On Non-Chiral Extension of Kerr/CFT

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[arXiv:1010.4291] & [arXiv:1102:3423]

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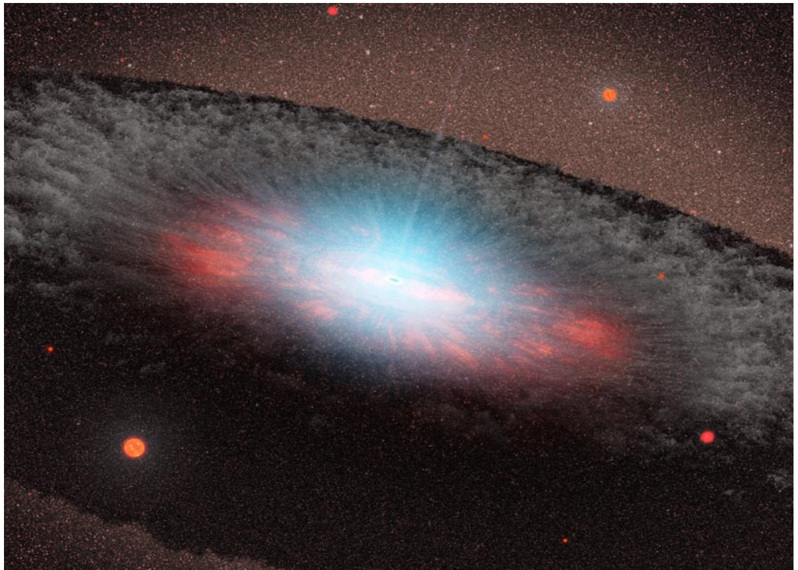
Today's theme: Extension of Kerr/CFT

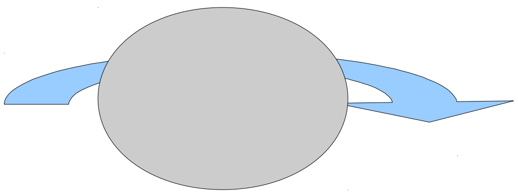
Kerr/CFT = AdS/CFT in extremal BH,
without excitations above it.

Where is non-extremal excitations ??

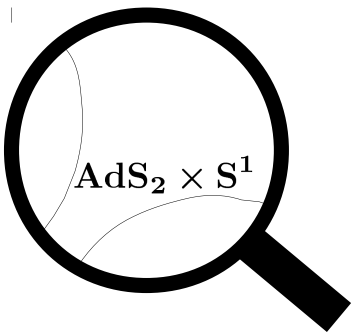
Toward holography in realistic BH's!



Strategy: Zero-entroy extremal BH and AdS_3/CFT_2

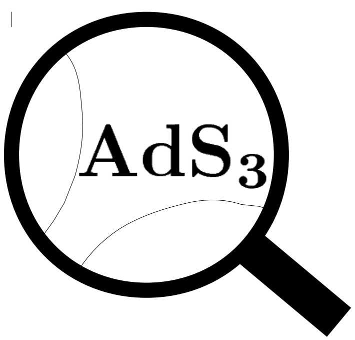


Extremal BH



Near Horizon Geometry

Zero entopy limit

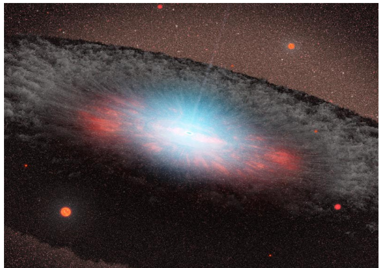


Local AdS_3 (BTZ) geometry

AdS_3/CFT_2

$c_R = c_L$

(Future extension...)

