Wind farm flow modeling using input-output dynamic mode decomposition

Jennifer Annoni, Pieter Gebraad, and Peter Seiler

Abstract—Wind turbines in a wind farm are operated individually to maximize their own power regardless of the impact of aerodynamic interactions on neighboring turbines. There is the potential to increase power and reduce overall structural loads by properly coordinating the turbines. To perform control design and analysis, a model needs to be of low computational complexity, but retain the necessary dynamics seen in high-fidelity models. The objective of this work is to obtain a reduced-order model that represents the full-order flow computed using a high-fidelity model. A variety of methods, including proper orthogonal decomposition (POD) and dynamic mode decomposition (DMD), can be used to extract the dominant flow structures and obtain a reduced-order model. In this paper, we use an extension to DMD that can handle problems with inputs and outputs. This method, termed input-output dynamic mode decomposition (IODMD), uses a subspace identification technique to obtain models of low complexity. This technique is used to construct a reduced-order model of the flow within a two-turbine array computed using large eddy simulations.

I. INTRODUCTION

In the United States, many states have a Renewable Portfolio Standard (RPS) or Goal. For example, Minnesota has a RPS target of 25% renewable energy by 2025 [1]. Wind energy will be a significant factor in achieving this goal. Wind farm control can be used to increase wind energy efficiency by maximizing power in wind farms that are already installed. It can also be used to mitigate structural loads to maximize the lifetime of the turbines and better integrate wind energy into the energy market.

Currently, turbines in a wind farm are operated to optimal operating point leads to sub-optimal performance. Properly coordinating turbines in a wind farm has the potential to increase the overall performance of a wind farm [2]. Designing wind farm control strategies requires a model of the wind farm that has a low computational complexity, but retains the necessary dynamics. A variety of wake models exist in literature that are useful for studying wind farm control. The simplest models are the Park model [3] and the eddy viscosity model [4]. These models provide a quick, preliminary description of the wake interactions in a wind farm. Several high-fidelity CFD models have been developed as well [5], [6]. These high-fidelity models are more accurate tools and can be used for evaluating wind farm controllers.

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However, they are computationally expensive. These low- and high-fidelity models have been used to evaluate wind farm control strategies. The analysis provides conflicting results based on the wake model chosen for control design. For example, control strategies designed using simple static models may report significant improvements in wind farm performance, but an analysis of such control strategies using high-fidelity simulations can result in minimal to no improvements in wind farm performance. An example of a comparison between control predictions given by a high-fidelity and simplified model is given in [7], where constant offsets of pitch and torque are used to change wake deficits. It is shown that extensions to the Park model are needed to match the results of high-fidelity models.

Improving models for wind farm control requires a better understanding of the aerodynamic interactions in a wind farm. While many studies have been performed using static models and constant offsets of the operating point of the wind turbines, more recently dynamic wake modeling and control approaches have been proposed. Previously proposed approaches use either high-order first-principle modeling, e.g. [8], or simplified heuristic data-driven modeling techniques [9], [10] to arrive at a dynamic wake model. In this paper, we focus on new techniques to construct a reduced-order wake model from data generated by simulations or experiments.

Techniques developed by the fluids and controls communities are both relevant for reduced-order wake modeling. First, some studies have been done to understand the dominant turbulent structures generated in CFD simulations and in experiments [11]. Proper orthogonal decomposition (POD) and dynamic mode decomposition (DMD) are two popular techniques in the fluids literature that compute the dominant modes of the flow. These modes have been used to construct reduced-order models that can be used for control, such as balanced POD and DMD with controls [12]–[15]. Some of these methods require computing the adjoint of the system which is not readily available in most CFD codes and is not available during experiments. The controls/systems community has an alternative set of techniques to identify models from input-output data, e.g. subspace identification techniques such as N4SID [16]. The methods generate reduced-order black box models to represent input-output measurements from the system. In this framework, the states have no physical meaning.

