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Research Article

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A new approach towards characterization of semicompactness of fuzzy topological space and its crisp subsets

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Abstract

It is widely accepted that one of the most satisfactory generalization of the concept of *compactness* to fuzzy topological spaces is $\alpha-compactness$, first introduced by Gantner et al. in 1978, followed by further investigations by many others. Chakraborty et al. introduced fuzzy *semicompact* set and investigated and characterized fuzzy *semicompact* spaces in terms of fuzzy *nets* and fuzzy $prefilterbases$ in 2005. In this paper, we propose to introduce a new approach to characterize the notion of $\alpha-semicompactness$ in terms of ordinary *nets* and $filters$. This paper deals also with the concept of $\alpha-semilimit$ points of crisp subsets of a fuzzy topological space X and the concept of $\alpha-semiclosed$ sets in X and these concepts are used to define and characterize $\alpha-semicompact$ crisp subsets of X.

Keywords: Fuzzy topological spaces, $\alpha-$semicompactness, $\alpha-$semicluster point of nets and filters, $\alpha-$adherent point, $\alpha-$semiadherent point, $\alpha-$limit point, $\alpha-$semilimit point, $\alpha-$closure, $\alpha-$semiclosure.

1 Introduction

The idea of fuzzy set was originated from the classical paper of L.A.Zadeh [18] in 1965. Subsequently many researchers have worked on various basic concepts from general topology using fuzzy sets and developed the theory of fuzzy topology. In recent years fuzzy topology has been found to be very useful in solving many practical problems. The notions of the sets and functions in fuzzy topological spaces are used extensively in many engineering problems, computational topology for geometric design, computer-aided geometric design, engineering design research and mathematical sciences. El-Naschie [8, 9] has shown that the notion of fuzzy topology may be applicable to quantum physics in connection with string theory and e^{∞} theory and fuzzy topology may be used to provide information about the elementary particles content of the standard model of high energy physics. Shihong Du. et al. [5] are currently working to fuzzify the 9-intersection Egenhofer model [6, 7] for describing topological relations in Geographic Information System(GIS) query. X.Tang [15] has used a slightly changed version of Chang's [2] fuzzy topological spaces to model spatial objects for GIS database and Structured Query Language(SQL) for GIS. In-depth analytical study of fuzzy set theory is still required to provide more and more information about any mathematical system and its subsystems to the modern scientific research in the arena of mathematical sciences and physical sciences. The concept of compactness is one of the central and important concepts of paramount interest to topologist and it seems to be the most celebrated type among all the covering properties. Its enormous use and potentiality for numerous applications induced mathematicians to generalize

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the concept in fuzzy setting. It was Chang [2] who first introduced the concept of compactness in fuzzy topological space. This attempt has been followed by Goguen [12] who after pointing out a drawback in the definition of Chang, proved Alexander's subbase theorem, but could establish Tychonoff theorem only for finite products. Thereafter, Wong [17], Weiss [16] and Lowen [13] treated compactness for fuzzy topological spaces in different ways. But it is seen that all those approaches contain many limitations. In 1978, Gantner, Steinlage and Warren [10] did a splendid work towards the process of fuzzifying the concept of *compactness* and initiated a new definition of fuzzy compactness, termed as $\alpha-compactness$, by which they could assign degrees to compactness and with this definition they finally generalized Tychonoff theorem and even $1-$point compactification. It seems from the subsequent investigations that the theory of $\alpha-compactness$ by Gantner et al. [10] is the most satisfactory one. This view has also been endorsed by Malghan and Benchalli [14] who also treated $\alpha-compactness$ vis-à-vis $\alpha-perfect$ maps in fuzzy topological spaces and studied a version of local $\alpha-compactness$ weaker than that of Gantner et al. [10]. Georgiou and Papadopoulos [11] dealt with $\alpha-compactness$ using the notion of fuzzy upper limit. In a very recent paper Chakraborty et al. [3] introduced fuzzy *semicompact* set and investigated and characterized fuzzy *semicompact* spaces in terms of fuzzy *nets* and fuzzy $prefilterbases$. In section 3, we propose to define and characterize $\alpha-semicompactness$ in terms of ordinary *nets* and $filters$. This seems to be quite new approach in as much as to our knowledge, in almost none of the theories concerning the investigations of numerous fuzzy topological concepts, ordinary *nets* and $filters$ have been involved so far. In section 4, we propose to introduce $\alpha-semiclosed$ sets and $\alpha-semicompactness$ for crisp subsets of fuzzy topological spaces and will carry on the investigation of $\alpha-semicompactness$ in a greater detail. At the same time, we shall strive to ultimately achieve certain expected results in analogy to those known for *semicompact* sets vis-à-vis *semiclosed* sets in a topological space.