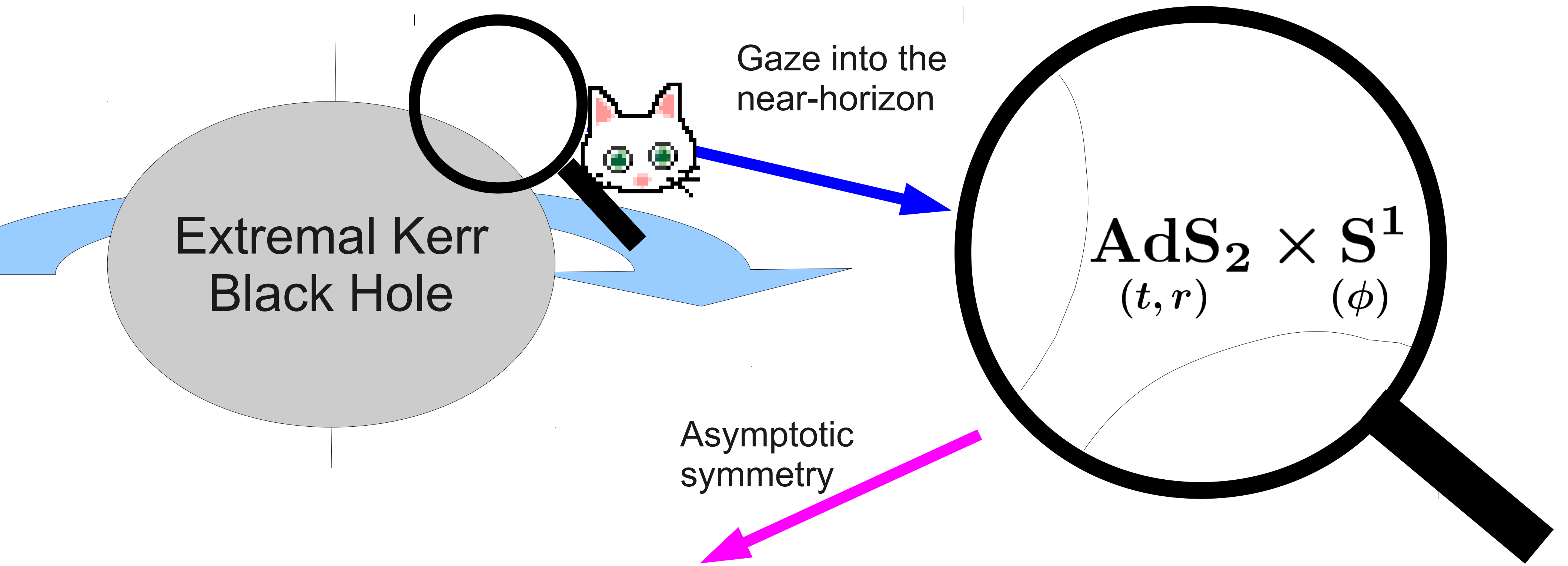


Thank you for your patience !

Appendix

AdS/CFT in Extremal Black Holes (Kerr/CFT)

[Guica-Hartman-Song-Strominger, 0809.4266]



Virasoro alg. x 1

AdS/CFT (Symmetry Correspondence)

2d chiral CFT !

Boundary conditon and asymptotic symmetry

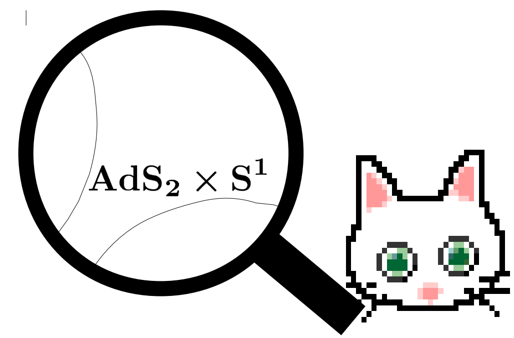
Near Hrizon Extremal Kerr(NHEK) metric

$$\boldsymbol{ds^2 = 2G_4 J\Omega(\theta)^2\Big[\underline{-r^2dt^2 + \frac{dr^2}{r^2} + \Lambda(\theta)^2(d\phi - r dt)^2} + d\theta^2\Big]}$$

θ-dep S^1-fibrated AdS_2 isometry = $\mathbf{SL(2,\mathbb{R}) \times U(1)_\phi}$

Bekenstein-Hawking entropy

$$S_{BH} = 2\pi J$$



Boundary condition

$$\boldsymbol{g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu},\ \ h_{\mu\nu} \sim} \begin{array}{cc} \begin{pmatrix} r^2 & r^{-2} & r^{-1} & 1 \\ & r^{-3} & r^{-2} & r^{-1} \\ & & r^{-1} & r^{-1} \\ & & & 1 \end{pmatrix} & \begin{array}{c} t \\ r \\ \theta \\ \phi \end{array} \end{array}$$

($\leftrightarrow$ choice of Hilbert space)

Asymptotic symmetry generator

$$\boldsymbol{\xi^L_n = -e^{-in\phi}(inr\partial_r + \partial_\phi)} \qquad \boldsymbol{\xi^R_0 = -\partial_t}$$

$$\boldsymbol{[\xi^L_m, \xi^L_n]_{Lie} = -i(m-n)\xi^L_{m+n}}$$

Virasoro algebra
= conformal sym of
2d (chiral) CFT !