In this paper, we use an extension of DMD to generate a reduced-order wake model. The model reduction approach has two main advantages. First, it relies on input-output data from a forced response and does not require the construction/simulation of the adjoint system. Second, the reduced-
order model is constructed a way that retains the physical meaning of the states. In other words, the reduced-order state can be mapped back to the approximate full-order state of the system. The method addressed in this paper projects the states onto a reduced-order subspace using the dominant modes of the system and then uses direct N4SID to define the reduced-order model dynamic model of the system. This paper will begin by reviewing the standard reduced-order modeling techniques in the fluids literature, i.e. POD and DMD, that focus on identifying dominant spatial and temporal modes in the flow of autonomous systems. Next, we will introduce our extension to DMD that is capable of producing reduced-order models of dynamic systems that include inputs and outputs (Section III). This technique has been applied to a high-fidelity CFD simulation of a wind turbine array described in Section IV. The results of obtaining a model using IODMD are presented in Section V. Finally, conclusions and suggestions for future work are given in Section VI.

II. BACKGROUND: REDUCED-ORDER MODELING

A. Proper Orthogonal Decomposition

POD provides a low-order approximation of a fluid system that is capable of capturing the dominant structures in the flow. Specifically, POD can be used to extract dominant spatial features from both simulation and experimental data that can be used to uncover the structures in the flow field [17], [18]. This can be done by projecting the velocity field that can be used to uncover the structures in the flow field onto a set of orthogonal basis functions.

Consider a system modeled by the following continuous-time nonlinear dynamics:

$$\dot{x}(t) = f(x(t), u(t))$$

(1)

where $x \in \mathbb{R}^n$ is the state vector and $u \in \mathbb{R}^p$ is the input vector. The POD modes of this system can be computed by simulating the system (1) forward in time and collecting snapshots of the nonlinear system, $x(t)$. A data matrix of the snapshots is formed by:

$$X_0 = [x(t_0), x(t_1), ..., x(t_m)]$$

(2)

where $m$ is the number of snapshots. The POD modes are then computed by taking the singular value decomposition of the data matrix:

$$X_0 = U\Sigma V^*$$

(3)

The POD modes are contained in the columns of $U$ and the relative energy of each mode is contained in the singular values in $\Sigma$. These modes provide the spatial component of the flow with the first POD mode being the spatial mode that contains the most energy. Note that a reduced-order model can be constructed using POD modes and the Galerkin projection. See [18] for details on the Galerkin projection. In addition, there have been a few other variations of POD that have been developed in literature to directly handle inputs, e.g. [12], [13]. POD modes are good at representing specific datasets. However, POD modes do not necessarily provide a good description of a dynamically evolving flow driven by a forcing input.

B. Dynamic Mode Decomposition

DMD extracts the dominant spatial and temporal information about the flow [19]–[22]. This method attempts to fit a discrete-time linear system to a set of snapshots from simulation or experiments. Consider a system modeled by the following discrete-time nonlinear dynamics:

$$x_{k+1} = f(x_k)$$

(4)

where $x \in \mathbb{R}^n$ is the state vector. A collection of snapshot measurements $\{x_k\}_{k=0}^m \in \mathbb{R}^n$ is obtained for the system either via simulation or experiments.

The objective of DMD is to approximate the system on a low dimensional subspace. Assume there is a matrix $A$ that relates the snapshots in time by:

$$x_{k+1} = Ax_k$$

(5)

The snapshots of the system are defined as:

$$X_0 = [x_0, x_1, ..., x_{m-1}]$$

(6)

$$X_1 = [x_1, x_2, ..., x_m]$$

(7)

where $x_k$ are the snapshots and $m$ is the number of snapshots. The full-order $A$ matrix can be computed such that:

$$A = X_1 X_0^T$$

(8)

where $X_0$ indicates the pseudoinverse of $X_0$. The DMD method attempts to fit the snapshots in time using a low rank matrix that captures the dynamics of the dataset. This matrix can be used to construct the DMD modes that correspond to specific temporal frequencies. A low-order representation of $x_k$ can be written as $z_k = Q^* x_k$, where $Q$ is the projection subspace. The truncated, reduced-order model takes the form:

$$z_{k+1} = (Q^* A Q) z_k := F z_k$$

(9)

The state matrix $F := Q^* A Q \in \mathbb{R}^{r \times r}$ describes the dynamics of the reduced-order subspace. Solutions to this reduced-order model can be used to construct the approximate solutions to the full-order model.