2 Preliminaries

Throughout this paper, by (X,τ) or simply by X we mean a fuzzy topological space (henceforth abbreviated as fts.) in Chang's [2] sense and to denote A to be a fuzzy set in X, we shall sometimes write $A \in I^X$, where $I = [0,1]$. For $A,B \in I^X$, we write $A \leq B$ if $A(x) \leq B(x)$, for each $x \in X$. For a fuzzy set A in X, $Cl(A)$, $Int(A)$ and $1-A$ will respectively denote the *closure*, *interior* and *complement* of A. For a family $\zeta = \{A_\alpha : \alpha \in \Lambda\}$ (here and henceforth also, Λ denotes an indexing set) of fuzzy sets in X, the union $\bigvee_{\alpha\in\Lambda} A_\alpha$ and the intersection $\bigwedge_{\alpha\in\Lambda} A_\alpha$ are fuzzy sets defined respectively by $(\bigvee_{\alpha\in\Lambda} A_\alpha)(x) = sup\{A_\alpha(x) : \alpha \in \Lambda\}$ and $(\bigwedge_{\alpha\in\Lambda} A_\alpha)(x) = inf\{A_\alpha(x) : \alpha \in \Lambda\}$, for each $x \in X$[18].

3 $\alpha-$Semicompact Fuzzy Topological Spaces

We recall from [10] the definition of $\alpha-$shading of a fts.

Definition 3.1. *Let X be a fts. A collection $\zeta \subset I^X$ is called an $\alpha-$shading of X, where $0<\alpha<1$, if for each $x \in X$ there exists some $U_x \in \zeta$ such that $U_x(x) > \alpha$.*
A subcollection ζ_0 of an $\alpha-$shading ζ of X, that is also an $\alpha-$shading of X, is called an $\alpha-$subshading of ζ.

Remark 3.1. *It is clear from the above definition that a collection ζ of fuzzy sets in a fts. X is an $\alpha-$shading iff $sup\{U(x) : U \in \zeta\} > \alpha$ for each $x \in X$.*

Definition 3.2. *A fts. X is said to be $\alpha-$semicompact if each $\alpha-$shading of X by fuzzy semiopen sets of X has a finite $\alpha-$subshading.*
The following definition is also given in [10] for an L-fuzzy space, in slightly different forms. We prefer to incorporate here, a complete proof of the said theorem in our setting.

Definition 3.3. *[10] A family $\{F_i : i \in \Lambda\}$ of fuzzy sets in a fts. X is said to have $\alpha-$finite intersection property (to be abbreviated as $\alpha-$FIP) if for each finite subset Λ_0 of Λ there is some $x \in X$ such that $inf\{F_i(x) : i \in \Lambda_0\} \geq 1-\alpha$.*

Theorem 3.1. *A fts. X is $\alpha-$semicompact iff for every family $\zeta = \{F_i : i \in \Lambda\}$ of fuzzy semiclosed sets in X with $\alpha-$FIP, there is some $x \in X$ such that $inf\{F_i(x) : i \in \Lambda\} \geq 1-\alpha$.*
Proof. Let *X be $\alpha-semicompact$* and let $\zeta = \{F_i : i \in \Lambda\}$ be a family of fuzzy *semiclosed* sets in X with $\alpha-FIP$. If possible, let for each $x \in X$, $(\bigwedge_{i\in\Lambda} F_i)(x) < 1-\alpha$. i.e, $\{\bigvee_{i\in\Lambda}(1-F_i)\}(x) > \alpha$, for each $x \in X$. Hence $\Omega = \{1-F_i : i \in \Lambda\}$ is an $\alpha-shading$ of X by fuzzy *semiopen* sets. *By $\alpha-semicompactness$* of *X*, there exists a finite subset Λ_0 of Λ such that, $\{\bigvee_{i\in\Lambda_0}(1-F_i)\}(x) > \alpha$, for each $x \in X$. i.e, $(\bigwedge_{i\in\Lambda_0} F_i)(x) < 1-\alpha$, for each $x \in X$. This implies ζ does not have $\alpha-FIP$, a contradiction. Hence $inf\{F_i(x) : i \in \Lambda\} \geq 1-\alpha$, for each $x \in X$.
Conversely, let $\zeta = \{F_i : i \in \Lambda\}$ be an $\alpha-$shading of X by fuzzy *semiopen* sets. That is, $(\bigvee_{i\in\Lambda} F_i))(x) > \alpha$. Then $\Omega = \{1-F_i : i \in \Lambda\}$

