisfying the following equality for a given \( (x_i, v_i) \in \mathbb{R}^n \times \mathbb{R}^m \):

\[
f_i(x_i, u_i^*) - v_i = 0
\]

As a result of the mean value theorem, there exists a constant \( \lambda \) in the range of \( 0 < \lambda < 1 \), such that the nonlinear function \( f_i(x_i, u_i) \) can be expressed around \( u_i^* \) as:

\[
f_i(x_i, u_i) = f_i(x_i, u_i^*) + (u_i - u_i^*)f_{iu_i} = f_i(x_i, u_i^*) + e_{iu_i}
\]

where \( f_{iu_i} = \partial f_i(x_i, u_i)/\partial u_i |_{u_i = u_i} \) and \( u_i = \lambda u_i + (1 - \lambda)u_i^* \).

Substituting equation (15) into the error equation (12) and using (14), we get

\[
\dot{e}_i = A_i e_i - b_i \left[ e_{iu_i} + m_i(x_1, x_2, \ldots, x_N) + d_i(t) + \beta \tanh( b_i^T P_i e_i/\varepsilon) + v_i^* \right]
\]

However, the implicit function theorem only guarantees the existence of the ideal controller \( u_i^*(x_i, v_i) \) for system (14), and does not recommend a technique for constructing solution even if the dynamics of the system are well known. In the following, a fuzzy system and classic controller will be used to obtain the unknown ideal controller.

### 3. Fuzzy Systems

Fig. 1 shows the basic configuration of the fuzzy systems considered in this paper. Here, we consider a multi-input, single-output fuzzy systems: \( x \in U \subset \mathbb{R}^n \rightarrow y \in V \subset \mathbb{R}^m \). Consider that a multi-output system can be separated into a group of single-output systems.

The fuzzifier performs a mapping from a crisp input vector \( x = [x_1, x_2, \ldots, x_n]^T \) to a fuzzy set, where the label of the fuzzy set are such as “small”, “medium”, “large”, etc.

The fuzzy rule base is consisted of a collection of fuzzy IF–THEN rules. Assume that there are \( M \) rules, and the \( i \)th rule is

\[
R_i(u): \text{if} (x_1 \text{ is } A_{i1} \ldots x_n \text{ is } A_{in}) \text{ then} (y \text{ is } B^i)_l = 1, 2, \ldots, M
\]
where $x = [x_1, x_2, \ldots, x_n]^T$ and $y$ are the crisp input and output of the fuzzy system, respectively. $A_j$ and $B_j$ are fuzzy membership function in $U_j$ and $V$, respectively.

The fuzzy inference performs a mapping from fuzzy sets in $U$ to fuzzy sets in $V$, based on the fuzzy IF–THEN rules in the fuzzy rule base.

The defuzzifier maps fuzzy sets in $V$ to a crisp value in $V$. The configuration of Fig. 1 represents a general framework of fuzzy systems, because many different choices are allowed for each block in Fig. 1, and various combinations of these choices will construct different fuzzy systems [26]. Here, we use the sum–product inference and the center–average defuzzifier. Therefore, the fuzzy system output can be expressed as

$$y(x) = \frac{\sum_{i=1}^{M} y' \prod_{i=1}^{n} \mu_{A_i}(x_i)}{\sum_{i=1}^{M} \prod_{i=1}^{n} \mu_{A_i}(x_i)}$$

where $\mu_{A_i}(x_i)$ is the membership degree of the input $x_i$ to fuzzy set $A_i$ and $y'$ is the point at which the membership function of fuzzy set $B_j$ achieves its maximum value.

The fuzzy systems in the form of (18) are proven in [27] to be a universal approximator if their parameters are properly chosen.

