A LINGUISTIC-VALUED WEIGHTED AGGREGATION OPERATOR TO MULTIPLE ATTRIBUTE GROUP DECISION MAKING WITH QUANTITATIVE AND QUALITATIVE INFORMATION

XIAOBING LI  
Intelligent Control Development Center, Southwest Jiaotong University,  
Chengdu 610031, P.R. China

DA RUAN  
Belgian Nuclear Research Centre (SCK-CEN),  
2400 Mol & Ghent University, 9000 Gent, Belgium

JUN LIU  
School of Computing and Mathematics, Faculty of Engineering, University of Ulster, Jordanstown, Newtownabbey BT37 0QB, Northern Ireland, UK

YANG XU  
Intelligent Control Development Center, Southwest Jiaotong University, Chengdu 610031, P.R. China

Received: 14-04-2008  
Revised: 30-06-2008

In selecting an optional alternative in an environment of multiple attribute group decision making, different attributes of the alternative are often considered as with quantitative and qualitative information. Consequently, decision making problems may include preference information in different formats. In this paper, a lattice-based linguistic-valued weighted aggregation (LVWA) operator is proposed for multiple attribute group decision making with non-totally ordered linguistic-valued information. Then some transformation functions for unifying different formats of preference information are reviewed and summarized. Finally, an example is illustrated how to use the LVWA operator and transformation functions for multiple attribute group decision making.

Keywords: Transformation functions, multiple attribute group decision making, linguistic-valued weighted aggregation (LVWA) operator, linguistic-valued lattice implication algebra, lattice theory

1. Introduction

Multiple attribute group decision making (MAGDM) addresses the problems of choosing an optimal choice that has the highest degree of satisfaction by multiple experts’ assessments from a set of alternatives that are characterized in terms of their attributes. Generally, multiple attribute group decision making problems follow a common scheme composed by the three phases:

1. Evaluation phase: Experts are asked to give preference values to each attribute of each alternative.
2. Aggregation phase: It combines individual preference values to obtain a collective preference value for each alternative.
3. Exploitation phase: It orders the collective preference values to obtain the best alternatives.

In the first phase, experts are asked to provide their preferences on each attribute of each alternative. Usually, the information is expressed by means of numerical values such as exact values, interval values, fuzzy numbers, etc. However, in real world, human beings are constantly making decisions under a linguistic environment. For example, when evaluating the “comfort” or “design” of a car, linguistic labels like “good”, “fair”, “poor” are usually used; evaluating a
the speed of a car, linguistic labels like “very fast”, “fast”, “slow” can be used, and evaluating students’ performances in their courses, linguistic labels like “bad”, “medium”, “good” can be used. As a result, it is necessary to consider aggregations of linguistic information.

To date, several methods have been proposed for dealing with linguistic information, for instance:

(1) The extension principle based method for operations on fuzzy numbers that support the semantics of the linguistic labels\(^\text{14, 15}\).

(2) The symbolic method for computations on the indexes of the linguistic terms\(^\text{16}\); both the methods (1) and (2) process the results in the initial expression domains, which produce the consequent loss of information and hence the lack of precision\(^\text{17}\).

(3) A fuzzy linguistic model based method for the linguistic information with a pair of values called 2-tuple, composed by a linguistic term and a number\(^\text{17-21}\). Along with the model, this method deals with the 2-tuple without loss of information.

(4) The direct computing with words method\(^\text{1-3}\).

In this paper, we follow the 4th method to aggregate linguistic-valued information for group decision making. At present, a number of researches have focused on group decision making with linguistic preference. Herrera et al. developed a consensus model for group decision making under linguistic assessments\(^\text{7}\) and combined the linguistic ordered weighted averaging (LOWA) operator with linguistic preference relations and the concept of dominance and non-dominance to show its use in the field of group decision making based on the LOWA operator\(^\text{8}\). Later, Herrera et al. presented a consensus model in complete linguistic framework for group decision making guided by consistency and consensus measures\(^\text{9}\). Z.S. Xu proposed an uncertain linguistic ordered weighted averaging (ULOWA) aggregation operator and uncertain linguistic hybrid aggregation (ULHA) operator, and developed an approach to multiple attribute group decision making with uncertain linguistic information based on the ULOWA and ULHA operators\(^\text{10}\).