Asymptotic charges and AdS/CFT

Conserved charges in the dual theory (ex. Virasoro charges)
= asymptitic charges in the gravity side

$$Q_\xi = \int_{\partial\Sigma} k_\xi[h;\bar{g}]$$

$$k_\xi[h;\bar{g}] = -\frac{\sqrt{-\bar{g}}}{16\pi}\Big[\bar{D}^{[\nu}(h\xi^{\mu]}) + \bar{D}_\sigma(h^{[\mu\sigma}\xi^{\nu]}) + \bar{D}^{[\mu}(h^{\nu]\sigma}\xi_\sigma) + \frac{3}{2}h\bar{D}^{[\mu}\xi^{\nu]} + \frac{3}{2}h^{\sigma[\mu}\bar{D}^{\nu]}\xi_\sigma + \frac{3}{2}h^{[\nu\sigma}\bar{D}_\sigma\xi^{\mu]}\Big](d^{D-2}x)_{\mu\nu}$$

(In the case of Einstein gravity, [Barnich-Brandt, hep-th/0111246])

For Virasoro charges, $L_n \simeq Q_{\xi_n^L}$.

Commutators $\leftrightarrow$ Poisson brackets

$$\{Q_\xi, Q_\eta\}_{PB} = Q_{[\xi,\eta]_{Lie}} + \underbrace{\int_{\partial\Sigma} k_\eta[\pounds_\xi \bar{g}, \bar{g}]}_{\text{Central extension}}$$

$$\Downarrow$$

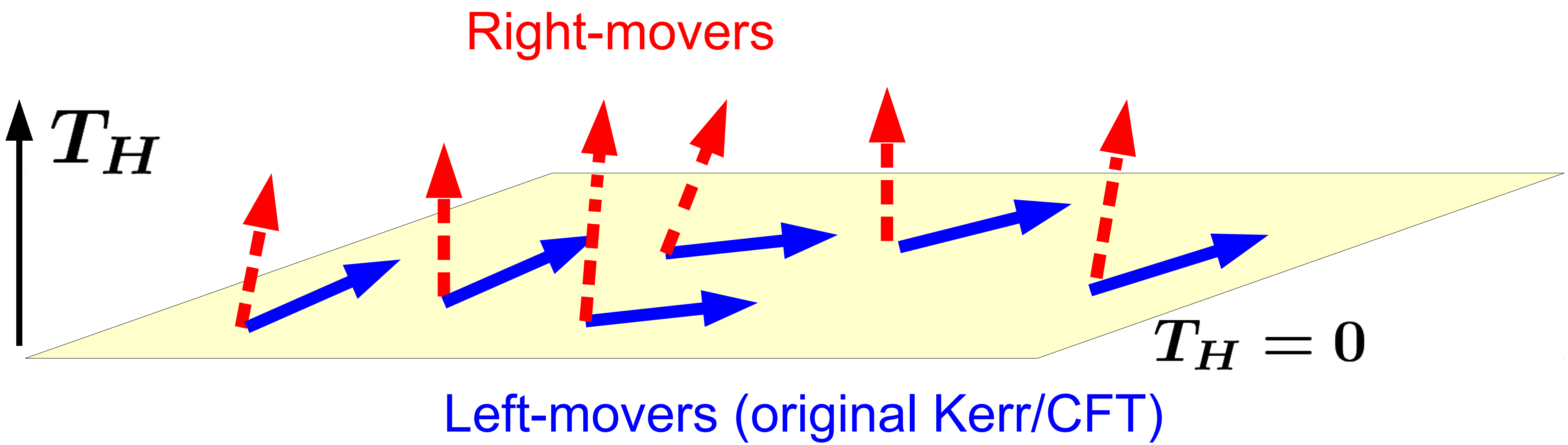
$$c = 12J$$

# of d.o.f
of the dual CFT

Non-chiral (→ Non-extremal) Kerr/CFT ?

There have been some suggestions that Kerr/CFT is extended to non-extremal BH, where the dual theory is non-chiral CFT_2.