**Theorem 1 [26]:** Suppose $f(x)$ is a continuous function on a compact set $U$. Then, for any $\varepsilon > 0$, there exists a fuzzy system like (18) satisfying:

$$\sup_{x \in U} |f(x) - y(x)| \leq \varepsilon$$

The output given by (18) can be rewritten in the following compact form:

$$y(x) = w(x)^T \theta$$

where $\theta = [y^1 y^2 \ldots y^M]$ is a vector grouping all consequent parameters, and $w(x) = [w_1(x) w_2(x) \ldots w_M(x)]^T$ is a set of fuzzy basis functions defined as:

$$w_j(x) = \frac{\prod_{i=1}^{n} \mu_{A_i}(x_i)}{\sum_{i=1}^{M} \prod_{i=1}^{n} \mu_{A_i}(x_i)}$$

The fuzzy system (18) is assumed to be well defined so that $\sum_{i=1}^{M} \mu_{A_i}(x_i) \neq 0$ for all $x \in U$.

### 4. FAC Design

In Section 2, it has been shown that there exists an ideal control for achieving control objectives. In this section, we show how to develop a fuzzy system to adaptively approximate the unknown ideal controller.

The ideal controller can be represented as:

$$u_i^* = f_i(z) + u_{pid} + \varepsilon_{iu}$$

where $f_i(z) = \theta_i^0 w_i(z)$, and $\theta_i^0$ and $w_i(z)$ are consequent parameters and a set of fuzzy basis functions, respectively. $\varepsilon_{iu}$ is an approximation error that satisfies $|\varepsilon_{iu}| \leq \varepsilon_{max}$ and $\varepsilon_{max} > 0$. The $u_{pid}$ is the primary controller that developed properly to initially control the underlying system and parameters $\theta_i^0$ are determined through the following optimization.

$$\theta_i^* = \arg \min_{\theta_i^0} [\sup |\theta_i^0 w_i(z) - f_i(z)|]$$

Denote the estimate of $\theta_i^0$ as $\hat{\theta}_i^0$ and $u_{irob}$ as a robust controller to compensate approximation error, uncertainties, disturbance and interconnection term to rewrite the controller given in (22) as:

$$u_i = \theta_i^T w_i(z) + u_{pid} + u_{irob}$$
In which \( u_{\text{rob}} \) is defined below.

\[
u_{\text{rob}} = \frac{|b_i^T P_i e_i|}{f_{\text{min}} b_i^T P_i e_i} (\theta_{i2}^T w_{i2}(x_i) + f_{\text{min}} u_{\text{icom}} + f_{\text{min}} u_r + \dot{v}_i)
\]

(25)

In the above, \( \theta_{i2}^T w_{i2}(x_i) \) approximates the ideal controller, \( \theta_{i1}^T w_{i1}(x_i) \) tries to compensate the interconnection term, \( u_{\text{icom}} \) compensates for approximation errors and uncertainties, \( u_r \) is designed to compensate for bounded external disturbances, and \( \dot{v}_i \) is estimation of \( v_i \). Define error vector \( \dot{\theta}_{i1} = \theta_{i1} - \theta_{i1}^T \) and use (24) and (25) to rewrite the error equation (16) as:

\[
\dot{\epsilon}_i = A_i \epsilon_i - b_i \{ \dot{\theta}_{i1}^T w_{i1}(z_i) + u_{\text{rob}} - \epsilon_{\text{ui}} \} f_{\text{ui}} + m_i(x_1, x_2, \ldots, x_N) + d_i(t) + \beta \tanh(b_i^T P_i e_i/\epsilon) + \dot{v}_i
\]

(26)

Consider the following update laws.

\[
\dot{\theta}_{i1} = \Gamma_1 b_i^T P_i e_i w_{i1}(z_i)
\]

\[
\dot{\theta}_{i2} = \Gamma_2 b_i^T P_i e_i w_{i2}(x_i)
\]

\[
\dot{u}_r = \gamma u_i b_i^T P_i e_i
\]

\[
\dot{u}_{\text{icom}} = \gamma_{\text{icom}} b_i^T P_i e_i
\]

\[
\dot{v}_i = \gamma_{v_i} b_i^T P_i e_i
\]

(27)

where \( \Gamma_1 = \Gamma_1^T > 0, \Gamma_2 = \Gamma_2^T > 0, \gamma u_i > 0, \gamma_{\text{icom}} > 0, \gamma_{v_i} > 0 \) are constant parameters.