Although there are many aggregation operators to aggregate linguistic information, they can only aggregate linearly ordered linguistic information. Note that there exist incomparable linguistic terms, such as slightly false and very true. So it is necessary to find an algebra for modeling the ordering relation of the natural language terms.

Lattice theory is a well-developed branch of an abstract algebra for modeling the ordering relation in the real world. Lattice-valued algebra for modeling linguistic values would be a possible choice. To establish theories and methods to simultaneously deal with fuzziness and incomparability of processed object itself and uncertainty in the course of information processing, Xu combined a lattice with implication algebra and established the lattice implication algebra\(^\text{24}\), which provides a necessary foundation for the processing of incomparable information. In addition, there are some research works on incomparable information processing. An evaluation method with incomparable information is presented in Ref. 13. Lattice-valued linguistic-based decision making method is discussed in Ref. 22. A model for handling linguistic terms in the framework of lattice-valued logic is presented in Ref. 4. In Ref. 30, the LVWA operator based on linguistic-valued lattice implication algebra is presented. In this paper, based on the LVWA operator, an approach to solve multiple attribute group decision making with incomparable linguistic-valued information is established. In Ref. 11, a new method for sensory evaluation of industrial products with uncertain information is presented. In this approach, sensory data provided by different evaluators are transformed into measures of consistency on fuzzy satisfaction degrees. Based on these measures of consistency, the aggregated information for all evaluators and all attributes and measure the dissimilarity between evaluators and between used evaluations attributes is obtained. The effectiveness of this method has been validated in the fabric hand evaluation for a number of samples of knitted cotton.

On the other hand, in multiple attribute group decision making, different types of attributes, either quantitative or qualitative, need to be considered. Therefore, the decision making problems may include many different types of preference information such as number, interval and linguistic values. In order to deal with these preference information in different formats, some researches have been done\(^\text{26-29}\). This paper also aims at developing a new method for unifying
A linguistic-valued weighted aggregation operator

Then \((L, \lor, \land', \rightarrow)\) is a lattice implication algebra.

**Example 2.2 (Łukasiewicz implication algebra on finite chains)** Consider a set \(L = \{a_i \mid i = 1, 2, \ldots, n\}\). For any \(1 \leq j, k \leq n\), define

\[
\begin{align*}
  a_j \lor a_k &= a_{\max\{j,k\}}, \\
  a_j \land a_k &= a_{\min\{j,k\}}, \\
  (a_j)' &= a_{n-j+1}, \\
  a_j \rightarrow a_k &= a_{\min\{n-j+k,n\}}.
\end{align*}
\]

Then \((L, \lor, \land', \rightarrow)\) is a lattice implication algebra.

In the following sections, the lattice implication algebra \((L, \lor, \land', \rightarrow)\) is denoted by \(L\) simply unless emphasized.

2.2. **Linguistic-valued lattice implication algebra**

**Definition 2.2** Let \((L, \lor, \land, \rightarrow)\) \((i = 1, \ldots, n)\) be a family of lattice implication algebras. Then

\[
L = \prod_{i=1}^{n} L_i = \{(a_1, a_2, \ldots, a_n) \mid a_i \in L_i\}
\]

is called a direct product of \(n\) lattice implication algebras.

**Theorem 2.2** Let \(L_i\) \((i = 1,\ldots, n)\) be a lattice implication algebra. If the operators \(\lor, \land, \rightarrow\) on \(L = \prod_{i=1}^{n} L_i\) are defined as follows respectively: for any

\[
(a_1, a_2, \ldots, a_n), (b_1, b_2, \ldots, b_n) \in \prod_{i=1}^{n} L_i,
\]

\[
\begin{align*}
(a_1, a_2, \ldots, a_n) \lor (b_1, b_2, \ldots, b_n) &= (a_1 \lor b_1, a_2 \lor b_2, \ldots, a_n \lor b_n), \\
(a_1, a_2, \ldots, a_n) \land (b_1, b_2, \ldots, b_n) &= (a_1 \land b_1, a_2 \land b_2, \ldots, a_n \land b_n), \\
(a_1, a_2, \ldots, a_n) \rightarrow (b_1, b_2, \ldots, b_n) &= (a_1 \rightarrow b_1, a_2 \rightarrow b_2, \ldots, a_n \rightarrow b_n),
\end{align*}
\]

then \((L, \lor, \land, \rightarrow)\) is also a lattice implication algebra.