is a family of fuzzy *semiclosed* sets such that $\{\bigwedge_{i\in\Lambda}(1-F_i)\}(x) < 1-\alpha$, for each $x \in X$. Then in view of the hypothesis, Ω cannot have $\alpha-$*FIP*. Thus for a finite subset Λ_0 of Λ and for each $x \in X$, we have $\{1-(\bigvee_{i\in\Lambda_0} F_i)(x)\} = \{\bigwedge_{i\in\Lambda_0}(1-F_i)\}(x) < 1-\alpha$, for each $x \in X$. This implies $(\bigvee_{i\in\Lambda_0} F_i)(x) > \alpha$, for each $x \in X$. Hence X is $\alpha-$*semicompact.*

We now define the concept of a sort of cluster points of ordinary *nets and filters* in a *fts.* and ultimately use them to characterize $\alpha-$*semicompactness* of a *fts.*

Definition 3.4. *A point $x \in X$ is said to be an $\alpha-$semicluster point of an ordinary net $\{S_n : n \in (D,\geq)\}$, ($(D,\geq)$ being any directed set) in a fts. X if for each fuzzy semiopen set U with $U(x) > \alpha$ and for each $m \in D$, there exists a $k \in D$ such that $k \geq m$ in D and $U(S_k) > \alpha$.*

Theorem 3.2. *A fts. X is $\alpha-$semicompact iff every ordinary net in X has an $\alpha-$semicluster point in X.*

Proof. Let X be $\alpha-$*semicompact.* If possible, let there be a net $\{x_n : n \in (D,\geq)\}$ in X having no $\alpha-$*semicluster* point in X. Then for each $x \in X$, there exists a fuzzy *semiopen* set U_x *with* $U_x(x) > \alpha$ and $m_x \in D$ such that $U_x(x_n) \leq \alpha$, for all $n \geq m_x$, $(n \in D)$. Then $\Omega = \{U_x : x \in X\}$ is a collection of fuzzy *semiopen* sets such that for any finite subcollection $\{U_{x_1}, U_{x_2}, U_{x_3}, \ldots\ldots.., U_{x_k}\}$ of Ω, there exists $m \in D$ such that $m \geq m_{x_1}, m_{x_2}, m_{x_3}, \ldots\ldots.., m_{x_k}$ and $(\bigvee_{i=1}^{k} U_{x_i})(x_n) \leq \alpha$, for $n \geq m$ $(n \in D)$. This implies $(\bigwedge_{i=1}^{k}(1-U_{x_i}))(x_n) \geq 1-\alpha$. Then the collection $\Omega = \{1-U_x : x \in X\}$ of fuzzy *semiclosed* sets has $\alpha-$*FIP.* Then by theorem (3.1), there exists $y \in X$ such that $(\bigwedge_x(1-U_x))(y) \geq 1-\alpha$, $x \in X$. i.e, $(\bigvee_x U_x)(y) \leq \alpha$, $x \in X$. i.e, $U_x(y) \leq \alpha$, for all $U_x \in \Omega$. In particular, $U_y(y) \leq \alpha$, contradicting the definition of U_y. Hence the given net in X has an $\alpha-$*semicluster* point in X.