In following equation, \( \lambda_{\text{max}(\cdot)} \) and \( \text{svd}_{\text{max}(\cdot)} \) are maximum eigenvalue and maximum singular value decomposition, respectively.

**Lemma 1:** The following inequality holds if \( \lambda_{\text{max}}(Q_i) \geq -f_{\text{dm}}/f_{\text{min}} \lambda_{\text{max}}(P_i) \).

\[
\frac{1}{f_{\text{ui}}} e_i^T Q_i e_i + \frac{1}{f_{\text{ui}}} e_i^T P_i e_i + \beta \frac{1}{f_{\text{ui}}} |b_i^T P_i e_i| \geq 0
\]

(28)

**Proof:** From assumption (1) and \( \beta > 0 \), we can have the following inequality has been satisfied.

\[
\beta \frac{1}{f_{\text{ui}}} |b_i^T P_i e_i| \geq 0
\]

(29)

From assumption (1) and the above lemma, it is obvious that

\[
(\lambda_{\text{min}}(Q_i)f_{\text{min}} + \lambda_{\text{min}}(P_i)f_{\text{dm}}) \geq 0
\]

(30)

This in turn leads to the following inequality.

\[
\frac{1}{f_{\text{ui}}} \left( \lambda_{\text{min}}(Q_i)f_{\text{min}} + \lambda_{\text{min}}(P_i)f_{\text{dm}} \right) ||e_i||^2 \geq 0
\]

(31)

After some algebraic manipulations, the following inequality is obtained.

\[
\frac{1}{f_{\text{ui}}} e_i^T Q_i e_i + \frac{1}{f_{\text{ui}}} e_i^T P_i e_i + \beta \frac{1}{f_{\text{ui}}} |b_i^T P_i e_i| \geq 0
\]

(32)

Use (29) and (32) to have the following which completes the proof.

\[
\frac{1}{f_{\text{ui}}} e_i^T Q_i e_i + \frac{1}{f_{\text{ui}}} e_i^T P_i e_i + \beta \frac{1}{f_{\text{ui}}} |b_i^T P_i e_i| \geq 0
\]

Q.E.D

**Lemma 2:** Based on lemma 1 and equation (10), the following inequality holds.

\[
svd_{\text{max}}(A_i) \leq \frac{f_{\text{dm}}}{2f_{\text{min}} \lambda_{\text{max}}(P_i)}
\]

(34)

**Proof:** Using equation (10) and after some algebraic manipulations, the following inequality is obtained.

\[
\|Q_i\| \leq ||A_i^T P_i|| + ||P_i A_i|| = 2||P_i A_i||
\]

(35)

Using the above equation, we get

\[
\|Q_i\| \leq 2||P_i|| ||A_i|| = 2\lambda_{\text{max}}(P_i) \text{svd}_{\text{max}}(A_i)
\]

(36)

Use (30) and (36) to have the following which completes the proof.

\[
svd_{\text{max}}(A_i) \leq \frac{f_{\text{dm}}}{2f_{\text{min}} \lambda_{\text{max}}(P_i)}
\]

Q.E.D

**Theorem 2:** Consider the error dynamical system given in (26) for the LSS (1) satisfying assumption (1), interconnection term satisfying assumption (3), the external disturbances satisfying assumption (4), and a desired trajectory satisfying assumption (2). Then the controller structure given in (24), (25) with adaptation laws (27) makes the tracking error converge asymptotically to a neighborhood of origin and all signals in the closed-loop system be bounded.