[Bredberg-Hartman-Song-Strominger (2009)], [Castro-Larsen (2009)], [Castro-Maloney-Strominger (2010)], ...



Can we derive it from asymptotic symmetry ?

Attempts to non-chiral Kerr/CFT

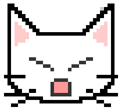
Several attempts have been done,
to enhance U(1)t to right-handed Virasoro...

A slightly modified b.c. for NHEK yields the ASG

$$\xi_n^L = -e^{-in\phi}(inr\partial_r + \partial_\phi) \qquad \xi_n^R = -e^{-int/\beta}(inr\partial_r + \beta\partial_t)$$

Two Virasoro algebras ! [Matsuo-Tsukioka-Yoo, 0907.4272]

However....



- The right-handed Virasoro charges are all divergent.
- The right-handed central charge = 0.

Non-chiral Kerr/CFT in extremal BTZ

Near-horizon extremal BTZ

$$ds^2 = \frac{L^2}{4}\left[-r^2dt^2 + \frac{dr^2}{r^2} + (d\phi - r dt)^2\right]$$

$$(t,\phi) \sim (t, \phi + 2\pi\ell)$$

Boundary cond.

$$h_{\mu\nu} \sim \begin{pmatrix} \color{red}{\mathbf{1}} & r^{-1} & 1 \\ & r^{-3} & r^{-1} \\ & & 1 \end{pmatrix}$$

Asymptotic sym.

$$\xi^L_n = -e^{-in\phi/\ell}(inr\partial_r + \ell\partial_\phi)$$

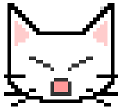
$$\xi^R_n = -e^{-int/\beta}(inr\partial_r + \beta\partial_t)$$

Two Virasoros as asymptotic symmetry.

All asymptotic charges are finite.

Central charges

$$c_L = \frac{3L}{2G_3}, \quad \color{red}{c_R = 0}$$



Vanishing central charge again...

Regularization of Time-slice

Look at the geometry again....

$$ds^2 = \frac{L^2}{4}\left[\frac{dr^2}{r^2} - 2r\,dt\,d\phi + d\phi^2\right]$$

Time-slice is light-like at the boundary ! 

(null-orbifold)

It can make the asymptotic charge ill-defined...

We make the time-slice space-like, by a regularization

$$t' \equiv t + \alpha\phi, \quad \phi' \equiv \phi, \quad (t', \phi') \sim (t', \phi' + 2\pi\ell)$$

(we changed the orbifold.)

(We take $\alpha \to 0$ limit at last.)

Regularization of Time-slice (2)

$$ds^2 = \frac{L^2}{4}\left[\frac{dr^2}{r^2} - 2r dt' d\phi' + (1 + 2\alpha r) d\phi'^2\right]$$

$t = t' - \alpha\phi$ $\rightarrow$ t becomes periodic automatically.

Explicit calculation yields

$$c_R = c_L = \frac{3L}{2G_3}$$

Equal & finite values !

(also they agrees with Brown-Henneaux)

Origin of the regularization

$$ds^2 = \frac{L^2}{4}\left[\frac{dr^2}{r^2} - 2r\,dt'd\phi' + (1+2\alpha r)d\phi'^2\right]$$

BTZ metric:

$$ds^2 = L^2\left[-\frac{\rho^4}{\rho^2+r_+^2}d\tau^2 + \frac{d\rho^2}{\rho^2} + (\rho^2+r_+^2)\left(d\psi - \frac{r_+^2}{\rho^2+r_+^2}d\tau\right)^2\right]$$

Near horizon transformation:

$$\rho^2 = \frac{\lambda r}{2}, \quad \tau = -\frac{r_+}{\lambda}t, \quad \psi = \frac{\phi}{2r_+} - \frac{r_+}{\lambda}t$$

Resulting metric before the limit:

$$ds^2 = \frac{L^2}{4}\left[\frac{dr^2}{r^2} - 2r\,dt'd\phi' + \left(1+\frac{\lambda}{2r_+^2}r\right)d\phi'^2\right]$$

Zero entropy limit for extremal BH

Near horizon geometry (4D):