**Remark 2.1** Let \(L = L_1 \times L_2\) where \(L_j\) \((j = 1, 2)\) be a finite-chain-type lattice implication algebra. Then \(L\) is a lattice implication algebra.

**Definition 2.3** Let \(ML = \{b_1, b_2\}\) be a linguistic term set, where \(b_1\) be an antonym of \(b_2\), and \(b_1 \leq b_2\) in term of their meanings in natural language, such as “poor” and “good”, “false” and “true” etc. Define the same
operators on ML as the ones in Example 2.1. Then ML is a lattice implication algebra, called a meta-linguistic lattice implication algebra.

**Example 2.3** Let $ML = \{\text{good, poor}\}$. The operators on ML are defined as the same in Example 2.1. Then ML is a meta-linguistic lattice implication algebra.

**Definition 2.4** Let $MW = \{a_1, a_2, \ldots, a_m\}$ and $a_i$ $(i = 1, 2, \ldots, n)$ be linguistic modifiers, used to modify the meta-linguistic terms. The set MW is ordered in the sense that $a_i \leq a_j$ if and only if $i \leq j$. The operators $\vee$, $\wedge$, $\rightarrow$, $\prime$ on MW are defined as the same in Example 2.2, then $(MW, \vee, \wedge, \rightarrow, \prime, 1)$ is a lattice implication algebra, called a lattice implication algebra with modifiers.

**Example 2.4** Let $MW = \{\text{absolutely (Abbr. to Ab), highly (Abbr. to Hi), very (Abbr. to Ve), quite (Abbr. to Qu), exactly (Abbr. to Ex), almost (Abbr. to Al), rather (Abbr. to Ra), somewhat (Abbr. to So), slightly (Abbr. to Sl)}\}$ be a set of linguistic modifiers. Then the chain $\text{Ab} \geq \text{Hi} \geq \text{Ve} \geq \text{Qu} \geq \text{Ex} \geq \text{Al} \geq \text{Ra} \geq \text{So} \geq \text{Sl}$ is a linguistic-modifier lattice implication algebra with operations defined as in Example 2.2.

**Definition 2.5** Let $MW = \{a_1, a_2, \ldots, a_m\}$ be a lattice implication algebra with modifiers, $ML = \{b_1, b_2\}$ be a meta lattice implication algebra, denote $L_{MW	imes ML} = \{(a_i, b_1)_L, (a_i, b_2)_L, (a_1, b_1)_L, (a_1, b_2)_L\}$, which Hasse diagram is shown as Fig.1.

The operations on $L_{MW	imes ML}$ is defined as follows:

$(a_i, b_j) = (a_i, b_j)$ ,

$(a_i, b_j) \rightarrow (a_k, b_j) = (a_i \rightarrow a_k, b_j \rightarrow b_j)$.

Define a mapping $f$:

$f : MW \times ML \rightarrow L_{MW	imes ML}$

$f((a_i, b_j)) = \begin{cases} (a_i, b_j), & b_j = b_1 \\ (a_i, b_j), & b_j = b_2 \end{cases}$

Then $f$ is a isomorphic mapping.

**Example 2.5** Let $MW = \{\text{absolutely, highly, very, quite, exactly, almost, rather, somewhat, slightly}\}$ be a set of linguistic modifiers, and $ML = \{\text{good, poor}\}$. So $MW \times ML = \{\text{absolutely good, highly good, very good, quite good, exactly good, almost good, rather good, somewhat good, slightly good, absolutely poor, highly poor, very poor, quite poor, exactly poor, almost poor, rather poor, somewhat poor, slightly poor}\}$. Then
I is a weight vector and \( n (i.e., \text{absolutely poor}) \) is a linguistic-valued lattice implication algebra. In the following section, we will use this linguistic-valued lattice implication algebra as a linguistic assessment set to represent the preference, or the important weight, denoted shortly as \( S \).

3. A linguistic-valued aggregation operator for multiple attribute group decision making

Yager introduced an ordered weighted averaging (OWA) operator defined as follows.