**Proof:** To prove convergence of the tracking error and boundedness of parameters error, we must apply the tracking error and the parameters error in Lyapunov function. Thus, the parameters such as \( e_i, \dot{\theta}_{i1}, \dot{\theta}_{i2}, \dot{u}_r, \dot{u}_{\text{icom}}, \) and \( \dot{v}_i \) are applied in Lyapunov function.

Consider the following Lyapunov function.
Decentralized Fuzzy MRAC

\[ V = \sum_{i=1}^{N} \frac{1}{2} \left( \frac{1}{f_{u,i}} e_i^T P_i e_i + \theta_{11}^T \Gamma_1^{-1} \dot{\theta}_{11} + \theta_{12}^T \Gamma_2^{-1} \dot{\theta}_{12} \right. \]
\[ + \frac{\tilde{u}_{ir}^2}{\gamma_{u,r}^2} + \frac{\tilde{u}_{icom}^2}{\gamma_{u,icom}^2} + \left. \frac{\tilde{v}_i^2}{\gamma_{v_i}^2} \right) \]

(38)

where \( \dot{\theta}_{11} = \theta_{11} - \theta_{11}^*, \dot{\theta}_{12} = \theta_{12} - \theta_{12}^*, \tilde{u}_{ir} = u_{ir} - d_{max}/f_{min}, \tilde{u}_{icom} = u_{icom} - \varepsilon_{max} - \tilde{\delta}_{max}/f_{min}, \) and \( \gamma'_{v_i} = \gamma_{v_i} - |v_i|. \)

The time derivative of the Lyapunov function becomes:

\[ \dot{V} = \sum_{i=1}^{N} \frac{1}{2} \left( \frac{1}{f_{u,i}} e_i^T (A_i^T P_i + P_i A_i) e_i + \frac{\tilde{u}_{ir} \tilde{u}_{ir}}{\gamma_{u,r}} + \frac{\tilde{u}_{icom} \tilde{u}_{icom}}{\gamma_{u,icom}} + \frac{\tilde{v}_i \tilde{v}_i}{\gamma_{v_i}} \right) \]

(39)

Use (26), to rewrite above equation as:

\[ \dot{V} = \sum_{i=1}^{N} \frac{1}{2} \left( \frac{1}{f_{u,i}} e_i^T (A_i^T P_i + P_i A_i) e_i + \frac{\beta}{f_{u,i}} b_i^T P_i e_i \tanh(b_i^T P_i e_i/\varepsilon) + \right. \]
\[ + \frac{1}{f_{u,i}} \left( b_i^T P_i e_i (v'_i + (\theta_{11}^T w_{i1}(z) + u_{rob} - \varepsilon_{in}) f_{u,i} ight. \]
\[ + m_i(x_1, x_2, \ldots, x_N) + d_i(t)) + \right. \]
\[ + \theta_{11}^T \Gamma_1^{-1} \dot{\theta}_{11} + \theta_{12}^T \Gamma_2^{-1} \dot{\theta}_{12} + \tilde{u}_{ir} \tilde{u}_{ir} \gamma_{u,r} \]
\[ + \tilde{u}_{icom} \tilde{u}_{icom} \gamma_{u,icom} + \tilde{v}_i \tilde{v}_i \gamma_{v_i} \]

(40)

Use \( b_i^T P_i e_i \tanh(b_i^T P_i e_i/\varepsilon) = |b_i^T P_i e_i| \) and (10), to rewrite (40) as follows:

\[ \dot{V} = \sum_{i=1}^{N} \frac{1}{2} \left( \frac{1}{f_{u,i}} e_i^T (A_i^T P_i + P_i A_i) e_i - \beta \frac{\tilde{u}_{ir} \tilde{u}_{ir}}{f_{u,i}} \right) \]
\[ + \frac{1}{f_{u,i}} \left( b_i^T P_i e_i (v'_i + (\theta_{11}^T w_{i1}(z) + u_{rob} - \varepsilon_{in}) f_{u,i} \right. \]
\[ + m_i(x_1, x_2, \ldots, x_N) + d_i(t)) + \right. \]
\[ + \theta_{11}^T \Gamma_1^{-1} \dot{\theta}_{11} + \theta_{12}^T \Gamma_2^{-1} \dot{\theta}_{12} + \tilde{u}_{ir} \tilde{u}_{ir} \gamma_{u,r} \gamma_{u,icom} \]
\[ + \tilde{u}_{icom} \tilde{u}_{icom} \gamma_{u,icom} + \tilde{v}_i \tilde{v}_i \gamma_{v_i} \]