The typical choice for the projection subspace, $Q$, is the POD modes of $X_0$, i.e. $Q = U_r$ where $r$ is the order of the reduced-order model. The optimal reduced-order state matrix, $F$, for this choice is:

$$F := U_r^* A U_r = U_r^* X_1 (U_r^* X_0)^\dagger$$

(10)

where the corresponding low rank approximation for the full-order state matrix is:

$$A \approx U_r F U_r^* = U_r U_r^* X_1 X_0^\dagger$$

(11)
If an eigenvalue decomposition is done on $F$, then $A$ is now:

$$A \approx U_r T A T^{-1} U^*_r$$

where $U_r, T$ are defined as the DMD modes and the corresponding values of $A$ provide the specific temporal frequency for each DMD mode.

One limitation of this approach is that DMD cannot produce input-output models. Specifically, the dynamics and the modes will be disrupted by external forcing, i.e. DMD is not robust to perturbations in the system. In addition, this technique is not robust to noise making it difficult to use for predictive modeling or control. A few studies have been conducted to address this issue [23, 24]. In these studies, different approaches have been taken to account for noise. In [23], a Kalman filter is used in addition to the reduced-order model obtained using DMD, which helps account for process noise as well as measurement noise. The approach taken in the next section uses external forcing to assist in identifying a reduced-order linear model that is more robust to noise. By introducing an external input, there is a stronger signal-to-noise ratio that is capable of better identifying a reduced-order model.

### III. Input-Output Dynamic Mode Decomposition

In this section, we introduce an extension of the DMD approach to include inputs and outputs. The approach will be summarized in this section for time-invariant system, but it can be extended to parameter-varying systems [25]. This paper addresses the use of this technique to develop an input-output model that does not require adjoints and is robust to noise. DMD has previously been studied in the context of control [15]. This approach projects the full-order model onto the output subspace. By using the output subspace, the inputs are accounted for when fitting the data to a linear system. The approach specified in this paper combines DMD with standard subspace identification [16], often used in the controls literature, to obtain an input-output characterization of the system. This input-output DMD implementation is referred to as IODMD for the remainder of the paper.

Now, consider a discrete-time nonlinear system with inputs:

$$x_{k+1} = f(x_k, u_k)$$

$$y_k = h(x_k, u_k)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, and $y \in \mathbb{R}^q$ are the state, input, and output vectors.

A collection of snapshot measurements are obtained via simulation or experiments by exciting the system. Snapshots are taken from the nonlinear system and the states, inputs, and outputs are recorded as:

$$X_0 = [x_1, x_2, ..., x_{m-1}]$$

$$X_1 = [x_2, x_3, ..., x_m]$$

$$U_0 = [u_1, u_2, ..., u_{m-1}]$$

$$Y_0 = [y_1, y_2, ..., y_{m-1}]$$

IODMD attempts to fit the snapshot measurements in time by:

$$x_{k+1} = A x_k + B u_k$$

$$y_k = C x_k + D u_k$$

The state matrices $(A, B, C, D)$ have the dimensions compatible to those of $(x, u, y)$. The state is projected onto a low dimensional subspace in order to make the computations tractable.

A truncated model can be expressed in terms of this reduced-order state, i.e. $z = Q^* x \in \mathbb{R}^r$ where $Q \in \mathbb{R}^{n \times r}$ is a generic orthonormal basis that forms the projection subspace.

$$z_{k+1} = (Q^* A Q) z_k + (Q^* B) u_k := F z_k + G u_k$$

$$y_k = (C Q) z_k + D u_k := H z_k + D u_k$$

The reduced-order state matrices $(F, G, H, D)$ are obtained by minimizing the error of the Frobenius norm:

$$\min_{F, G, H, D} \left\| \begin{bmatrix} X_1 - [Q \ 0] \ 0 \ I \end{bmatrix} F \ G \ H \ D \ [Q^* \ 0] \ 0 \ I \ [X_0] \ [U_0] \right\|_F^2$$

