Cutoff Wave Number for Shear Waves in a Two-Dimensional Yukawa System (Dusty Plasma)

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The cutoff wave number for shear waves in a liquid-state strongly coupled plasma was measured experimentally. The phonon spectra of random particle motion were measured at various temperatures in a monolayer dusty plasma, where microspheres interact with a Yukawa potential. In the liquid state of this particle suspension, shear waves were detected only for wavelengths smaller than 20 to 40 Wigner-Seitz radii, depending on the Coulomb coupling parameter. The temperature of the suspension was controlled using a laser-heating method.

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The waves in molecular solids or liquids can rarely be studied experimentally at an atomistic level. The reasons include small distances between the atoms or molecules in regular matter, high characteristic frequencies, and lack of experimental techniques of visualizing the motion of individual atoms or molecules. A suitable model system to experimentally study waves at an atomistic level is a dusty plasma. A dusty plasma is a suspension of highly charged micron-size particles in a plasma. When these particles are confined, their mutual repulsion causes them to self-organize in a structure that can have a crystalline or liquid order. In a dusty plasma the interparticle distance can be of the order of 1 mm, characteristic frequency of the order of 10 s\(^{-1}\), and the speed of sound of the order of 10 mm/s.

This ordered structure is vastly softer than molecular materials and even colloidal crystals, so that it can be manipulated easily using even the very weak force of radiation pressure applied by a laser beam. These unique characteristics, plus a very helpful possibility of direct imaging, make it possible to study complex phenomena like wave propagation, phase transitions, and transport phenomena, all at an atomistic level.

Waves in dusty plasmas have recently attracted much attention. The theory of different modes has been developed for two-dimensional (2D) [1–4] and three-dimensional (3D) [3,5–8] dusty plasmas in solid and liquid states. For strongly coupled dusty plasmas in the solid state, the linear modes for compressional and shear motion [9,10] have been extensively studied experimentally. For dusty plasmas in a liquid state, however, only a few experimental studies of wave modes have been reported so far [11–13]. For 2D dusty plasmas, one reason for this scarcity is that 2D dusty plasmas tend to crystallize under normal experimental conditions.

 Liquids generally support compressional (longitudinal) waves. However, shear (transverse) waves can only propagate if their wavelength is as short as a few molecular spacings. Thus, the dispersion relation of the shear waves in a liquid has a cutoff wave number. This cutoff is well-known for molecular liquids [14,15], and it has been predicted theoretically for strongly coupled plasmas as well [4,7]. To the best of our knowledge, however, the cutoff wave number has never been observed experimentally in strongly coupled plasmas. Unlike the scattering methods used to detect phonons in molecular liquids, here we will use direct observation of random particle motion at an atomistic level, and by analyzing this random motion we will measure a phonon spectrum that reveals the wave number cutoff.

Using strongly coupled dusty plasmas, experiments to observe shear waves in a liquid state have only recently begun. In Ref. [12], a liquid state of a 2D dusty plasma was achieved by placing a varying amount of perturbing particles in a lower incomplete layer. Compressional and shear waves polarized in the plane of the particle suspension were studied in the solid and liquid states. The dispersion relation of the shear mode did not resolve a cutoff. In Ref. [11], the shear waves with a vertical polarization were observed in a 3D liquid-state dusty plasma. Their dispersion relation was measured and found to agree with a viscoelastic theory. Because of the lack of experimental data points near \(\omega = 0\), no conclusion can be made on the existence of the cutoff wave number.

In this Letter, we use our laser-heating method [16] to melt a 2D dusty plasma crystal and to control the temperature of the resulting liquid. This allows us to study waves in a 2D dusty plasma in solid and liquid states, at various temperatures. The waves correspond to random particle motion at a given temperature. Unlike our experiment in Ref. [17], where we launched sinusoidal waves, here we did not use any additional external means to excite these waves.