Conversely, let every net in X has an $\alpha-$*semicluster* point in X. *Let* $\Omega = \{U_i : i \in \Lambda\}$ be an arbitrary collection of fuzzy *semiclosed* sets in X *with* $\alpha-$*FIP.* Let Λ^* denotes the collection of all finite subsets of Λ. Then $(\Lambda^*,\geq)$ is a directed set, where $\mu,\lambda \in \Lambda^*$, $\mu \geq \lambda$ *if* $\mu \supseteq \lambda$. Let us put $F_\mu = \bigwedge_{i\in\mu} U_i$ for each $\mu \in \Lambda^*$. As Ω has $\alpha-$*FIP* for each $\mu \in \Lambda^*$, there is some x_μ(say) in X such that $inf\{U_i(x_\mu) : i \in \mu\} \geq 1-\alpha$. By virtue of theorem (3.1), X will be $\alpha-$*semicompact* if $inf\{U_i(z) : i \in \Lambda\} \geq 1-\alpha$, for some $z \in X$. If possible let $inf\{U_i(z) : i \in \Lambda\} < 1-\alpha$, for each $z \in X$. *Now* $\{x_\mu : \mu \in (\Lambda^*,\geq)\}$ is clearly a net in X and hence by hypothesis, it has an $\alpha-$*semicluster* point $y \in X$. Then $inf\{U_i(y) : i \in \Lambda\} < 1-\alpha$ and hence there exists $i_0 \in \Lambda$ such that $U_{i_0}(y) < 1-\alpha$. That is, $(1-U_{i_0})(y) > \alpha$. Since $\{i_0\} \in \Lambda^*$, there exists $\mu_0 \in \Lambda^*$ with $\mu_0 \geq \{i_0\}$ *(i.e,* $i_0 \in \mu_0$*)* such that $(1-U_{i_0})(x_{\mu_0}) > \alpha$. Then $U_{i_0}(x_{\mu_0}) < 1-\alpha$. As $i_0 \in \mu_0$, $inf\{U_i(x_{\mu_0})\} \leq \{U_{i_0}(x_{\mu_0})\} < 1-\alpha$, a contradiction. Hence $inf\{U_i(z) : i \in \Lambda\} \geq 1-\alpha$, for some $z \in X$. Then by virtue of theorem (3.1), X is $\alpha-$*semicompact.*

Definition 3.5. *A point $x \in X$, where X is a fts, is said to be an $\alpha-$semicluster point of a filter base ζ on X if for each fuzzy semiopen set U with $U(x) > \alpha$ and for each $F \in \zeta$ there exists $x_F \in F$ such that $U(x_F) > \alpha$.*

Theorem 3.3. *A fts X is $\alpha-$semicompact iff every filterbase ζ on X has an $\alpha-$semicluster point in X.*

Proof. Let be X be $\alpha-$*semicompact.* Let there exists, if possible, a *filterbase* ζ on X having no $\alpha-$*semicluster* point in X. Then for each $x \in X$, there exists a fuzzy *semiopen set* U_x with $U_x(x) > \alpha$ and there exists $F_x \in \zeta$ such that $U_x(y) \leq \alpha$ for each $y \in F_x$. Thus $\Omega = \{U_x : x \in X\}$ is an $\alpha-$*shading* of X by fuzzy *semiopen* sets of X. By $\alpha-$*semicompactness* of X, there exists finitely many points $x_1, x_2, x_3, \ldots\ldots.., x_n \in X$ such that $\Omega_0 = \{U_{x_1}, U_{x_2}, U_{x_3}, \ldots\ldots., U_{x_n}\}$ is again an α-*shading* of X. *Now let* $F \in \zeta$ such that $F \subseteq \{F_{x_1} \cap F_{x_2} \cap F_{x_3} \cap \ldots\ldots\ldots. \cap F_{x_n}\}$ [ζ *is filterbase*]. Then $U_{x_i}(y) \leq \alpha$ for each $y \in F$ and for i = *1,2,......,n*. Thus Ω_0 fails to be an $\alpha-$*shading* of X, a contradiction. Hence every *filterbase* on X has an $\alpha-$*semicluster* point.

Conversely, let every *filterbase* ζ on X has an $\alpha-$*semicluster* point in X. If possible let $\{y_n : n \in (D,\geq)\}$ be a net in X having no $\alpha-$*semicluster* point in X. Then for each $x \in X$, there is a fuzzy *semiopen* set U_x with $U_x(x) > \alpha$ and there exists some $m_x \in D$ such that $U_x(y_n) \leq \alpha$ for all $n \geq m_x$ $(n \in D)$. Thus $B = \{F_x : x \in X\}$, where $F_x = \{y_n : n \geq m_x\}$, is a subbase for a *filterbase* ζ on X, where ζ consists of all finite intersections of members of B. By hypothesis, ζ *has an* $\alpha-$*semicluster* point $z \in X$. But there is a fuzzy *semiopen* set U_z with $U_z(z) > \alpha$ and there exists $m_z \in D$ such that $U_z(y_n) \leq \alpha$ for all $n \geq m_z$. That is, for all $p \in F_z$, where $F_z \in B \subseteq \zeta$, $U_z(p) \leq \alpha$. Hence z cannot be an $\alpha-$*semicluster* point of the *filterbase* ζ, a contradiction. Hence the net $\{y_n : n \in (D,\geq)\}$ in X has an $\alpha-$*semicluster* point. By theorem (3.2), $\alpha-$*semicompactness* of X follows.