This is the direct N4SID subspace method for estimating state matrices given measurements of the (reduced-order) state, input, and output. A useful choice for the projection space is given by the POD modes of $X_0$, see (3). The state of the linear system can be approximated on a subspace defined by the first $r$ POD modes of $X_0$. The optimal reduced-order state matrices for this choice is:

$$\begin{bmatrix} F & G \\ H & D \end{bmatrix}_{opt} = \begin{bmatrix} U^*_r X_1 \\ Y_0 \end{bmatrix} \Sigma_r V^*_r \Sigma_r$$

As with standard DMD, an eigenvalue decomposition of $F_{opt}$ can be used to construct DMD modes that provide spatial modes associated with a specific temporal frequency for the system. This new methodology also yields input-output information for the model. This proposed method is a tractable implementation of the existing direct N4SID (subspace) method [16] that can be applied for very large systems. This is not simply a black-box (input-output) approach because the state of the reduced-order system $z_k$ can be used to approximately reconstruct the full-order state by:

$$x_k \approx U_r z_k$$

Moreover, the approach only requires input/output/state data from the model. Construction and simulation of an adjoint system is not required.
IV. SIMULATION SETUP

IODMD, as described in Section III, was used to obtain a reduced-order model from SOWFA simulations. The goal of this study is to construct a reduced-order model that can be used to reconstruct the dominant characteristics of the flow field seen in high-fidelity models.

A. Simulator of On and Offshore Wind Farm Applications

SOWFA is a high-fidelity large eddy simulation tool that was developed at NREL for wind farm studies. SOWFA is a CFD solver based on OpenFOAM coupled with NREL’s FAST wind turbine simulator. SOWFA has been used in previous wind farm control studies [5], [26], [27].

SOWFA uses an actuator line model coupled with FAST to study turbines in the atmospheric boundary layer. Specifically, SOWFA solves the three-dimensional incompressible Navier-Stokes equations and transport of potential temperature equations, which take into account the thermal buoyancy and Earth rotation (Coriolis) effects in the atmosphere.

SOWFA calculates the unsteady flow field to compute the time-varying power, velocity deficits, and loads at each turbine in a wind farm. This level of computation, with high-fidelity accuracy, takes on the order of days to run on a supercomputer using a few hundred to a few thousand processors, depending on the size of the wind farm. The simulations run for this study were performed on NREL’s high-performance computer Peregrine.

B. Two-Turbine Setup

A high-fidelity simulation of a two-turbine scenario is performed with SOWFA to provide the data for IODMD reduced-order modeling. The turbines are aligned with a spacing of 5\(D\). The simulated turbines are NREL 5-MW baseline turbines [28], which have a rotor diameter of \(D = 126m\). Details about the positioning of the turbines in the domain are given in Figure 1. The spatial discretization for CFD is refined in two steps in a rectangular region, with the smallest cells containing the turbine rotors, the axial-induction zones of the rotor, and a large part of the wake. Farther from the turbines, the mesh is coarser to reduce computation time.

The conditions simulated in SOWFA are based on the study reported in [26], [27]. They consist of a neutral atmospheric boundary layer with a low aerodynamic surface roughness value of 0.001 m, typical for offshore conditions. The generated inflow, coming from the southwest (300°), has a horizontally-averaged wind speed of 8 m/s and a turbulence intensity of 6% at the turbine hub height. We used a simulation time of 1800 s to let the wakes develop through the domain.

C. IODMD for SOWFA

A forced input was applied to the upstream turbine by changing the collective blade pitch angle from 0° to 4° using a pseudorandom binary sequence (PRBS), see Figure 2. By changing the blade pitch from 0° to 4° at varying frequencies, various dynamics of the system are excited. This aids in accurately identifying a reduced-order model of this system.

The data from the simulations is sampled at 1s intervals, i.e. snapshots of the flow are recorded every 1s. This was determined by doing a frequency analysis of the flow in a two-turbine array. In particular, Figure 3 shows the frequency content within a wake 4 rotor diameters downstream of the upstream turbine. In addition, Figure 3 shows that a majority of the frequency content in the rotor-swept area (63 m around the origin) of the wake does not exceed 0.5 Hz. As a result, we sample the flow, i.e. the states, the inputs, and the outputs every 1s. The inputs of interest for this problem are the blade pitch angle of the upstream turbine, the generator torque of the upstream turbine, and the generator torque of the downstream turbine. The outputs of interest include the power from the upstream turbine and the power from the downstream turbine. The purpose of this model is to be able to approximately reconstruct the flow corresponding to these
inputs and outputs.