(41)

Function \( \zeta(||x_i||) \) is smooth and it can be approximated with the fuzzy system as the following equation.

\[ \zeta(||x_i||) = \theta_{21}^T w_{i2}(x_i) + \delta_i \]

(42)

where \( \delta_i \) is approximation error and satisfying \( |\delta_i| \leq \delta_{max} \). Using assumption (1) yields \( 1/f_{u,i} \leq 1/f_{min} \) and by assumptions (3), (4) and equations (42), to rewrite (41) as follow.

\[ \dot{V} \leq \sum_{i=1}^{N} \frac{1}{2} \left( \frac{1}{f_{u,i}} e_i^T (A_i^T P_i + P_i A_i) e_i - \beta \frac{\tilde{u}_{ir} \tilde{u}_{ir}}{f_{u,i}} \right) \]
\[ + \frac{b_i^T P_i e_i}{f_{min}} \left| v'_i - |v'_i| - b_i^T P_i e_i \theta_{11}^T w_{i1}(z) - \right. \]
\[ \left. \frac{b_i^T P_i e_i}{f_{min}} \theta_{21}^T w_{i2}(x_i) + \frac{b_i^T P_i e_i}{f_{min}} \delta_{max} \right) \]
\[ + \frac{b_i^T P_i e_i}{f_{min}} d_{max} + \frac{\theta_{11}^T \Gamma_1^{-1} \dot{\theta}_{11} + \theta_{12}^T \Gamma_2^{-1} \dot{\theta}_{12}}{\gamma_{u,r}} \]
\[ + \frac{\tilde{u}_{ir} \tilde{u}_{ir}}{\gamma_{u,r}} + \frac{\tilde{u}_{icom} \tilde{u}_{icom}}{\gamma_{u,icom}} + \frac{\tilde{v}_i \tilde{v}_i}{\gamma_{v_i}} \]

(43)

The equation (43) can be rewritten as below.

\[ \dot{V} \leq \sum_{i=1}^{N} \frac{1}{2} \left( \frac{1}{f_{u,i}} e_i^T (A_i^T P_i + P_i A_i) e_i - \beta \frac{\tilde{u}_{ir} \tilde{u}_{ir}}{f_{u,i}} \right) \]
\[ + \frac{b_i^T P_i e_i}{f_{min}} \left| v'_i - \left| v'_i \right| - b_i^T P_i e_i \theta_{11}^T w_{i1}(z) \right. \]
\[ \left. - \frac{b_i^T P_i e_i}{f_{min}} \theta_{21}^T w_{i2}(x) + \frac{b_i^T P_i e_i}{f_{min}} \delta_{max} \right) \]
\[ + \frac{b_i^T P_i e_i}{f_{min}} d_{max} + \frac{\theta_{11}^T \Gamma_1^{-1} \dot{\theta}_{11} + \theta_{12}^T \Gamma_2^{-1} \dot{\theta}_{12}}{\gamma_{u,r}} \]
\[ + \frac{\tilde{u}_{ir} \tilde{u}_{ir}}{\gamma_{u,r}} + \frac{\tilde{u}_{icom} \tilde{u}_{icom}}{\gamma_{u,icom}} + \frac{\tilde{v}_i \tilde{v}_i}{\gamma_{v_i}} \]

(44)