**Definition 3.1** An OWA operator of dimension \( n \) is a mapping OWA: \( R^n \rightarrow R \) that has associated an \( n \) vector \( w = (w_1, w_2, \ldots, w_n) \) such that \( w_j \in [0,1], j = 1,2, \ldots, n, \) and \( \sum_{j=1}^{n} w_j = 1. \) Furthermore,

\[
\text{OWA}_w(\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n) = \sum_{j=1}^{n} w_j b_j ,
\]

where \( b_j \) is the \( j \)th largest of the \( a_j \).

However, the OWA operator can only be used in the situations where the input arguments are the exact numerical values. In the real world, human beings are constantly making decisions under a linguistic environment. Hence, it is necessary to investigate linguistic-valued information aggregation.

**Remark 3.1:** There have been some existing works on linguistic-valued information aggregation including Yager’s work, such as Refs. 1, 2, 4, 5, but they are all based on the totally ordered linguistic term set.

In the following, we shall investigate a linguistic-valued weighted aggregation operator, which can be used in situations where the aggregated arguments are given in the form of linguistic values which may be incomparable.

**Definition 3.2** A mapping LVWA: \( S^n \rightarrow S \) is called a linguistic-valued weighted aggregation (LVWA) operator, if

\[
\text{LVWA}_w(\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n) = \text{ow}_w(\mathbf{a}_j),
\]

where \( S \) is an evaluation set which is a linguistic-valued lattice implication algebra and includes both comparable and incomparable linguistic terms commonly used in natural language, where \( w = (w_1, w_2, \ldots, w_n) \) is a weight vector and \( w_j \) is the weight of linguistic-valued \( a_j \) with \( w_j \in S \), \( a_j \in S \) and \( j = 1,2, \ldots, n \).

**Remark 3.2:** Yager’s aggregation method in Ref. 23 is a special case of the proposed method which is limited to the totally ordered linguistic term set.

The LVWA operator has the following properties:

**Theorem 3.1** (Monotonicity) Let \( A = (a_1, a_2, L, a_n) \) and \( C = (c_1, c_2, L, c_n) \) be argument vectors. If for each \( j \) \( (j = 1,2, \ldots, n) \), \( a_j \geq c_j \), then \( \text{LVWA}_w(A) \geq \text{LVWA}_w(C) \).

**Proof.** Since

\[
\text{LVWA}_w(A) = \text{ow}_w(\mathbf{a}_j), \quad \text{and}
\]

\[
\text{LVWA}_w(C) = \text{ow}_w(\mathbf{c}_j),
\]

then the result follows directly from the property \( a_j \geq c_j \).

**Theorem 3.2** (Commutativity) Let \( A = (a_1, a_2, L, a_n) \) be an ordered argument vector, \( \tilde{A} = (\tilde{a}_1, \tilde{a}_2, L, \tilde{a}_n) \) is any permutation of the elements in \( A \), then

\[
\text{LVWA}_w(A) = \text{LVWA}_w(\tilde{A}) .
\]

**Proof.** Suppose that \( w = (w_1, w_2, L, w_n) \) is the weighting vector of linguistic-valued \( a_j \). Then

\[
\text{LVWA}_w(\mathbf{a}_j) = \text{ow}_w(\mathbf{a}_j) \quad \text{and}
\]

\[
\text{LVWA}_w(\mathbf{c}_j) = \text{ow}_w(\mathbf{c}_j) .
\]

Hence,

\[
\text{LVWA}_w(A) = \text{LVWA}_w(\tilde{A}) .
\]

**Theorem 3.3** (Idempotence) If \( \forall j \in S \) \( w_j = I \), and \( a_j = a \) \( (j = 1,2, L, n) \), then

\[
\text{LVWA}_w(\mathbf{a}_j) = a .
\]

**Proof.** Since \( a_j = a \), it follows that

\[
\text{LVWA}_w(\mathbf{a}_j) = a .
\]
Theorem 3.4 Let \( w = (I, I, L, I) \). Then
\[
L V W A_w (a_1, a_2, L, a_n) = \inf [a_j].
\]

4. Transformation schemes for unifying different formats of preference information

To obtain evaluation results of all the alternatives in multiple attribute group decision making, different formats of preference information need to be unified into a common format. The linguistic-valued preference information set \( S \) is chosen as the common format. The unifying steps are given as follows:

**Step 1**: normalization of quantitative preference information

**Step 2**: normalization of qualitative preference information

**Step 3**: transformation for numerical preference information to linguistic-valued preference information.

The concrete transformation methods will be given in the following subsections.