4 $\alpha-$Semiclosed sets, $\alpha-$Semicompact sets

In this section we introduced a class of special type of crisp subsets of X which are defined by the fuzzy sets in a fts (X,τ). The introduced concept is developed to some extent, which is used as a supporting tool for our ultimate aim of introducing and studying $\alpha-$semicompactness for crisp subsets along with $\alpha-$semiclosedness.

We recall from [4] the following definitions in order to introduce our proposed definition of $\alpha-$semiclosed sets.

Definition 4.1. *Let (X,τ) be a fts and $A \subseteq X$. An element $x \in X$ is said to be an $\alpha-$adherent ($\alpha-$limit) point of A $(0 < \alpha < 1)$ if for each fuzzy open set U in X with $U(x) > \alpha$, there exists $y \in A$ ($y \in A \setminus \{x\}$) such that $U(y) > \alpha$.*
The set of all $\alpha-$limit points of A is denoted by A^{α}.

Definition 4.2. *A subset C of X is said to be $\alpha-$closed if it contains all its $\alpha-$limit points.*

Definition 4.3. *α-closure of a set $A \subseteq X$, denoted by α-ClA, is defined as $\alpha - ClA = A \cup A^{\alpha}$.*

Remark 4.1. *Obviously $A \subseteq \alpha - ClA$ and if $A^{\alpha} \subseteq A$ then $\alpha - ClA \subseteq A$.*
A set $A \subseteq X$ is said to be $\alpha-$closed iff $\alpha-ClA = A$.
Now we define $\alpha-$*semiclosed* set by defining $\alpha-$*semiadherent* point and $\alpha-$*semilimit* point of a crisp subset *A of a fts.* (X,τ).

Definition 4.4. *Let (X,τ) be a fts and $A \subseteq X$. An element $x \in X$ is said to be an $\alpha-$semiadherent ($\alpha-$semilimit) point of A $(0 < \alpha < 1)$ if for each fuzzy semiopen set U in X with $U(x) > \alpha$, there exists $y \in A$ ($y \in A \setminus \{x\}$) such that $U(y) > \alpha$. The set of all $\alpha-$semilimit points of A will be denoted by $A^{\alpha-slp}$.*

Definition 4.5. *A subset C of X is said to be $\alpha-$semiclosed if it contains all its $\alpha-$semilimit points.*

Definition 4.6. *$\alpha-$semiclosure of a set $A \subseteq X$, denoted by $\alpha-$SclA, is defined as $\alpha-SclA = A \cup A^{\alpha-slp}$.*

Remark 4.2. *Obviously $A \subseteq \alpha-SclA$ and if $A^{\alpha-slp} \subseteq A$ then $\alpha-SclA \subseteq A$.*
A set $A \subseteq X$ is said to be $\alpha-$semiclosed iff $\alpha-SclA = A$.

Lemma 4.1. *(a) $A \subseteq B \Rightarrow A^{\alpha-slp} \subseteq B^{\alpha-slp}$ and hence $\alpha - SclA \subseteq \alpha - SclB$.*

(b) $\alpha - Scl(A \cup B) \supseteq \alpha - SclA \cup \alpha - SclB$.

Proof. (a) Let $A \subseteq X$ and let $x \in A^{\alpha-slp}$. This implies x is an $\alpha-$*semilimit* point of *A*. That is, for each fuzzy *semiopen* set U in X with $U(x) > \alpha$, there exists $y \in A \setminus \{x\}$ such that $U(y) > \alpha$. As $A \subseteq B$, it follows that $y \in B \setminus \{x\}$. Therefore, x is an $\alpha-$*semilimit* point of *B*. That is, $x \in B^{\alpha-slp}$. Hence $A^{\alpha-slp} \subseteq B^{\alpha-slp}$. Also $\alpha - SclA = A \cup A^{\alpha-slp} \subseteq B \cup B^{\alpha-slp} = \alpha - SclB$. Hence $\alpha - SclA \subseteq \alpha - SclB$.
(b) Proof is obvious

Definition 4.7. *[1] Let (X,τ) and (Y,τ_1) be two fts. A function $f: (X,\tau) \longrightarrow (Y,\tau_1)$ is said to be fuzzy semicontinuous if for every fuzzy open set V in τ_1, $f^{-1}(V)$ is fuzzy semiopen set in X.*