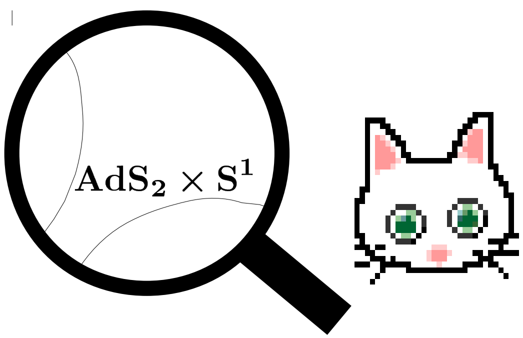
[Kunduri-Lucietti-Reall, 0705.4214]

$$ds^2 = A(\theta)^2\left[\underbrace{-r^2dt^2 + \frac{dr^2}{r^2} + B(\theta)^2(d\phi - krdt)^2}_{\theta\text{-dep } S^1\text{-fibrated } AdS_2}\right] + F(\theta)^2 d\theta^2$$

isometry = $\mathbf{SL(2,\mathbb{R}) \times U(1)_\phi}$

Bekenstein-Hawking entropy

$$S_{BH} = \frac{\pi}{2G_4}\int d\theta A(\theta)B(\theta)F(\theta)$$



We take zero-entropy limit for this geometry.

In this limit,
if AdS2 unbroken & existence of dual theory
$\rightarrow$ no singular behaviors expected.
$\rightarrow$ The geometry is expected to be regular.

BTZ-structure Emergence

$$ds^2 = A(\theta)^2\left[-r^2dt^2 + \frac{dr^2}{r^2} + B(\theta)^2(d\phi - krdt)^2\right] + F(\theta)^2 d\theta^2$$

S = 0 and regularity

Scalings of the parameters:

$$B(\theta) = \epsilon B'(\theta), \quad k = \frac{k'}{\epsilon}, \quad B(\theta)k \; (= B'(\theta)k') \; = 1 + \epsilon^2 b(\theta)$$

At the same time, we rescale the angular coordinate:

$$\phi' = \frac{\phi}{k} \quad (\phi' \sim \phi' + \frac{2\pi}{k})$$

In the $\epsilon \to 0$ limit,

$$ds^2 = A(\theta)^2\left[-r^2dt^2 + \frac{dr^2}{r^2} + (d\phi' - rdt)^2\right] + F(\theta)^2 d\theta^2$$

Structure of (massless) extremal BTZ emarges.

(Many concrete examples are known: [Guica-Strominger], etc...)

Non-chiral Kerr/CFT for zero-entropy extremal BH

$$ds^2 = A(\theta)^2\left[-r^2dt^2 + \frac{dr^2}{r^2} + (d\phi - rdt)^2\right] + F(\theta)^2 d\theta^2$$

$$(t,\phi) \sim (t, \phi + 2\pi\delta)$$

$\Downarrow$ regularization

$$ds^2 = A(\theta)^2\left[\frac{dr^2}{r^2} - 2r dt' d\phi' + (1 + 2\alpha r) d\phi'^2\right] + F(\theta)^2 d\theta^2$$

Boundary condition

$$h_{\mu\nu} \sim \begin{pmatrix} 1 & r^{-1} & 1 & 1 \\ & r^{-3} & r^{-2} & r^{-1} \\ & & r^{-1} & 1 \\ & & & 1 \end{pmatrix}$$

Asymptotic symmetry

$$\xi_n^L = -e^{-in\phi/\delta}(inr\partial_r + \delta\partial_\phi)$$

$$\xi_n^R = -e^{-int/\alpha\delta}(inr\partial_r + \alpha\delta\partial_t)$$

$$c_R = c_L = \frac{3}{G_4}\int d\theta\, A(\theta)F(\theta)$$

Origin of our regularization (2)

When we take the limits of zero-entropy and near-horizon at the same time,

$$ds^2 = A(\theta)^2 \left[\frac{dr^2}{r^2} - 2r dt' d\phi' + (1 + Cr) d\phi'^2 \right] + F(\theta)^2 d\theta^2$$

$$C \equiv \lim \frac{\lambda}{\epsilon}$$

λ : scaling parameter of near-horizon

ε : scaling parameter of zero-entropy

We can identify $\alpha = C/2$

Our regularization is automatically introduced. 

Summary

- Kerr/CFT is extended to a non-chiral form,
  for extremal BTZ and zero-entropy BH. 
- Naive prescription leads to $c_R = 0$
  $\rightarrow$ A regularization yields $c_R = c_L$. 
- The regularization appears automatically and naturally,
  as a remnant of the near horizon parameter. 
- Generalization to nonzero-entropy BH
  would be challenging.... 

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