4.1. Normalization of quantitative preference information

Generally, there exist six kinds of attributes: profit, cost, fixation, interval, deviation, and deviating interval. In Refs. 27-29, the methods of normalizing the above six kinds of attributes are given and expressed as follows:

- **Profit attribute**
  \[
  r_y = \frac{a_y}{\max_{i} [a_y]}
  \]

- **Cost attribute**
  \[
  r_y = \frac{a_y}{\min_{i} [a_y]}
  \]

- **Fixation attribute**
  \[
  r_y = 1 - \frac{a_y - \alpha_j}{\max_{i} [a_y - \alpha_j]}
  \]

- **Interval attribute**
  \[
  r_y = \frac{|a_y - \beta_j|}{\max_{i} [a_y - \beta_j] - \min_{i} [a_y - \beta_j]}
  \]

- **Deviation attribute**
  \[
  r_y = \begin{cases} 
  \frac{\max(q'_j - a_y, a_y - q''_j)}{\max_{i} [q'_i - \min_{i} (a_y), \max_{i} (a_y) - q''_i]}, & a_y \notin [q'_j, q''_j] \\
  1, & a_y \in [q'_j, q''_j]
  \end{cases}
  \]

where \( a_y \) denotes the original value of attribute \( A_j \) for alternative \( X_i \), \( r_y \) represents the normalized value of \( a_y \), \( \alpha_j \) and \( \beta_j \) are fixed values, \( [q'_j, q''_j] \) is an interval. The larger the profit attribute value, the better the attribute, while the larger the cost attribute value, the worse the attribute. The fixation attribute means that the closer to a fixed value \( \alpha_j \) attribute value, the better the attribute. Further, we can know that the closer to or included in an interval \( [q'_j, q''_j] \) values, the better the attribute. The larger of the distance of deviation attribute values to a fixed value are, the better of the attribute is. Deviation interval attribute means that the larger the distance of deviation attribute values to a fixed interval, the better the attribute.

**Remark 4.1** According to the above formula, the original values can be normalized within the interval \([0, 1]\).

4.2. Normalization of qualitative preference information

The linguistic values are designed to express preference information of qualitative attributes by decision makers. In this paper, all linguistic values are selected from linguistic-valued lattice implication algebra \( S \) defined in Example 2.5. Two kinds of attributes, profit and cost, are considered. The methods of qualitative attributes are given as follows:

- **Profit attribute**: in this case, as the preference information is expressed by linguistic values, we keep the original values as the normalized values.

- **Cost attribute**:
  \[
  r_y = a_y' \]

where \( a_y \) denotes the original value of attribute \( A_j \) for alternative \( X_i \), “’” is a negation operator, \( r_y \) represents the normalized value of \( a_y \), \( a_y \) and \( r_y \) are all linguistic values in \( S \).

4.3. Transformation function for numerical attribute values to linguistic-valued attribute values

After the transformation of the original attribute values, normalized attribute values are expressed by number in
interval \([0, 1]\) or linguistic values in \(S\). We need to unify these two kinds of preference information. Since the attribute values belonging to the interval \([0, 1]\) are comparable, we select the subset \(S' = \{s_0, s_1, L, s_n\}\) of \(S\) such that \(S'\) only contains comparable linguistic values and these linguistic values satisfy the following conditions:

- A negation operator "\(\neg\)" such that \(j = n - i (n + 1 \text{ is the cardinality})\)
- A min and a max operator in the linguistic term set: \(s_i \leq s_j \iff i \leq j\)

To aggregate the preference information, a transformation function for numerical attribute values to linguistic-valued attribute values is given as follows:

\[\tau : [0,1] \rightarrow S^*, \tau(a) = s_{[n]}\]

where \([\cdot]\) is the rounding operation, \(n + 1\) is the cardinality of \(S^*\). Utilizing the transformation function, the numerical attribute values can be transferred into linguistic-valued attribute values.