Theorem 4.1. *Let (X,τ) and (Y,τ_1) be two fts. If a function $f: (X,\tau) \longrightarrow (Y,\tau_1)$ is fuzzy semicontinuous then $f^{-1}(C)$ is a $\alpha-$semiclosed set in X for every $\alpha-$closed set C in Y.*
Proof. Let C be a $\alpha-$*closed* set in *Y*. Let $x \in X \setminus f^{-1}(C)$. Then $f(x) \notin C$ [$x \notin f^{-1}C$]. As C is a $\alpha-$*closed* set in *Y*, $f(x)$ is not a $\alpha-$*limit* point of *C*. Then there exists $V \in \tau_1$, such that $V(f(x)) > \alpha$ and $V(y) \leq \alpha$ for all $y \in C$ [actually for all $y \in C \setminus \{f(x)\}$]. Since f is fuzzy *semicontinuous,* $f^{-1}(V)$ is fuzzy *semiopen* set in *X. Let* $z \in f^{-1}(C)$. Then $f(z) \in C$. Now $f^{-1}(V)(x) = V(f(x)) > \alpha$ and $f^{-1}(V)(z) = V(f(z)) \leq \alpha$ for all $z \in f^{-1}(C)$. Thus x is not a $\alpha-$*semilimit* point of $f^{-1}(C)$. Hence $f^{-1}(C)$ is $\alpha-$*semiclosed* set in *X*.

Definition 4.8. *Let (X,τ) and (Y,τ_1) be two fts. A function $f: (X,\tau) \longrightarrow (Y,\tau_1)$ is said to be fuzzy semiopen map if for each fuzzy open set A in X , $f(A)$ is fuzzy semiopen set in Y.*

Theorem 4.2. *If $f: (X,\tau) \longrightarrow (Y,\tau_1)$ be a bijective fuzzy semiopen map, then the image of a $\alpha-$closed set in X is a $\alpha-$semiclosed set in Y.*
Proof. Let $C \subseteq X$ be a $\alpha-$*closed* set in *X*. Let $y \in Y \setminus f(C)$. Then there exists a unique $z \in X$ such that *f(z) = y* [since f is bijective]. As $y \notin f(C)$, $z \notin C$. Now as C is $\alpha-$*closed* set in *X*, there exists fuzzy open set V in τ such that $V(z) > \alpha$ *and* $V(p) \leq \alpha$ for all $p \in C$ [actually for all $p \in C \setminus \{z\}$]. As f is fuzzy *semiopen* map, we have $f(V)$ is fuzzy *semiopen* set in *Y*. Now $(V)(y) = V(z) > \alpha$. That is , $f(V)(y) > \alpha$. Let $t \in f(C)$. Then there exists $t_0 \in C$ such that $f(t_0) = t$. As $f(V)(t) = V(t_0) \leq \alpha$ for all $t_0 \in C$. Therefore, $f(V)(t) \leq \alpha$ for all $t \in f(C)$ [actually for all $t \in f(C) \setminus \{y\}$]. So y is not a $\alpha-$*semilimit* point of $f(C)$. Hence $f(C)$ is $\alpha-$*semiclosed* set in *Y*.
We now recall from [4] the definition of $\alpha-$*compact* (crisp) set.

Definition 4.9. *A crisp subset A of a fts. X is said to be $\alpha-$compact if each $\alpha-$shading of A by fuzzy open sets of X has finite $\alpha-$subshading.*
We now propose to define $\alpha-$*semicompact* subset of a fts.

Definition 4.10. *A crisp subset A of a fts. X is said to be $\alpha-$semicompact if each $\alpha-$shading of A by fuzzy semiopen sets of X has finite α-subshading.*

Theorem 4.3. *A $\alpha-$semiclosed subset A of a $\alpha-$semicompact space X is $\alpha-$semicompact.*
Proof. Let (X,τ) be a $\alpha-$*semicompact* space and A be a $\alpha-$*semiclosed* subset of X. If $x \in X \setminus A$, then x is not a $\alpha-$*semilimit* point of A. So there exists a fuzzy *semiopen* set U_x in X such that $U_x(x) > \alpha$ but $U_x(y) \leq \alpha$ for all $y \in A$ [actually for all $y \in A \setminus \{x\}$]. Let $\Omega = \{U_x : x \in X \setminus A\}$. Let S be a $\alpha-$*shading* of A by fuzzy *semiopen* sets of X. Let $W = \Omega \cup S$. Clearly W is a $\alpha-$*shading* of X by fuzzy *semiopen* sets of X. As X is $\alpha-$*semicompact,* there exists a finite subset $\{W_1,W_2,W_3,.......,W_n\}$ of W which is again a $\alpha-$*shading* of X. Now the subset $\{W_1,W_2,W_3,.......,W_n\}$ of W may contain some W_i which are members of Ω. If we omit them we get a finite $\alpha-$*subshading* of S by fuzzy *semiopen* sets of X consisting of the remaining members of W. Hence A is $\alpha-$*semicompact.*

Corollary 4.1. Every $\alpha-$*semiclosed* subset A of a $\alpha-$*compact* space X is $\alpha-$*semicompact.*
Proof. As X is $\alpha-$*semicompact,* it is $\alpha-$*compact.* Hence by theorem (4.3), the corollary follows.