To derive updates law, the above equation has been rewritten as below.
\[ \dot{V} \leq \sum_{i=1}^{N} \frac{1}{2} \left( -\frac{1}{f_{si}} e_i^T Q e_i - \hat{f}_{si} e_i^T P e_i - \beta \frac{\dot{V}}{f_{si}} \right) + \]
\[ - \dot{V} \left( \frac{b_i^T P e_i}{f_{min}} - \frac{\dot{V}}{\gamma_{\dot{V}}} \right) - \dot{\theta}_{1i}^2 \left( \frac{b_i^T P e_i w_{1i}(z) - \Gamma_{-1} \dot{\theta}_{1i}}{\gamma_{\dot{\theta}_{1i}}} \right) = 0 \]
\[ - \dot{\theta}_{2i}^2 \left( \frac{b_i^T P e_i w_{2i}(x) - \Gamma_{-1} \dot{\theta}_{1i}}{\gamma_{\dot{\theta}_{1i}}} \right) = 0 \]
\[ - \ddot{u}_{icom} \left( \frac{b_i^T P e_i}{\gamma_{\dot{u}_{icom}}} \right) - \ddot{u}_{ir} \left( \frac{b_i^T P e_i}{\gamma_{\dot{u}_{ir}}} \right) = 0 \]

Using (27), the above inequality rewrites as:
\[ \dot{V} \leq \sum_{i=1}^{N} \frac{1}{2} \left( -\frac{1}{f_{si}} e_i^T Q e_i + \hat{f}_{si} e_i^T P e_i + \beta \frac{\dot{V}}{f_{si}} \right) \]

Remark 3: To guarantee the boundedness of the parameters in the presence of the approximation error, which is unavoidable, the proposed adaptive laws (27) is modified by introducing a \( \sigma \)-modification term as follows:
\[ \dot{\theta}_{1i} = \Gamma_1 b_i^T P e_i w_{1i}(z_i) - \sigma \Gamma_1 \dot{\theta}_{1i} \]
\[ \dot{\theta}_{2i} = \Gamma_2 b_i^T P e_i w_{2i}(x_i) - \sigma \Gamma_2 \dot{\theta}_{2i} \]
\[ \dot{u}_{ir} = \gamma_{u_r} \left( \frac{b_i^T P e_i}{\gamma_{\dot{u}_{ir}}} \right) - \sigma \gamma_{u_r} u_{ir} \]
\[ \dot{u}_{icom} = \gamma_{u_{icom}} \left( \frac{b_i^T P e_i}{\gamma_{\dot{u}_{icom}}} \right) - \sigma \gamma_{u_{icom}} u_{icom} \]
\[ \dot{v}_i = \gamma_{v_i} \left( \frac{b_i^T P e_i}{\gamma_{\dot{v}_i}} \right) - \sigma \gamma_{v_i} v_i \]

5. Simulation Results

In this section, we apply the proposed decentralized fuzzy model reference adaptive controller to a two-inverted pendulum problem [14] in which the pendulums are connected by a spring as shown in Fig. 2. Each pendulum may be positioned by a torque input \( u_i \) applied by a servomotor and its base. It is assumed that the angular position of pendulum and its angular rate are available and can be used as the controller inputs. The pendulum dynamics are described by the following nonlinear equations.

\[
\begin{align*}
\dot{x}_{11} &= x_{12} \\
\dot{x}_{12} &= \left( \frac{m_1 g r}{j_1} - \frac{k r^2}{4 j_1} \right) \sin(x_{11}) + \frac{k r}{2 j_1} (l - b) + \frac{\alpha}{j_1} \text{sat}(u_1) + \frac{k r^2}{j_1} \sin(x_{21}) \\
y_1 &= x_{11}
\end{align*}
\]

\[
\begin{align*}
\dot{x}_{21} &= x_{22} \\
\dot{x}_{22} &= \left( \frac{m_2 g r}{j_2} - \frac{k r^2}{4 j_2} \right) \sin(x_{21}) + \frac{k r}{2 j_2} (l - b) + \frac{\alpha}{j_2} \text{sat}(u_2) + \frac{k r^2}{j_2} \sin(x_{12}) \\
y_2 &= x_{21}
\end{align*}
\]