Fig. 3. Frequency content of the flow velocity field at 4D downstream of the upstream turbine. Specifically, this shows the -20 dB bandwidth of the Fast Fourier Transform of the velocity signal for sample points at 4D downstream. Note that the velocity signal was sampled at 1 Hz. The pure yellow may indicate that the frequency at these points exceeds 0.5 Hz.

V. RESULTS

The results shown in this section were obtained using IODMD on the simulation scenarios described in Section IV-B. Note that the resulting flow fields have been rotated in this section so that the flow is shown moving from left to right. In these rotated coordinates, the upstream turbine is located at 13.75D in the x-direction and 4.35D in the y-direction. The downstream turbine is located at 18.75D in the x-direction and 4.35D in the y-direction. The upstream and downstream turbines are indicated by black lines in the figures in this section. In particular, we will focus on reconstructing the flow, i.e. the state evolution equation

\[ x_{k+1} = Ax_k + Bu_k \]

where \( x \) is the flow velocity at the sampled grid points, \( u := [\beta_1, \tau_{g1}, \tau_{g2}] \) where \( \beta_1 \) is the blade pitch angle at the upstream turbine, \( \tau_{g1} \) and \( \tau_{g2} \) are the generator torques at the upstream and downstream turbines, respectively. The blade pitch angle at the downstream turbine is 0 in this example and is excluded from the inputs. These inputs are associated with the force that the turbines are applying to the flow and are essential to include in the reduced-order model.

For this particular example, there are approximately 1.2 million sampled grid points and the 3 velocity components are recorded at each grid point. Hence the dimension of \( x \) is approximately 3.6 million.

The POD modes of the simulation were computed using the MapReduce approach [29]. POD modes 1, 2, 10, 20, 50, and 100 of the streamwise velocity component at hub height (90 m) can be seen in Figure 4. It should be noted that these modes were computed after subtracting out the baseflow, i.e. the mean flow. The first mode contains low frequency spatial information and is the most energetic mode of the system. 100 has high frequency spatial information and represents a small amount of energy of the system.

The full flow field is computed from the results of the reduced-order model using (25) and is shown in Figure 5. For this reconstruction, we selected 20 modes and we specifically looked at the streamwise velocity component at hub height. By only selecting 20 modes, this reduced-order model will not be able to capture the high-frequency turbulence. The order of this model was selected by evaluating the mean error accumulated when comparing the SOWFA flow field with the reduced-order flow field.

Lastly, this model was applied to a set of validation data to verify that this model would work under similar wind conditions but with a different forced input. The same model that was used to construct Figure 5 is used to reconstruct the flow for the validation data. Figure 6 shows that this model is able to similarly reconstruct the dominant characteristics of the flow provided in the validation case. This indicates that a reduced-order model can be obtained using IODMD and can be used as a predictive model for another similar dataset. The time-averaged error between SOWFA and the reduced-order model in the validation case is shown in Figure 7. The error is determined by subtracting streamwise velocity obtained from the reduced-order model from the streamwise velocity computed in SOWFA. The maximum error occurs on the edges of the wakes and may be a result of excess mixing far downstream (4D) of the upstream turbine. However, this error is less than 10%. This figure further demonstrates that the reduced-order model is able to capture a majority of the flow characteristics by comparing the streamwise velocity computed in SOWFA and in the reduced-order model obtained from IODMD.

VI. CONCLUSIONS AND FUTURE WORK

This paper introduces an extension to DMD, termed IODMD, and its application to a high-fidelity wake model, SOWFA. We were able to identify a 20-state model that can reconstruct the dominant characteristics of the flow. Future work will include constructing a reduced-order model from SOWFA using IODMD to design and analyze a closed-loop
controller that can be used for wind farm control in high-fidelity simulations.

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