Definition 4.11. *A function $f: (X,\tau) \longrightarrow (Y,\tau_1)$ is said to be $\alpha-$semicontinuous if $f^{-1}(A)$ is $\alpha-$semiclosed set in X for every $\alpha-$closed set A in Y.*

Remark 4.3. In view of theorem (4.1) every fuzzy *semicontinuous* function is $\alpha-$*semicontinuous* function.

Theorem 4.4. *Let $f: (X,\tau) \longrightarrow (Y,\tau_1)$ be a $\alpha-$semicontinuous function. If $A \subseteq X$ is $\alpha-$semicompact set in (X,τ) then $f(A)$ is $\alpha-$compact in (Y,τ_1).*
Proof. Let A be a $\alpha-$*semicompact* set in (X,τ). Let S be a $\alpha-$*shading* of $f(A)$ by fuzzy open sets in Y. For $x \in A \subseteq X$, $f(x) \in f(A)$ and there exists $U_{f(x)} \in S$ such that $U_{f(x)}(f(x)) > \alpha$. Let $z \notin U^{-1}_{f(x)}[0,\alpha]$ then $U_{f(x)} \in S$ with $U_{f(x)}(z) > \alpha$ and $U_{f(x)}(y) \leq \alpha$ for all $y \in U^{-1}_{f(x)}[0,\alpha]$. Thus z is not a $\alpha-$*limit* point of $U^{-1}_{f(x)}[0,\alpha]$ and hence $U^{-1}_{f(x)}[0,\alpha]$ is $\alpha-$*closed* in Y. As $f: X \longrightarrow Y$ is $\alpha-$*semicontinuous,* $f^{-1}(U^{-1}_{f(x)}[0,\alpha])$ is $\alpha-$*semiclosed* in X. Clearly $x \notin f^{-1}(U^{-1}_{f(x)}[0,\alpha])$. Indeed if $x \in f^{-1}(U^{-1}_{f(x)}[0,\alpha])$ then $f(x) \in U^{-1}_{f(x)}[0,\alpha]$ which implies $U_{f(x)}(f(x)) \leq \alpha$ which contradicts the fact that $U_{f(x)} \in S$ [S is $\alpha-$*shading* of $f(A)$]. Let $C = U^{-1}_{f(x)}[0,\alpha]$, a $\alpha-$*closed* set in Y. *So* $f^{-1}(C)$ is $\alpha-$*semiclosed* set in X and as $x \notin f^{-1}(C)$, x is not a $\alpha-$*semilimit* point of $f^{-1}(C)$. So there exists *fuzzy semiopen* set V_x in X such that $V_x(x) > \alpha$ and $V_x(z) \leq \alpha$ for all $z \in f^{-1}(C)$. Now $W = \{V_x : x \in A\}$is a $\alpha-$*shading* of A by fuzzy *semiopen* sets in X. As A is $\alpha-$*semicompact,* W has a finite subset $\{V_{x_1},V_{x_2},V_{x_3},........,V_{x_n}\}$ which is also a $\alpha-$*shading* of A. That is, for every $p \in A$, $V_{x_i}(p) > \alpha$ for some i= 1,2,........,n. We claim that $\{U_{f(x_i)} : i = 1,2,......,n\}$ is a finite $\alpha-$*subshading* for $f(A)$. Let $s \in f(A)$, so *f(t) = s* for some $t \in A$. *So* $V_{x_i}(t) > \alpha$ for some i= 1,2,......,n. As $t \notin f^{-1}(C)$, that is, as $t \notin f^{-1}(U^{-1}_{f(x_i)}[0,\alpha])$, we have $f(t) \notin U^{-1}_{f(x_i)}[0,\alpha]$, which implies that $U_{f(x_i)}(f(t)) \notin [0,\alpha]$. That is, $U_{f(x_i)}(s) \notin [0,\alpha]$. That is, $U_{f(x_i)}(s) > \alpha$. Hence $\{U_{f(x_i)} : i = 1,2,.......,n\}$ is a finite $\alpha-$*subshading* for $f(A)$ by fuzzy open sets in Y. Hence $f(A)$ is $\alpha-$*compact.*

Corollary 4.2. *If $f: (X,\tau) \longrightarrow (Y,\tau_1)$ be a fuzzy semicontinuous function and if $A \subseteq X$ is $\alpha-$semicompact in X then f(A) is $\alpha-$compact.*
Proof. Every fuzzy *semicontinuous* function is $\alpha-$*semicontinuous.* So by theorem (4.4) the corollary follows.