where \( y_1, y_2 \) are the angular displacements of the pendulums from vertical position. \( m_1 = 2 \text{kg}, m_2 = 2.5 \text{kg} \) are the pendulum end masses, \( j_1 = 0.5 \text{kg} \cdot \text{m}^2, j_2 = 0.62 \text{kg} \cdot \text{m}^2 \) are the moment of inertia, \( k = 100 \text{N/m} \) is spring constant, \( r = 0.5 \text{m} \) is the height of the pendulum, \( g = 9.81 \text{m/s}^2 \) shows the gravitational acceleration, \( l = 0.5 \text{m} \) is the natural length of spring, \( \alpha_1, \alpha_2 = 25 \) are the control input gains and \( b = 0.4 \text{m} \) presents distance between the pendulum hinges.

We consider the desired value of the outputs be zero (\( y_{id} = 0 \) for \( i = 1, 2 \)). As discussed in Section 4 the following primary PID controller are obtained after some trials and errors.

\[ u_{pid} = 40(e_i + 1/4 \int_0^t e_i d\tau + \dot{e}_i) \]
Figs. 3 and 4 present the outputs of the system where only the controller defined in equation (49) is applied to the system.

Obviously the primary controller by itself is not admissible. Now we applied the proposed controller defined in (24), (25). Initially the PID controller keeps the states of system $x_{i1}, x_{i2}$ in the range of $[-1,1], [-5,5]$. Let $x_i = [x_{i1}, x_{i2}]^T$, $z_i = [x_{i1}, x_{i2}, v_i]^T$ and $v_i$ are defined over $[-45,45]$. For each fuzzy system input, we define 6 membership functions over the defined sets. Consider that all of the membership functions are defined by the Gaussian function

$$

\mu_j(\chi) = \exp \left( -\frac{(\chi - c)^2}{2\delta^2} \right),

$$

where $c$ is center of the membership function and $\delta$ is its variance. We assume

---

Fig. 2. Two inverted pendulum connected by a spring.

Fig. 3. Performance of the PID controller in first subsystem.

Fig. 4. Performance of the PID controller in second subsystem.
that the initial value of $\theta_1(0), \theta_2(0), u_c(0), u_{com}(0)$, and $\varphi_i(0)$ be zero. Furthermore, it has been assumed that $f_{\min} = 1$, $\Gamma_1 = 10$, $\Gamma_2 = 10$, $\gamma_{u_{com}} = 5$, $\gamma_{u_r} = 5$, $\gamma_{\varphi} = 5$. In equation (47) and remark (1), we assume that $\sigma = 0.01$, $\varepsilon = 0.01$. The parameters $f_{\min}, f_{\min}$ and the vector $k_i = [k_{i,1}, k_{i,2}, \ldots, k_{i,n_i}]^T$ has been chosen so that the lemma 2 holds.

As shown in Figs. 3–6, it is obvious that the performance of the proposed controller is promising. Figs. 7 and 8 shown the total input of each subsystem.

![Fig. 5. Performance of the proposed controller in first subsystem.](image)

![Fig. 6. Performance of the proposed controller in second subsystem.](image)

![Fig. 7. Control input $u_1$.](image)
6. Conclusion

In this paper, we propose a new decentralized fuzzy model reference adaptive output tracking controller for a class of large-scale nonaffine nonlinear systems. Fuzzy systems used to approximate the part of controller let us to use the knowledge of the experts in the controller design procedure. Using the Lyapunov’s stability analysis, it has been shown that the derived adaptation laws guaranty the stability of closed-loop system, and asymptotic convergence of the tracking error to a neighborhood of zero. Robustness against external disturbances and approximation errors, relaxing the conditions and using knowledge of experts are the merits of the proposed controller.

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Fig. 8. Control input u2.