The experimental setup was as in Refs. [16,17], using similar experimental parameters. Argon plasma was produced using a capacitively-coupled rf discharge. The discharge was sustained by 35 W of rf power at 13.56 MHz. To
reduce the gas friction, Ar was used at a relatively low pressure of 5 mTorr. In this case, the neutral-gas damping rate $\nu$ is accurately modeled [18] by the Epstein expression with a leading coefficient of 1.26. In our experiment, $\nu = 0.87 \, \text{s}^{-1}$, so that particle motion on the time scales studied here was not overdamped.

A monolayer of microspheres was suspended in the plasma. The particles were highly charged as they absorbed more electrons than ions and they were levitated against gravity in the sheath above the lower rf electrode. The particles had a diameter of 8.09 $\pm$ 0.18 $\mu$m [18] and a mass $m = 4.2 \times 10^{-13}$ kg. The suspension included $\approx 6700$ particles, had a diameter of 50–60 mm, and rotated slowly in the horizontal plane.

The interparticle potential for particles arranged in a single plane, like ours, was experimentally shown [19] to be nearly Yukawa: $U(r) = Q/(4\pi\varepsilon_0 r)^{-1} \exp(-r/\lambda_D)$, where $Q$ is the particle charge and $\lambda_D$ is the screening length. The particle suspension is characterized by screen-
The wave number resolution mode, i.e., the motion of the particle suspension as a whole, appears in Fig. 1(d) as a concentration of energy at \( f = 0 \) and \( k = 0 \). This mode is unrelated to the shear wave, and it does not interfere with calculating the cutoff wave number \( k_c \). Note that in this Letter the sloshing mode is less prominent than in our previous experiment of Ref. [17], where we externally excited compressional waves and observed the sloshing mode in the direction of excitation.

The shear waves, therefore, can propagate in the liquid state of our particle suspension only if their wavelength is shorter than a critical value \( 2\pi/k_c \). This is well-known for simple liquids [14,15]. For strongly coupled plasmas, the cutoff wave number has been predicted in several simulations of liquids, both 3D [7,23] and 2D [4], but it apparently has never been observed experimentally.

The experimental observation of the cutoff wave number \( k_c \) for the shear waves in a 2D liquid dusty plasma is a chief result of this Letter. Next, we discuss the dependence of \( k_c \) on the coupling parameter \( \Gamma \), Fig. 3. We calculated \( k_c \) by beginning with the first four data points in the low-frequency portion of the dispersion relation [open circles in Fig. 1(d)] and fitting them to a straight line. Extrapolating this line to zero frequency yields our value of \( k_c \). The error bars indicate the width of the shear mode at the level of 75% of its peak value, as in Ref. [7]. The normalized cutoff values lie in the range of \( k_c a = 0.16-0.31 \) with perhaps a trend to increase for lower values of \( \Gamma \). Our data generally agree with the molecular dynamics simulation of Ref. [4], where a cutoff wave number of \( k_c a = 0.186 \) was observed for the shear waves in a 2D Yukawa liquid, for \( \kappa = 1 \) and \( \Gamma = 160 \).

The damping mechanism that precludes the existence of the shear waves in a liquid below \( k_c \) is identified as

\[ T(k,f) = \frac{m^2 f(k,f)}{k^2 T} \]

in Fig. 2. We measured the \( k \) for the peak in each cross section by computing its first moment. Repeating for each frequency yields the dispersion relation \( f(k) \), shown as open circles in Fig. 1. For the solid state, the dispersion relation in Fig. 1(c) passes through the origin at \( f = 0 \) and \( k_c a = 0 \), as expected. However, for the liquid state in Fig. 1(d), it does not. Instead, the dispersion relation reaches \( f = 0 \) at a finite cutoff wave number \( k_c a = 0.27 \).

The shear waves, therefore, can propagate in the liquid state of our particle suspension only if their wavelength is
For shear waves in a liquid, we verified the existence of the cutoff wave number. Its value was \( k_c \approx 0.16 - 0.31 \) depending on the coupling strength. In other words, shear waves were only able to propagate in the liquid dusty plasma when their wavelength was smaller than (20–40)\( \lambda \). This result illustrates why it is difficult to observe shear waves in molecular liquids: their wavelength must be very short, perhaps shorter than a few tens of intermolecular spacings.

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