Theorem 4.5. *Let (X,τ) be a fuzzy topological space. Then X is $\alpha-$semicompact iff every collection R of $\alpha-$semiclosed sets in X satisfying the finite intersection property has non-null intersection.*
Proof. Let (X,τ) be $\alpha-$*semicompact.* If possible, let there exists a collection R of $\alpha-$*semiclosed* sets having finite intersection property but $\bigcap_{C \in R} C = \phi$. Then $\bigcup_{C \in R} (X \setminus C) = X$. Let $x \in X$. Then $x \in X \setminus C_x$ for some $C_x \in R$. Since C_x is $\alpha-$*semiclosed* and $x \notin C_x$, we have some fuzzy *semiopen* sets U_x in X such that $U_x(x) > \alpha$ *and* $U_x(z) \leq \alpha$ for all $z \in C_x$. Then $S = \{U_x : x \in X\}$ is an $\alpha-$*shading* of X by fuzzy *semiopen* sets in X. As (X,τ) is $\alpha-$*semicompact,* we have a finite *subcollection* $\{U_{x_1},U_{x_2},U_{x_3},........,U_{x_n}\}$ of S such that for every $z \in X$, we have $U_{x_i}(z) > \alpha$ for some i $(1 \leq i \leq n)$.Thus $X = \bigcup_{i=1}^{n} U^{-1}_{x_i}(\alpha,1]$ and hence $\bigcap_{i=1}^{n} \{X \setminus U^{-1}_{x_i}(\alpha,1]\} = \phi$. *Let* $z \in C_{x_i}$. Then $U_{x_i}(z) \leq \alpha$ and hence $z \notin U^{-1}_{x_i}(\alpha,1]$. This implies $z \in \{X \setminus U^{-1}_{x_i}(\alpha,1]\}$. So $C_{x_i} \subseteq \{X \setminus U^{-1}_{x_i}(\alpha,1]\}$.
Hence $\bigcap_{i=1}^{n} C_{x_i} = \phi$ goes against the *finite intersection propety* of R.
Conversely, let S be an $\alpha-$*shading* of X by fuzzy *semiopen* sets in X. Let $U_\alpha = \{x \in X : U(x) \leq \alpha, U \in S\}$. If $y \notin U_\alpha$ then $U(y) > \alpha$. So we get a fuzzy *semiopen* set U such that $U(y) > \alpha$ and $U(z) \leq \alpha$ for every $z \in U_\alpha$. So y is not a $\alpha-$*semilimit* point of U_α. Hence U_α *is* $\alpha-$*semiclosed* set for each $U \in S$. Let $R = \{U_\alpha : U \in S\}$. Then R is a collection of $\alpha-$*semiclosed* sets. As S is a $\alpha-$*shading* of X by fuzzy *semiopen* sets of X, we have for each $x \in X$, there exists $V \in S$ such that $V(x) > \alpha$. Hence $x \notin V_\alpha$ for some $V \in S$. So $\bigcap_{U \in S} U_\alpha = \phi$. Now by hypothesis R does not satisfy the finite intersection property. So there exists a finite subcollection S_0 of S such that $\bigcap_{U \in S_0} U_\alpha = \phi$. So for every $x \in X$ there exists $U \in S_0$ such that $x \notin U_\alpha$ and hence $U(x) > \alpha$. Thus X is $\alpha-$*semicompact.*

5 Conclusion

The notions of the sets and functions in fuzzy topological spaces are highly developed and several characterizations have already been obtained. These are used extensively in many practical and engineering problems, computational topology for geometric design, computer-aided geometric design, engineering design research, Geographic Information System (GIS) and mathematical sciences. As the concept of *compactness* of a fuzzy topological space and its fuzzy subsets as well as its crisp subsets is one of the central and important concepts, the notions and results given in this paper may lead to some interesting in-depth analytical study and research from the view point of fuzzy mathematics.

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