5. An approach based on the LVWA operator to multiple attribute decision making with linguistic-valued information

Consider a multiple attribute group decision making problem with different formats of preference information. Assume that \(S\) is an evaluation set that is a linguistic-valued lattice implication algebra and includes both comparable and incomparable natural linguistic terms used to indicate preference information. Let \(X = \{x_1, x_2, L, x_n\}\) be a discrete set of alternatives, and \(U = \{u_1, u_2, L, u_m\}\) be a set of attributes. Let \(D = \{d_1, d_2, L, d_j\}\) be a set of decision makers, and \(w = (w_1, w_2, L, w_n)\) be the weight vector of decision makers, where \(w_j \in S, k = 1,2, L, I\).

Suppose that \(\mathcal{D}^0 = (a^{ij})_{mn}\) is the decision matrix, where \(a^{ij}_{kj}\) is a preference value, which takes the forms of number, interval, or linguistic value, given by the decision maker \(d_k \in D\), for alternative \(x_j \in X\) with respect to attributes \(u_i \in U\). Group decision making problems are composed by the following four phases:

1. Evaluation phase: The experts are asked to give the preference values to each attribute of each alternative.
2. Transformation phase: All the preference values are expressed in a unique linguistic-valued domain.
3. Aggregation phase: It combines the individual preferences to obtain a collective preference value for each alternative.
4. Exploitation phase: It orders the collective preference values to obtain the best alternatives.

In the following an approach to multiple attribute group decision making with linguistic-valued information is given based on the LVWA operator.

**Step 1:** Experts give preference information \(a^{i,j}_{k} \), \(i = 1, 2, L, m, j = 1, 2, L, n, k = 1, 2, L, I\).

**Step 2:** Utilize the decision information given in matrix \(\mathcal{D}^0\) and the methods of transformation in Section 4 to derive all the normalized linguistic values.

**Step 3:** Utilize the LVWA operator:

\[\tilde{a}^{i,j} = \text{LVWA}_{\tau}(a^{i,j}_{1}, a^{i,j}_{2}, L, a^{i,j}_{n}), \]

\(k = 1, 2, L, I, j = 1, 2, L, n\) to derive the individual overall preference value \(\tilde{a}^{i,j}\) of alternative \(x_j\), where \(w = (w_1, w_2, L, w_n)\) is a weight vector and \(w_j\) is the weight of linguistic-valued \(a_j\) with \(w_j \in S, j = 1, 2, L, I\).

**Step 4:** Utilize the LVWA operator:

\[\tilde{a}^{i,j} = \text{LVWA}_{\tau}(a^{i,j}_{1}, a^{i,j}_{2}, L, a^{i,j}_{n}), j = 1, 2, L, n\]

to derive the collective overall preference value \(\tilde{b}^{i,j}\) of alternative \(x_j\), where \(w = (w_1, w_2, L, w_n)\) is the weight vector of decision makers, with \(w_j \in S, j = 1, 2, L, n\).

**Step 5:** Rank all the alternatives \(x_j\), and select the optimal one(s) in according to \(\tilde{b}^{ij}\). The optimal alternative is \(x_j \in X\) that \(\tilde{b}^{ij}\) is maximal.

Step 6: End.

6. An illustrative example

To illustrate how the proposed method works, we will give a simple example\(^3\) to evaluate the set of cars \(A = \{x_1 = \text{Chevrolet}, x_2 = \text{Buick}, x_3 = \text{Toyota}\}\). Let \(U = \{u_1, u_2, u_3, u_4\}\) where \(u_1 = \text{comfort}, u_2 = \text{fuel efficiency}, u_3 = \text{cost}, u_4 = \text{maximum speed}\) (whose vector weights be \(w = (w_1, w_2, L, w_n)\)) and values of attributes \(u_1\) and \(u_2\) are linguistic values while values of attributes \(u_3\) and \(u_4\) are numerical values. Three kinds of cars (alternatives) \(x_j\) \((j = 1, 2, 3)\) are to be evaluated using the term set...
Step 1: Unify the attribute values into a linguistic values

Step 1.1: Utilize the transformation functions given in Section 4.1

\[ r_j = \frac{a_j}{\max_j a_j} \quad \text{and} \quad r_j = \frac{\min_j a_j}{a_j} \]

to derive the normalized preference information of attribute \( u_1 \) and \( u_4 \) respectively.

Step 1.2: Utilize the transformation functions given in Section 4.2 to derive the normalized preference information of attribute \( u_2 \) and \( u_3 \) respectively.

### Table 1. Preference information given by decision maker \( d_1 \)

<table>
<thead>
<tr>
<th>( u_1 )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_1 )</td>
<td>((a_{9,1}))</td>
<td>((a_{7,1}))</td>
<td>((a_{6,1}))</td>
</tr>
<tr>
<td>( u_2 )</td>
<td>((a_{4,0}))</td>
<td>((a_{2,0}))</td>
<td>((a_{2,1}))</td>
</tr>
<tr>
<td>( u_3 )</td>
<td>150</td>
<td>120</td>
<td>200</td>
</tr>
<tr>
<td>( u_4 )</td>
<td>180</td>
<td>200</td>
<td>160</td>
</tr>
</tbody>
</table>

### Table 2. Preference information given by decision maker \( d_2 \)

<table>
<thead>
<tr>
<th>( u_1 )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_1 )</td>
<td>((a_{7,1}))</td>
<td>((a_{6,0}))</td>
<td>((a_{6,0}))</td>
</tr>
<tr>
<td>( u_2 )</td>
<td>((a_{4,0}))</td>
<td>((a_{2,1}))</td>
<td>((a_{2,1}))</td>
</tr>
<tr>
<td>( u_3 )</td>
<td>150</td>
<td>120</td>
<td>200</td>
</tr>
<tr>
<td>( u_4 )</td>
<td>180</td>
<td>200</td>
<td>160</td>
</tr>
</tbody>
</table>

### Table 3. Preference information given by decision maker \( d_3 \)

<table>
<thead>
<tr>
<th>( u_1 )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_1 )</td>
<td>((a_{9,1}))</td>
<td>((a_{6,0}))</td>
<td>((a_{6,1}))</td>
</tr>
<tr>
<td>( u_2 )</td>
<td>((a_{4,0}))</td>
<td>((a_{2,0}))</td>
<td>((a_{2,0}))</td>
</tr>
<tr>
<td>( u_3 )</td>
<td>150</td>
<td>120</td>
<td>200</td>
</tr>
<tr>
<td>( u_4 )</td>
<td>180</td>
<td>200</td>
<td>160</td>
</tr>
</tbody>
</table>

### Table 4. Preference information given by decision maker \( d_4 \)

<table>
<thead>
<tr>
<th>( u_1 )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_1 )</td>
<td>((a_{7,1}))</td>
<td>((a_{6,1}))</td>
<td>((a_{6,0}))</td>
</tr>
<tr>
<td>( u_2 )</td>
<td>((a_{2,1}))</td>
<td>((a_{2,0}))</td>
<td>((a_{2,0}))</td>
</tr>
<tr>
<td>( u_3 )</td>
<td>150</td>
<td>120</td>
<td>200</td>
</tr>
<tr>
<td>( u_4 )</td>
<td>180</td>
<td>200</td>
<td>160</td>
</tr>
</tbody>
</table>
Table 5. Normalized reference information of Table 1

<table>
<thead>
<tr>
<th>$u_i$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>$(a_6,1)$</td>
<td>$(a_7,1)$</td>
<td>$(a_8,0)$</td>
</tr>
<tr>
<td>$u_2$</td>
<td>$(a_6,1)$</td>
<td>$(a_7,1)$</td>
<td>$(a_8,0)$</td>
</tr>
<tr>
<td>$u_3$</td>
<td>$(a_6,1)$</td>
<td>$(a_7,1)$</td>
<td>$(a_8,0)$</td>
</tr>
<tr>
<td>$u_4$</td>
<td>$(a_6,1)$</td>
<td>$(a_7,1)$</td>
<td>$(a_8,0)$</td>
</tr>
</tbody>
</table>

Table 6. Normalized reference information of Table 2

<table>
<thead>
<tr>
<th>$u_i$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>$(a_7,1)$</td>
<td>$(a_6,0)$</td>
<td>$(a_9,0)$</td>
</tr>
<tr>
<td>$u_2$</td>
<td>$(a_6,1)$</td>
<td>$(a_9,0)$</td>
<td>$(a_8,0)$</td>
</tr>
<tr>
<td>$u_3$</td>
<td>$(a_7,1)$</td>
<td>$(a_9,1)$</td>
<td>$(a_8,1)$</td>
</tr>
<tr>
<td>$u_4$</td>
<td>$(a_6,1)$</td>
<td>$(a_9,1)$</td>
<td>$(a_8,1)$</td>
</tr>
</tbody>
</table>

Table 7. Normalized reference information of Table 3

<table>
<thead>
<tr>
<th>$u_i$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>$(a_6,1)$</td>
<td>$(a_9,0)$</td>
<td>$(a_6,1)$</td>
</tr>
<tr>
<td>$u_2$</td>
<td>$(a_6,1)$</td>
<td>$(a_9,0)$</td>
<td>$(a_6,1)$</td>
</tr>
<tr>
<td>$u_3$</td>
<td>$(a_6,1)$</td>
<td>$(a_9,0)$</td>
<td>$(a_6,1)$</td>
</tr>
<tr>
<td>$u_4$</td>
<td>$(a_6,1)$</td>
<td>$(a_9,0)$</td>
<td>$(a_6,1)$</td>
</tr>
</tbody>
</table>

Table 8. Normalized reference information of Table 4

<table>
<thead>
<tr>
<th>$u_i$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>$(a_7,1)$</td>
<td>$(a_6,0)$</td>
<td>$(a_9,0)$</td>
</tr>
<tr>
<td>$u_2$</td>
<td>$(a_6,1)$</td>
<td>$(a_9,0)$</td>
<td>$(a_8,0)$</td>
</tr>
<tr>
<td>$u_3$</td>
<td>$(a_7,1)$</td>
<td>$(a_9,1)$</td>
<td>$(a_8,1)$</td>
</tr>
<tr>
<td>$u_4$</td>
<td>$(a_6,1)$</td>
<td>$(a_9,1)$</td>
<td>$(a_8,1)$</td>
</tr>
</tbody>
</table>

\[ \hat{\eta}_2^{(2)} = (a_6,0), \hat{\eta}_3^{(1)} = (a_6,1), \hat{\eta}_3^{(2)} = (a_7,0), \hat{\eta}_3^{(3)} = (a_7,1), \hat{\eta}_3^{(4)} = (a_8,0) \]

Step 3: Utilize the weight vector of decision makers, $\omega = ((a_5,0),(a_7,1),(a_8,0),(a_9,1))$, and the LVWA operator:

\[ \hat{\eta}_3 = \text{LVWA}_{\omega}(\hat{\eta}_1^{(1)}, \hat{\eta}_2^{(2)}, \hat{\eta}_3^{(3)}, \hat{\eta}_4^{(4)}) (j = 1, 2, 3) \]

to aggregate the individual overall preference values $\hat{\eta}_j^{(k)}$ for $k = 1, 2, 3, 4$ and thus get the collective overall preference value $\hat{\eta}_3$ of alternative $x_j$:

\[ \hat{\eta}_3 = \text{LVWA}_{\omega}(\hat{\eta}_1^{(1)}, \hat{\eta}_2^{(2)}, \hat{\eta}_3^{(3)}, \hat{\eta}_4^{(4)}) = ((a_2,0) \rightarrow (a_8,1)) \land ((a_7,1) \rightarrow (a_6,1)) \land ((a_9,0) \rightarrow (a_6,1)) = (a_6,0) \]

Similarly, we have

\[ \hat{\eta}_2 = (a_5,0), \hat{\eta}_4 = (a_6,0). \]

Step 4: Rank all the alternatives $x_j$, and select the optimal one(s) in accordance with $\hat{\eta}_3$. The optimal alternative is $x_j \in X$ that $\hat{\eta}_3$ is maximal. Thus the optimal one is $x_3$, that is Chevrolet.

6. Conclusions

In this paper, a linguistic-valued weighted aggregation operator was proposed, which can be used in the situations where the evaluation value set is a non-totally ordered linguistic term set, based on a linguistic-valued lattice implication algebra. In order to deal with preference information in different formats, transformation methods were summarized. Finally, a method for a multiple attribute group decision making is developed based on the LVWA operator and transformation methods. Advantages of this approach are as follows:

1. It does not require all linguistic terms to have a total order.
2. It permits to compute with preference information in different formats.

Acknowledgements

We would like to express our thanks to the support of the China-Flanders Bilateral Project (Grant No. O11S1105), the National Natural Science Foundation of China (Grant No. 60474022) and the Specialized Research Foundation for the Doctoral Program of Higher Education of China (Grant No. 20060613007).

References

Xiaobing Li, Da Ruan, Jun Liu, Yang Xu


